Corso di aggiornamento professionale:

Progetto di aste e collegamenti in acciaio

secondo le indicazioni del capitolo 4 delle NTC 2008

6 – Flessione composta

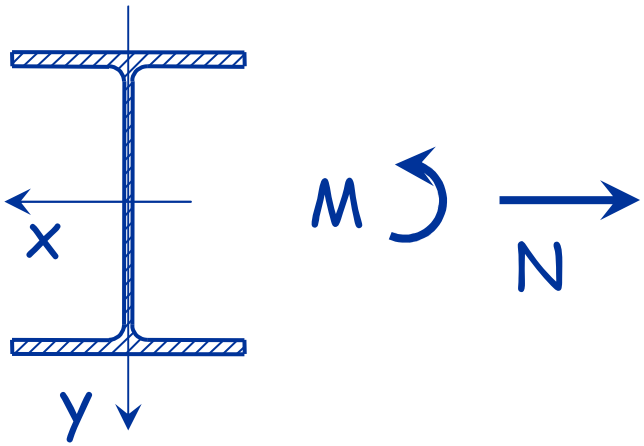
Parma

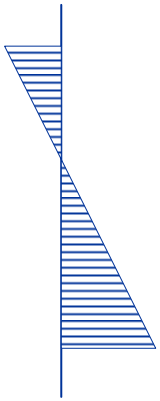
11, 12 e 25 novembre 2011

Edoardo M. Marino

Flessione composta
tensoflessione

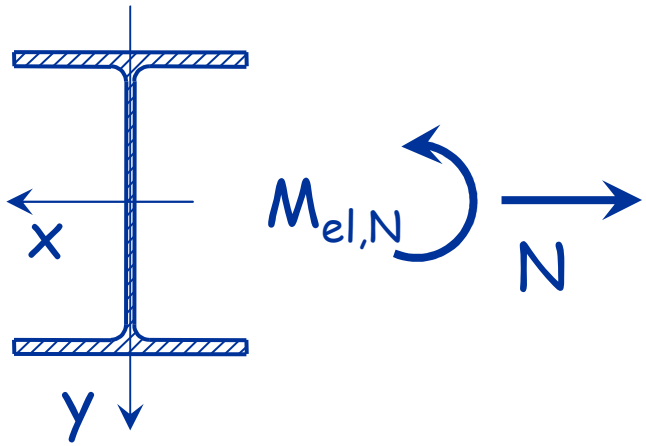
Comportamento ultimo

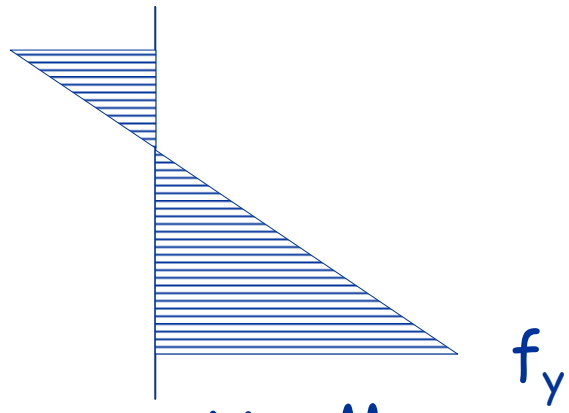




$$\sigma = \frac{N}{A} + \frac{M}{I} y$$

Comportamento ultimo



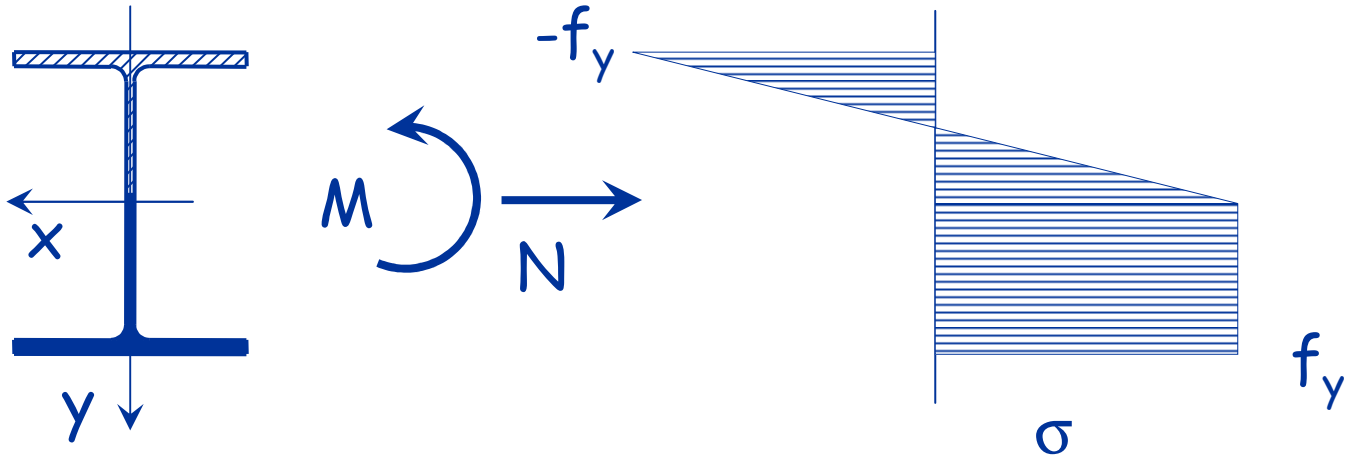


$$\sigma = \frac{N}{A} + \frac{M_{el,N}}{I}\, y$$

Per calcolare $M_{el,N}$:

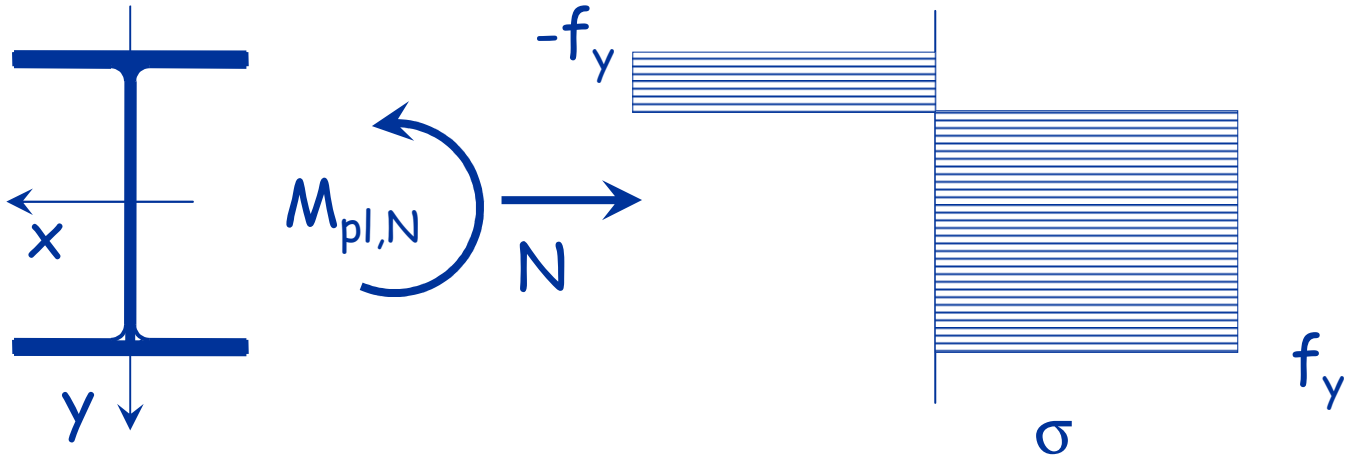
$$f_y = \frac{N}{A} + \frac{M_{el,N}}{W_{el}} \quad \Rightarrow \quad M_{el,N} = \left(f_y - \frac{N}{A} \right) W_{el}$$

Comportamento ultimo



Incrementando il momento flettente le deformazioni plastiche si propagano fino alla completa plasticizzazione della sezione

Comportamento ultimo

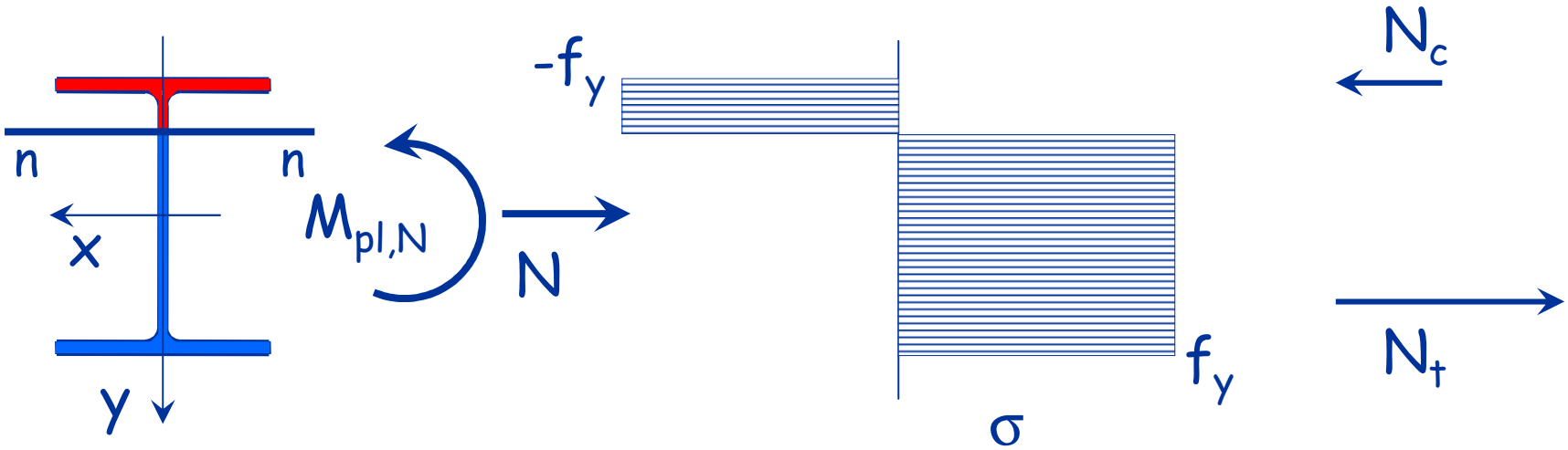


Per calcolare $M_{pl,N}$:

1 - Bisogna prima determinare la posizione dell'asse neutro, dall'equilibrio alla traslazione;

2 - Imponendo l'equilibrio alla rotazione rispetto all'asse baricentrico si determina poi $M_{pl,N}$

Comportamento ultimo



1 – Determinazione dell'asse neutro

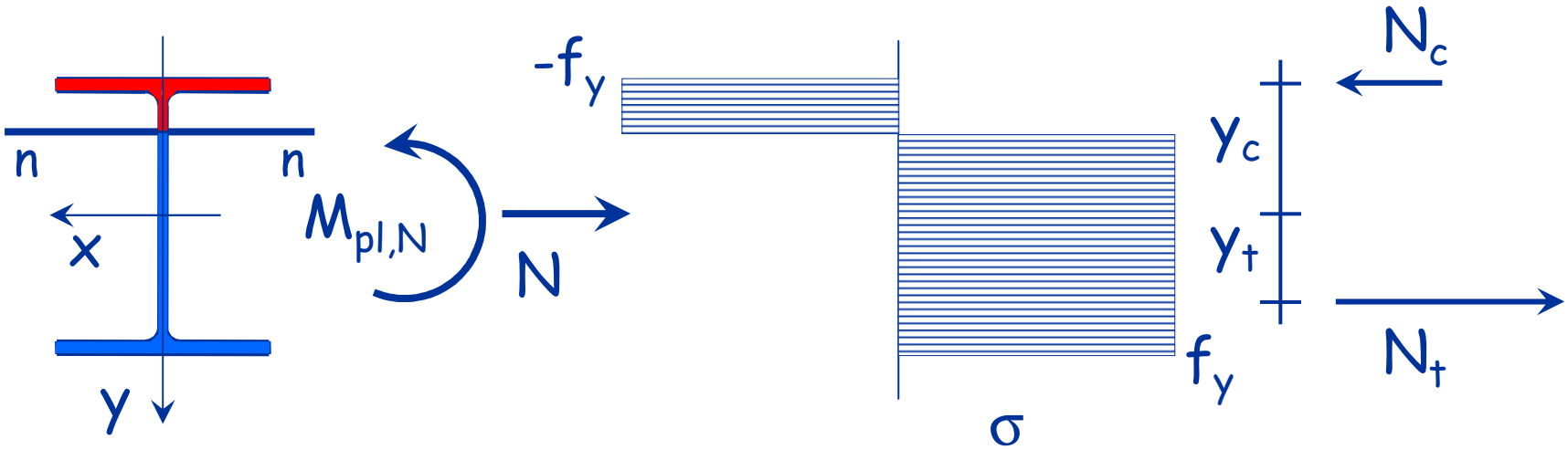
$N_c + N_t = N$
(equilibrio alla traslazione)

⇨ Asse neutro

$N_c = -f_y A_c$

$N_t = f_y A_t$

Comportamento ultimo



2 - Calcolo di $M_{pl,N}$

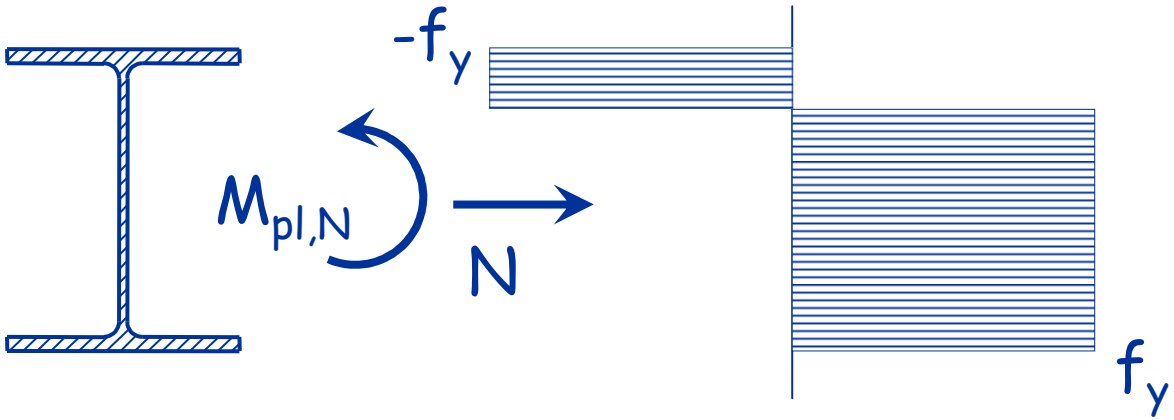
$$M_{pl,N} = N_t\, y_t - N_c\, y_c = f_y(A_t\, y_t - A_c\, y_c)$$

$$N_c = -f_y\, A_c$$

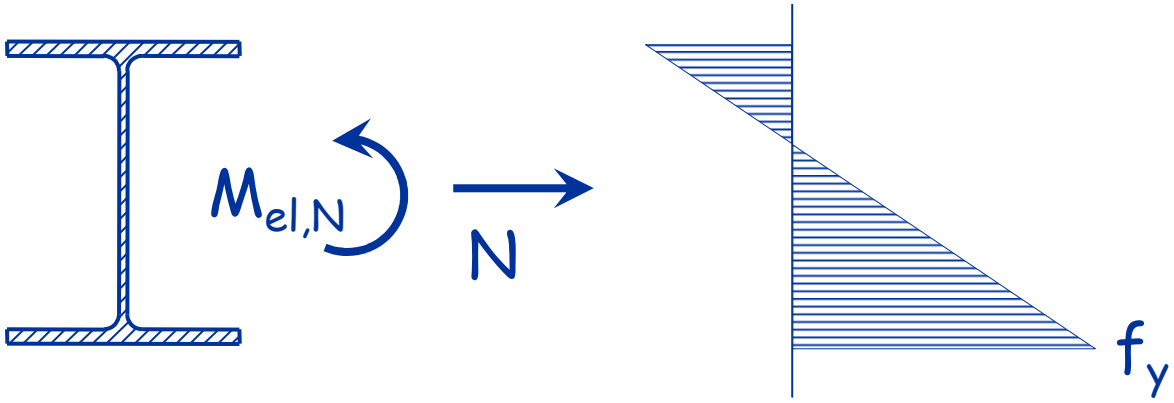
$$N_t = f_y\, A_t$$

Verifica - stato limite ultimo

Classe 1 e 2

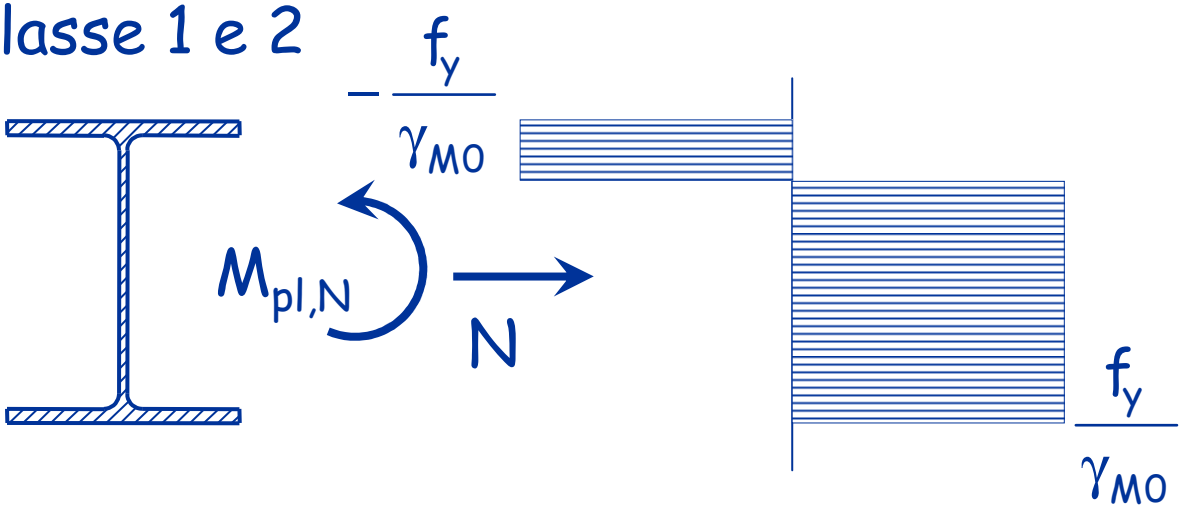


Classe 3



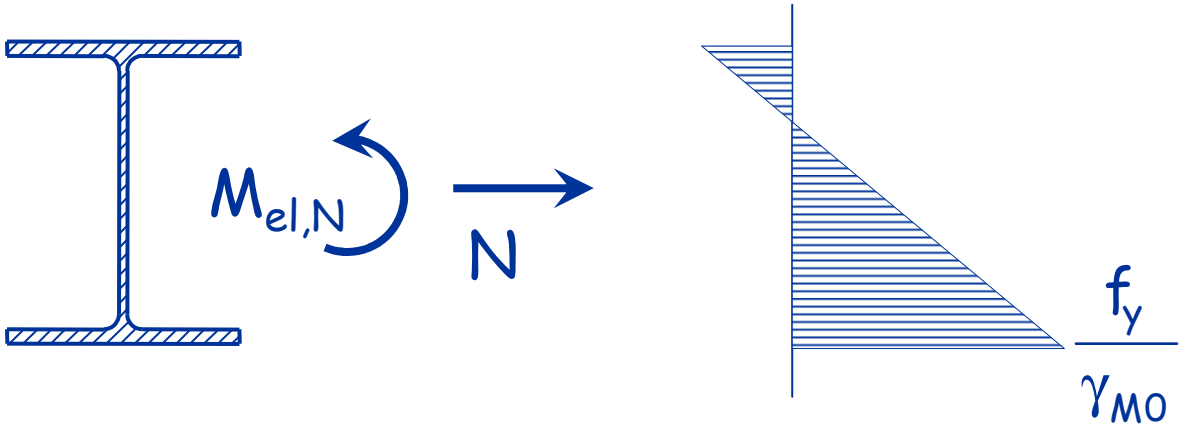
Verifica - stato limite ultimo

Classe 1 e 2



$$M_{Ed} \leq M_{pl,N,Rd}$$

Classe 3



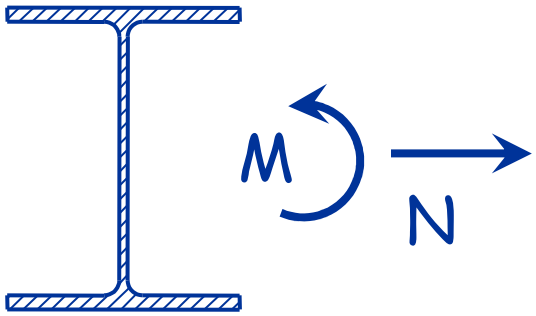
$$M_{Ed} \leq M_{el,N,Rd}$$

Domini di resistenza - stato limite ultimo

Dominio di resistenza, o curva di interazione = insieme delle coppie M-N per cui si ottiene lo stato limite ultimo della sezione

Per ricavare una coppia M-N del dominio

Sezione



Si assegna una posizione dell'asse neutro

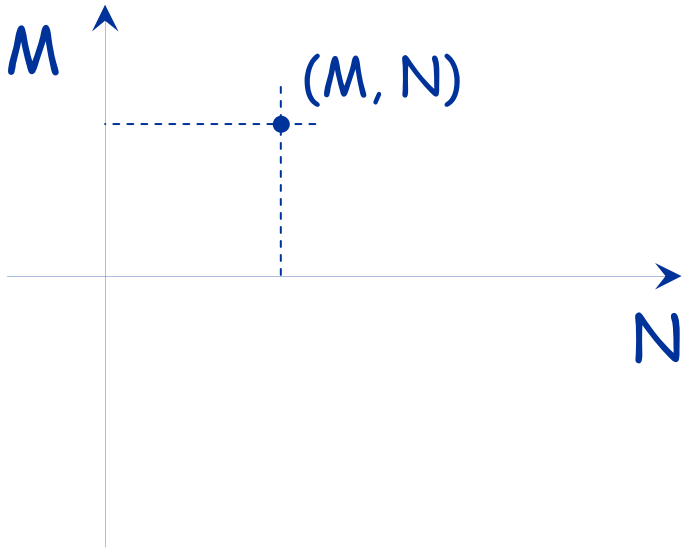
Si determina N

Si determina M ($M_{pl,N}$ o $M_{el,N}$)

Domini di resistenza - stato limite ultimo

Dominio di resistenza, o curva di interazione = insieme delle coppie M-N per cui si ottiene lo stato limite ultimo della sezione

Per ricavare una coppia M-N del dominio



Si assegna l'asse neutro

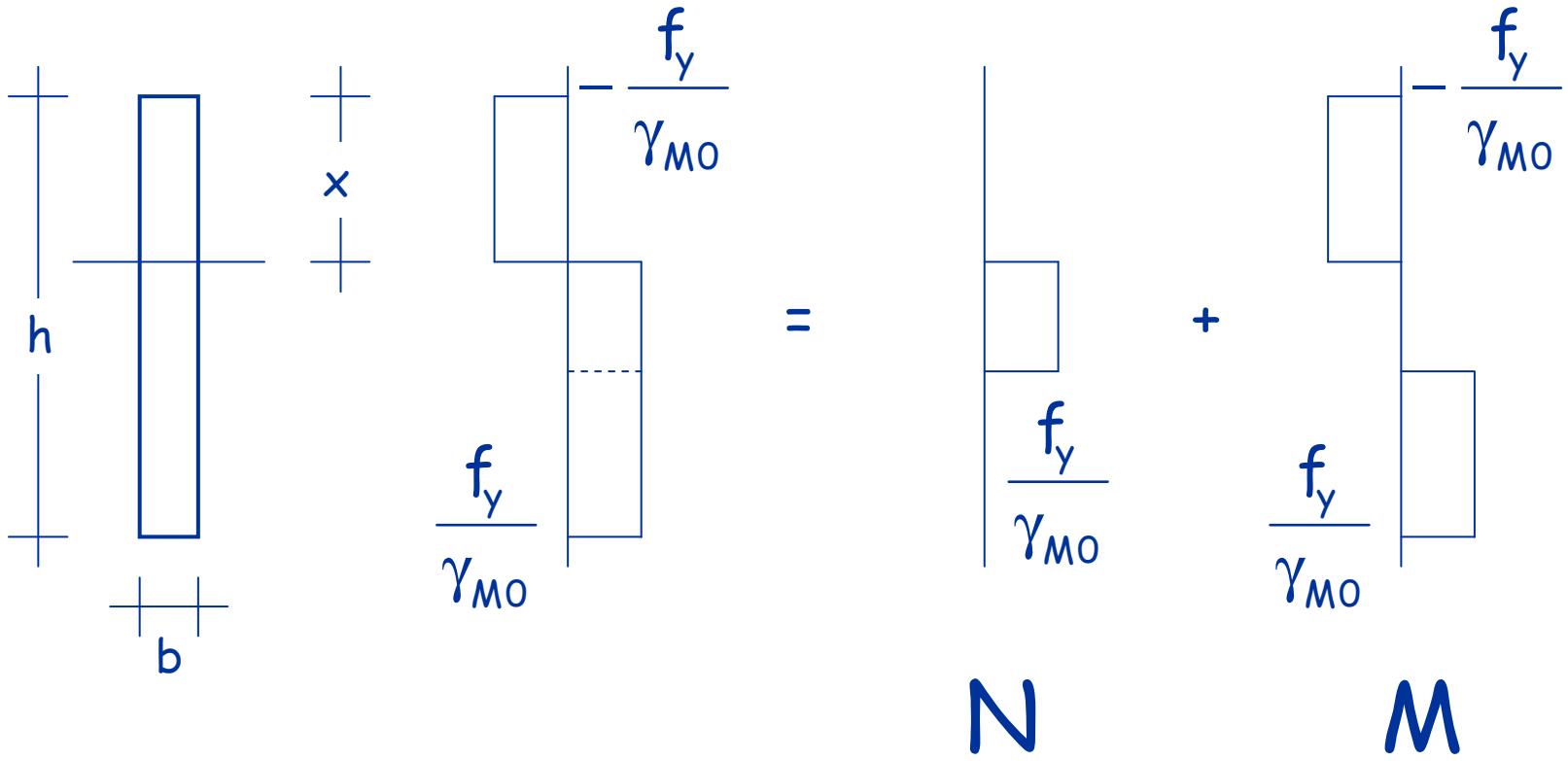
Si determina N

Si determina M ($M_{pl,N}$ o $M_{el,N}$)

e si riporta la coppia M – N nel diagramma

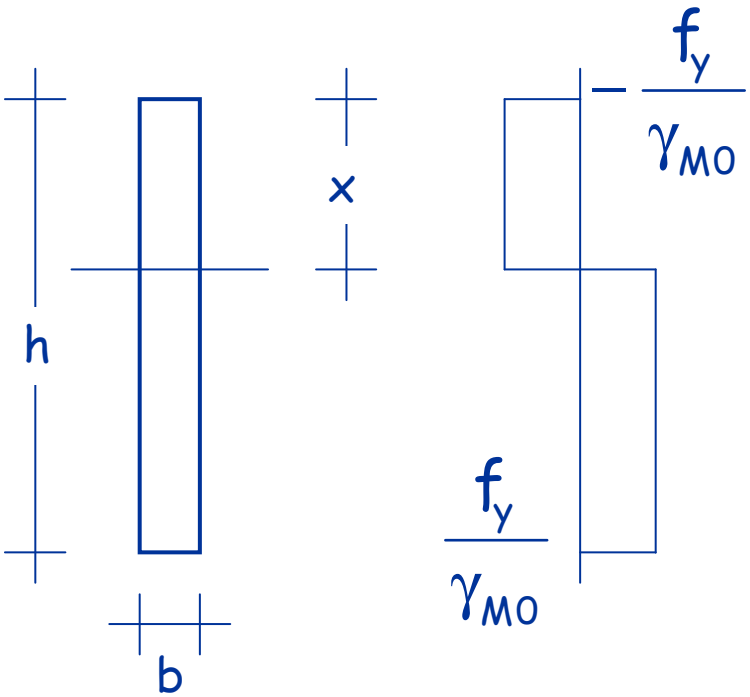
Esempio

Sezione rettangolare (solo ideale)



Esempio

Sezione rettangolare (solo ideale)



$$N = b\,(h - 2\,x)\,\frac{f_y}{\gamma_{M0}}$$

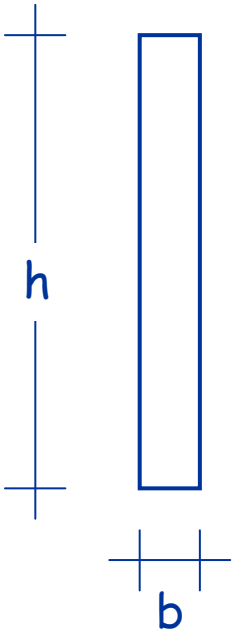
$$M = b\,x\,(h - x)\,\frac{f_y}{\gamma_{M0}}$$

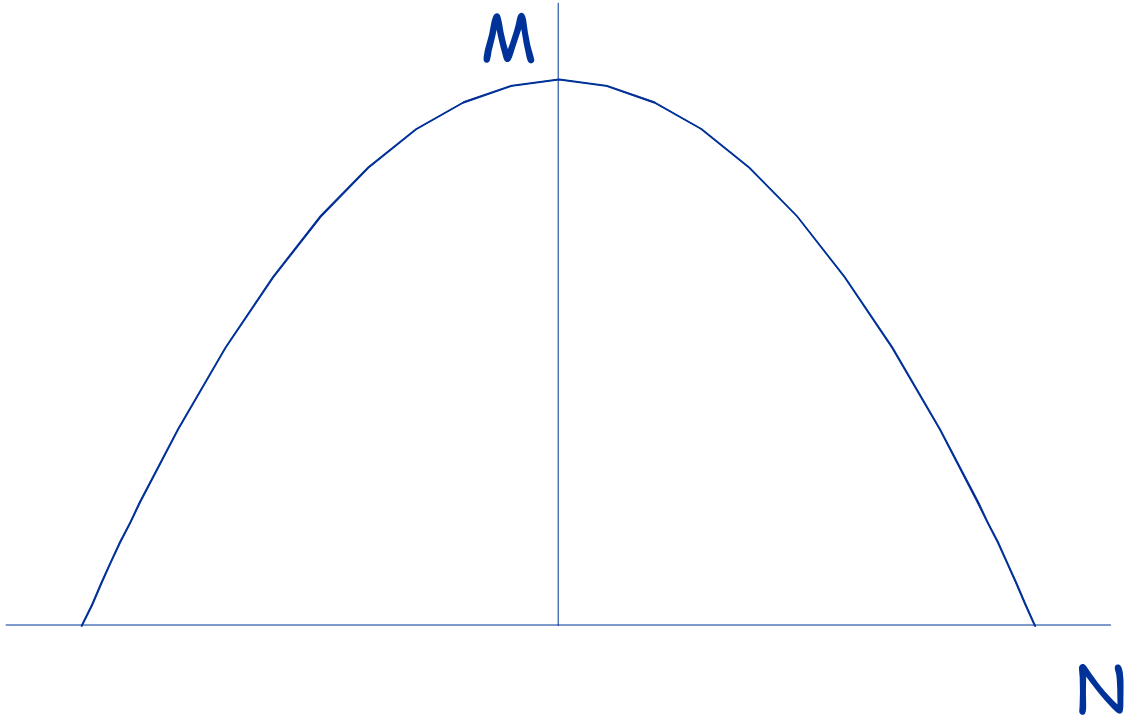
$$\Downarrow$$

$$M = \left[\frac{bh^2}{4}\,\frac{f_y^2}{\gamma_{M0}^2} - \frac{1}{4b}N^2\right]\frac{\gamma_{M0}}{f_y}$$

Esempio

Sezione rettangolare (solo ideale)

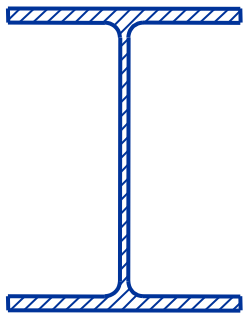




Il dominio ha un andamento parabolico

Questo vale in sostanza per un doppio T caricato nel piano debole

Domini di resistenza - stato limite ultimo per sezioni di classe 1 e 2

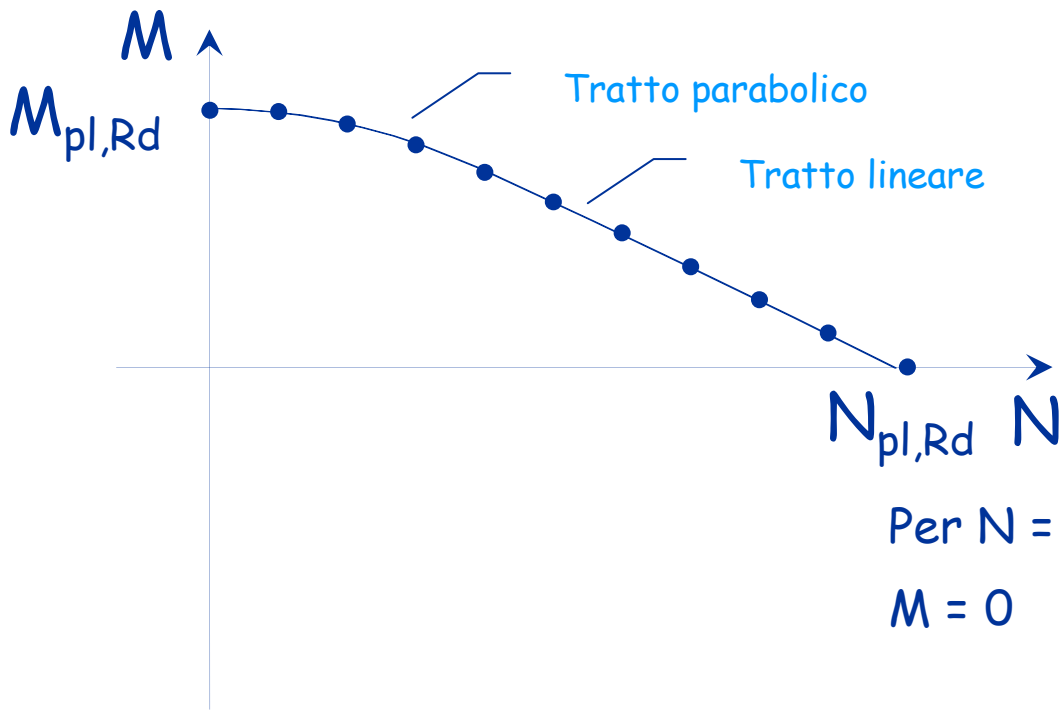


Per N = 0

$M = M_{pl,Rd}$

$$M_{pl,Rd} = \frac{W_{pl}\ f_y}{\gamma_{M0}}$$

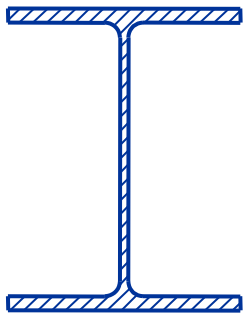
$$N_{pl,Rd} = \frac{A\ f_y}{\gamma_{M0}}$$



Per $N = N_{pl,Rd}$

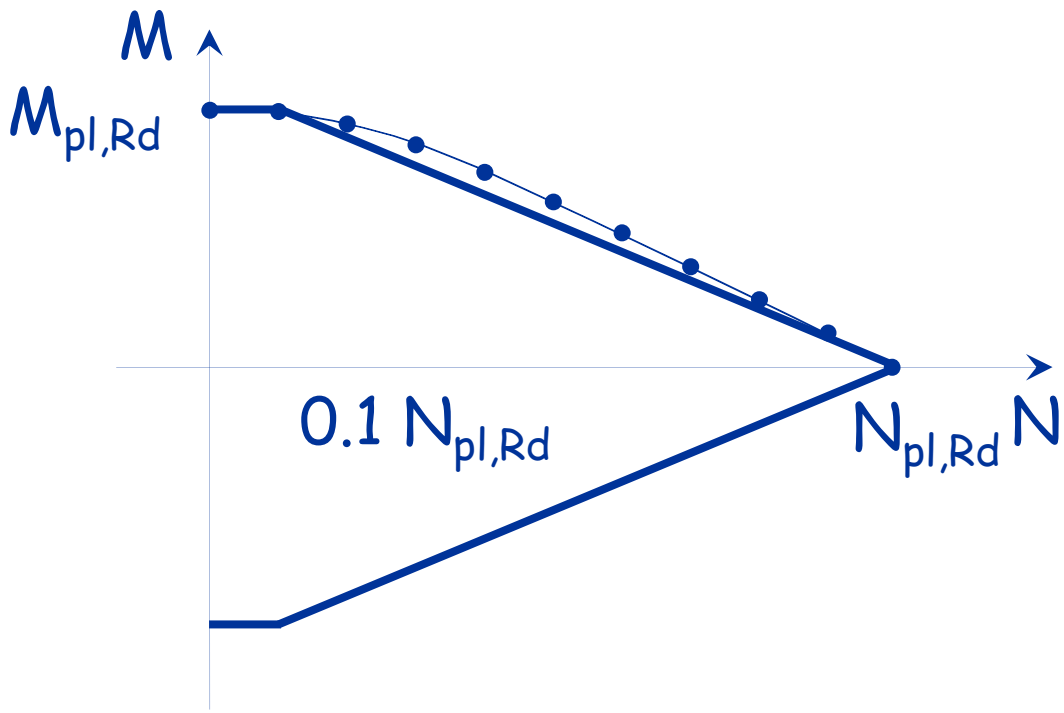
M = 0

Domini di resistenza - stato limite ultimo per sezioni di classe 1 e 2



$$M_{pl,N,Rd} = M_{pl,Rd} \qquad N \le 0.1\, N_{pl,Rd}$$

$$M_{pl,N,Rd} = 1.11\, M_{pl,Rd}\left(1 - \frac{N}{N_{pl,Rd}}\right) \qquad N > 0.1\, N_{pl,Rd}$$



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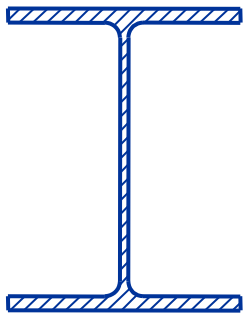
$$N_{pl,Rd} = \frac{A\, f_y}{\gamma_{M0}}$$

Domini di resistenza - stato limite ultimo per sezioni di classe 1 e 2



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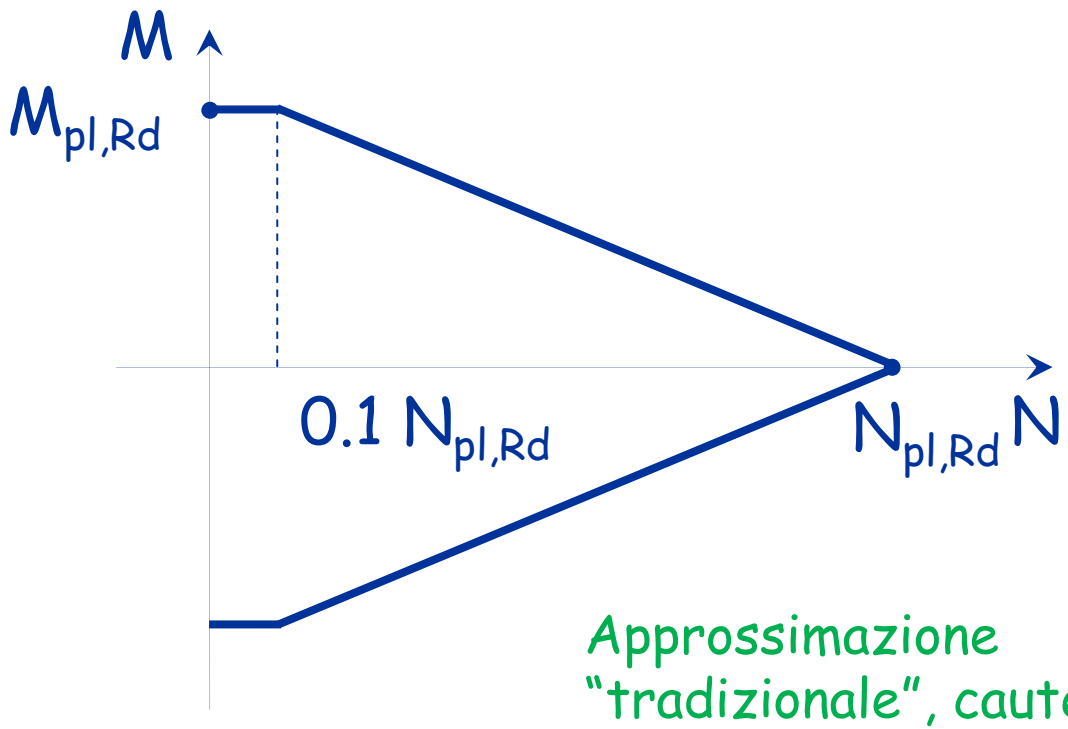


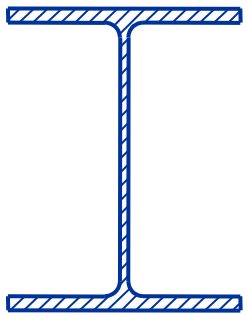
$$M_{pl,Rd} = \frac{W_{pl}\, f_y}{\gamma_{M0}}$$

$$N_{pl,Rd} = \frac{A\, f_y}{\gamma_{M0}}$$

Approssimazione "tradizionale", cautelativa

Domini di resistenza - stato limite ultimo

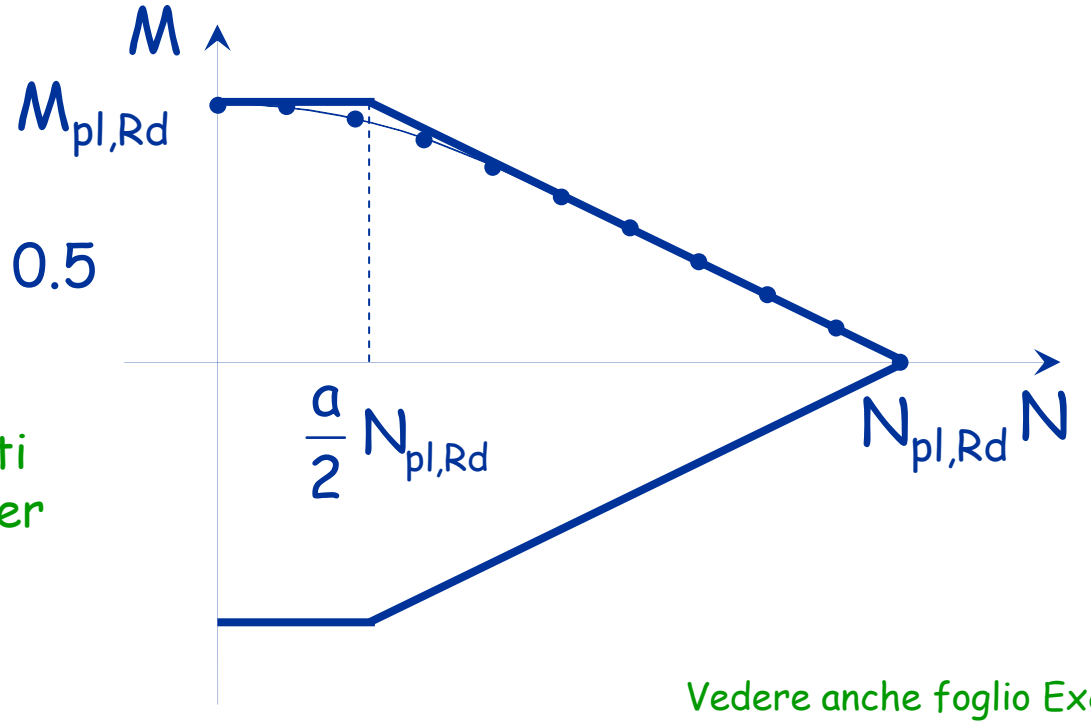
più preciso, meno cautelativo



$$M_{pl,N,Rd} = M_{pl,Rd} \qquad N \le \frac{a}{2} N_{pl,Rd}$$

$$M_{pl,N,Rd} = M_{pl,Rd}\left(1 - \frac{N}{N_{pl,Rd}}\right)\frac{1}{1 - 0.5\,a} \qquad N > \frac{a}{2} N_{pl,Rd}$$

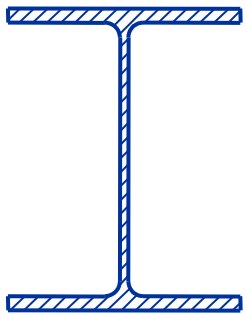
$$a = \frac{A - 2\,b\,t_f}{A} \le 0.5$$



Differenze più forti per gli IPE, meno per gli HE

Vedere anche foglio Excel Sagomario

Domini di resistenza - stato limite ultimo per sezioni di classe 3

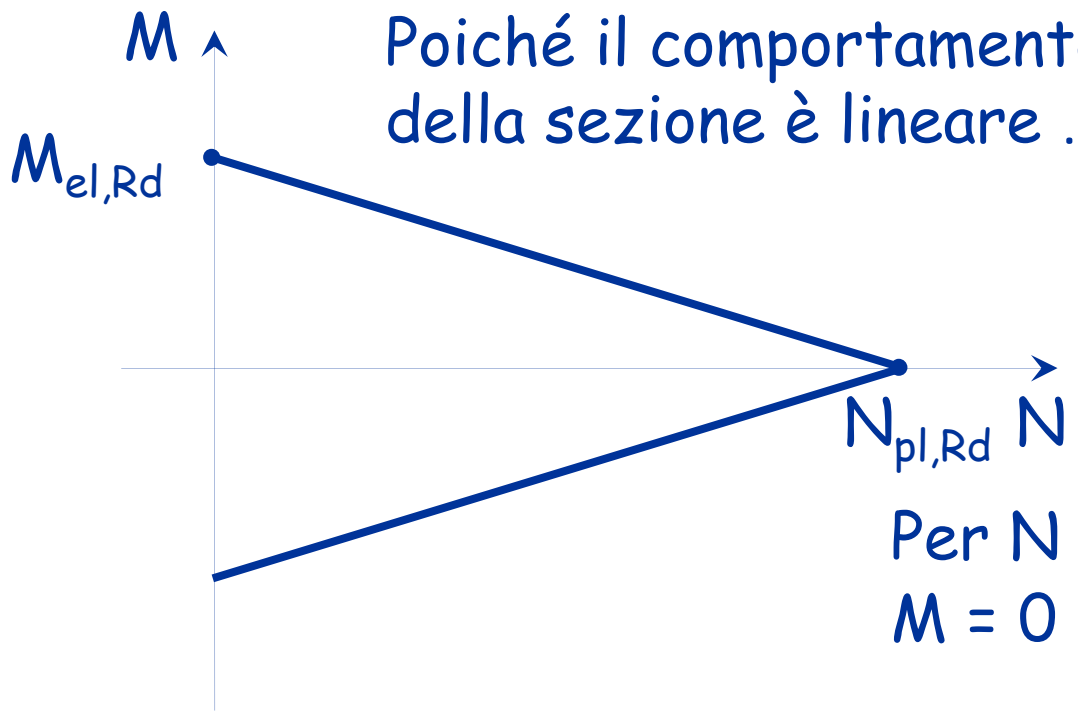


$$M_{el,N,Rd} = M_{el,Rd}\left(1 - \frac{N}{N_{pl,Rd}}\right)$$

Per N = 0
$M = M_{el,Rd}$

$$M_{el,Rd} = \frac{W_{el}\ f_y}{\gamma_{M0}}$$

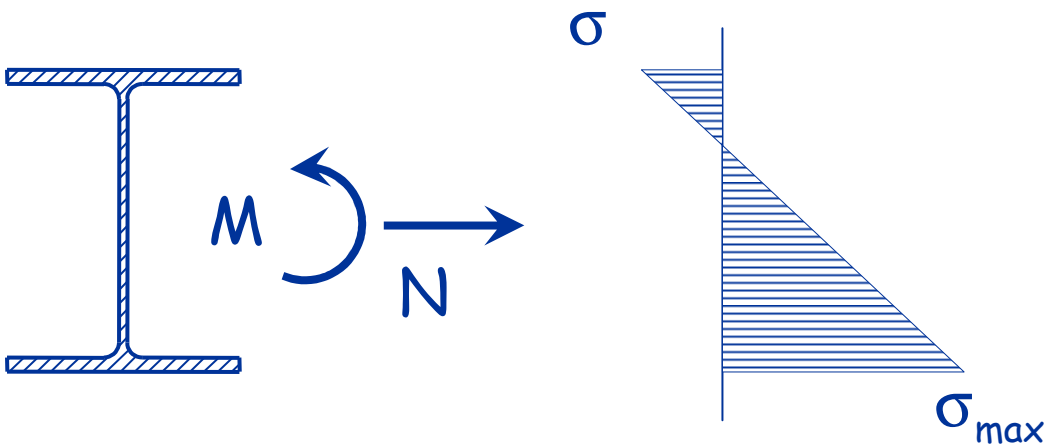
$$N_{pl,Rd} = \frac{A\ f_y}{\gamma_{M0}}$$



Poiché il comportamento della sezione è lineare ...

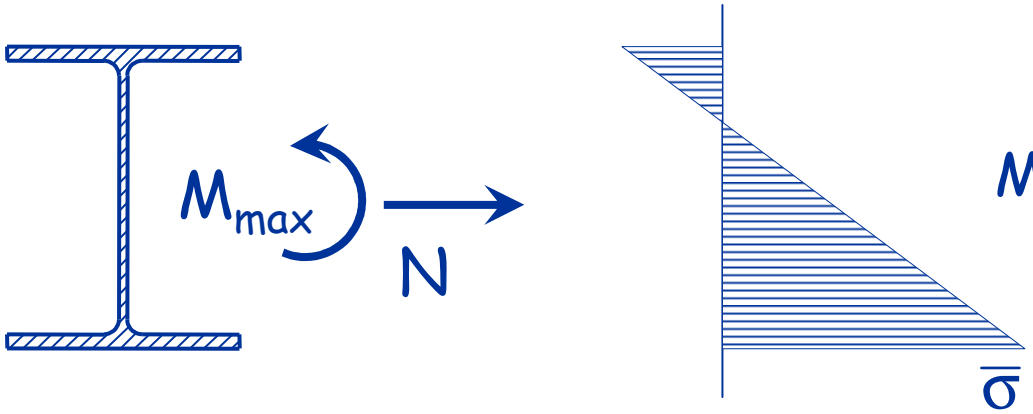
Per $N = N_{pl,Rd}$
$M = 0$

Verifica - tensioni ammissibili



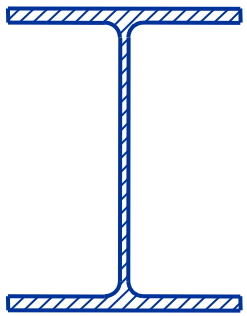
$$\sigma_{max} = \frac{N}{A} + \frac{M}{W_{el}} \leq \bar{\sigma}$$

oppure



$$M_{max} = W_{el}\,\bar{\sigma}\left(1 - \frac{N}{A\,\bar{\sigma}}\right) \geq M$$

Domini di resistenza - tensioni ammissibili

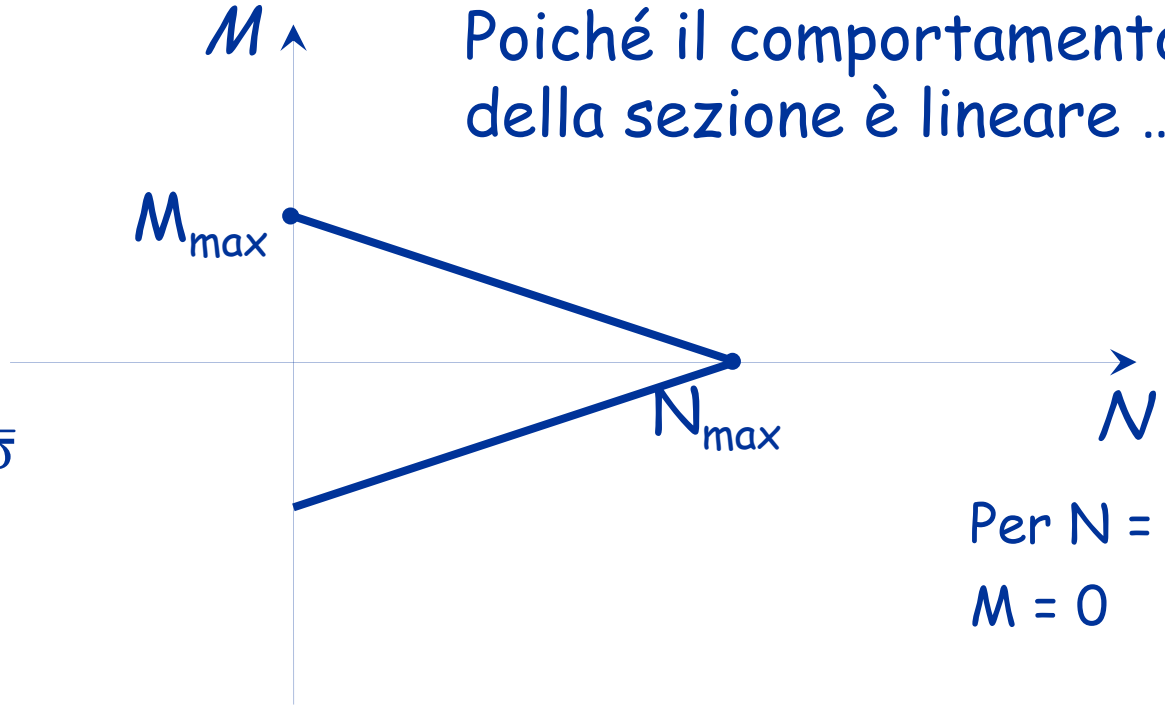


$$M_{max,N} = M_{max}\left(1 - \frac{N}{N_{max}}\right)$$

Poiché il comportamento della sezione è lineare ...

Per N = 0

$M = M_{max}$



$$M_{max} = W_{el}\,\overline{\sigma}$$

$$N_{max} = A\,\overline{\sigma}$$

Per $N = N_{max}$

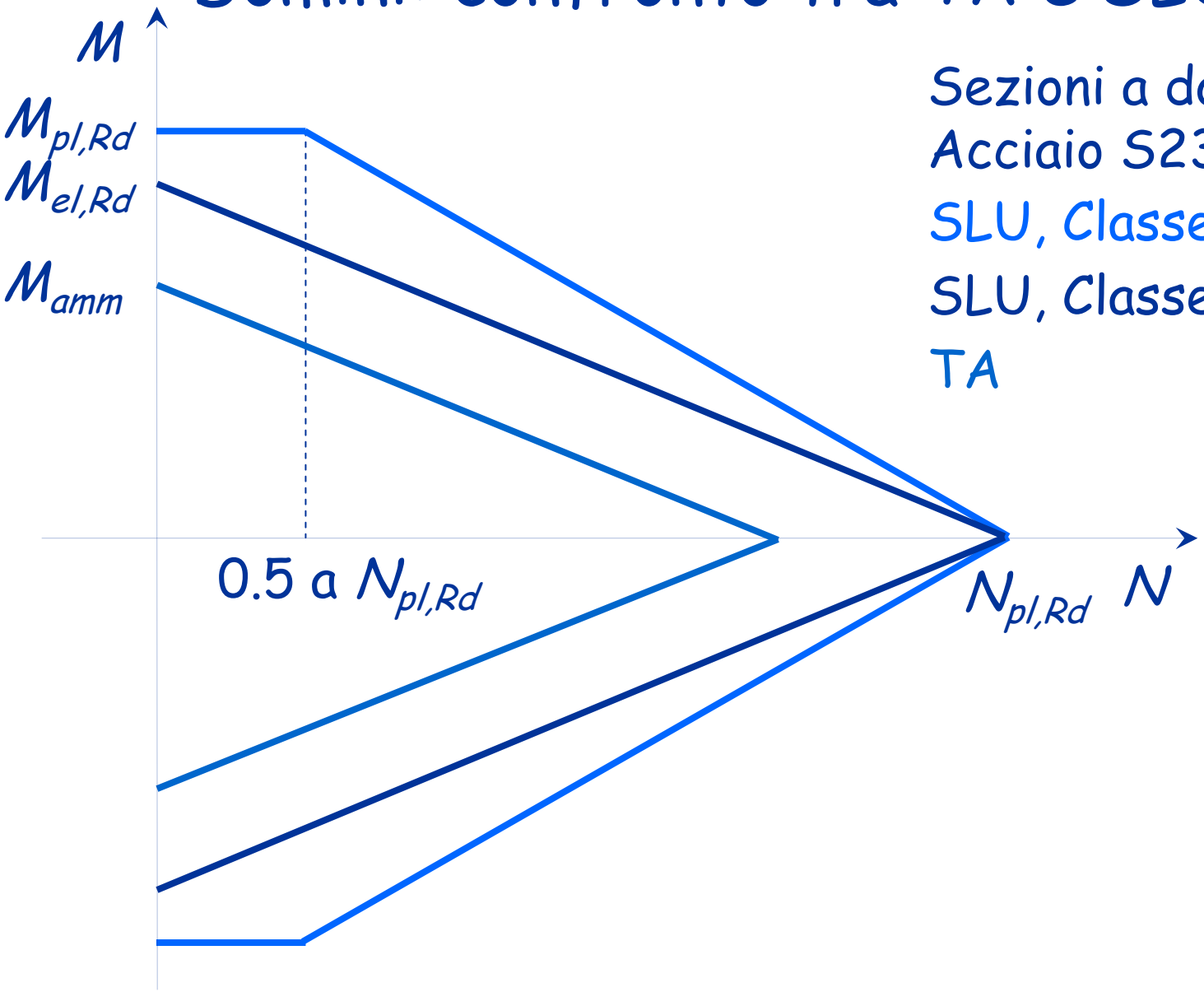
$M = 0$

Domini: confronto tra TA e SLU

Il confronto può essere effettuato sovrapponendo i domini ricavati per TA e SLU

Poiché i carichi allo SLU sono maggiori (di 1.3 ÷ 1.5) di quelli alle TA, il dominio relativo alle TA deve essere opportunamente scalato (ad esempio x 1,4)

Domini: confronto tra TA e SLU



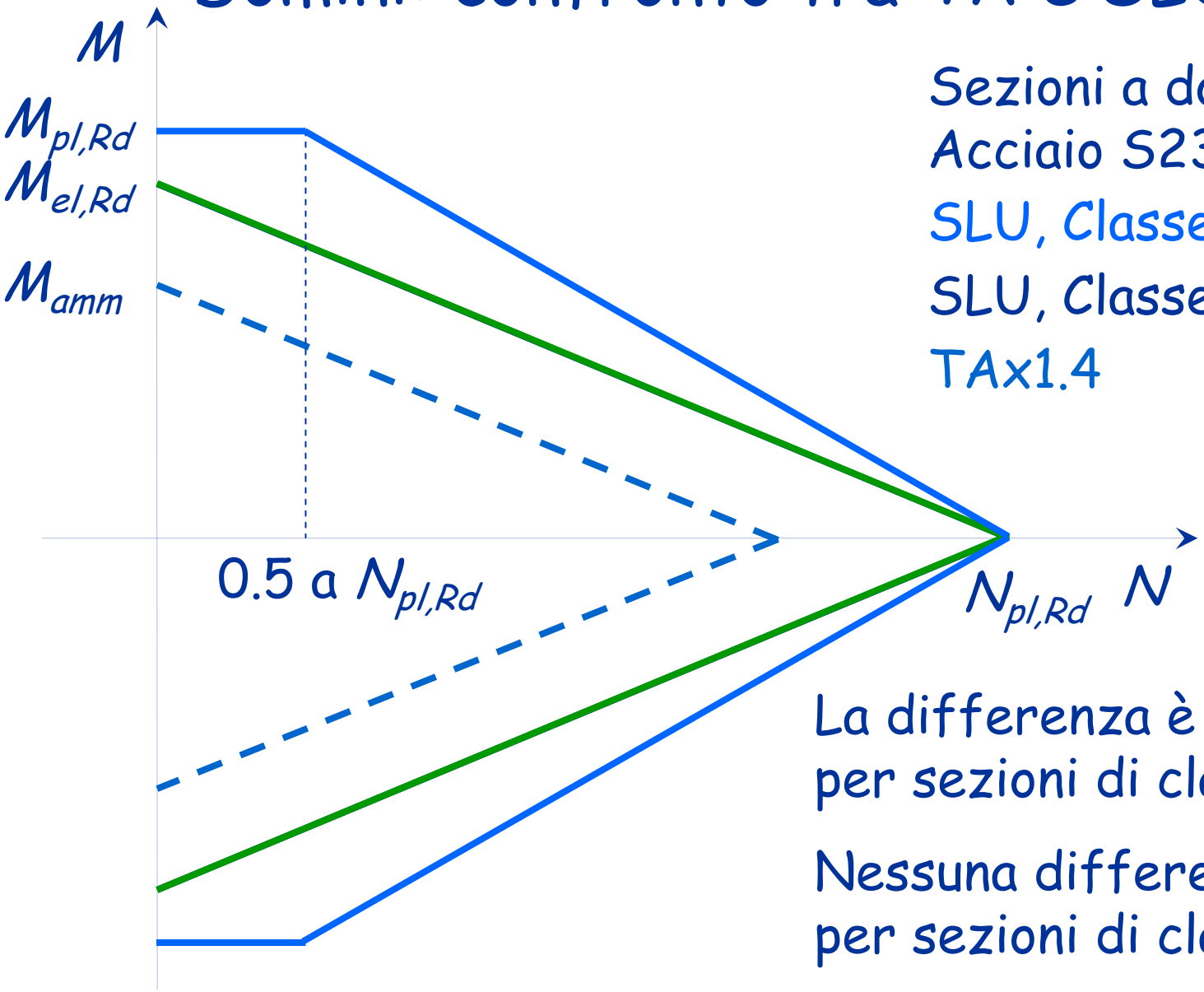
Sezioni a doppio T
Acciaio S235

SLU, Classe 1 e 2

SLU, Classe 3

TA

Domini: confronto tra TA e SLU



Sezioni a doppio T
Acciaio S235

SLU, Classe 1 e 2

SLU, Classe 3

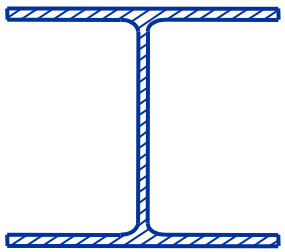
TAx1.4

La differenza è sensibile per sezioni di classe 1 e 2

Nessuna differenza per sezioni di classe 3

Esempio

Dati:



M_{Ed} = 210 kNm

N_{Ed} =1500 kN

Sezione	HEB300
A	149 cm^2
W_{pl}	1868 cm^3
Acciaio	S235

1 - Classe della sezione

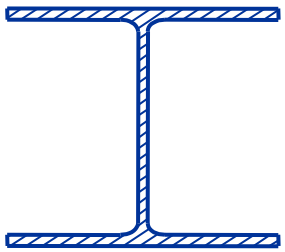
Anima: $\frac{c_w}{t_w} = \frac{208}{11} = 19 \leq 72\,\varepsilon = 72$

Flangia: $\frac{c}{t_f} = \frac{117.5}{19} = 6.2 \leq 9\,\varepsilon = 9$

La sezione appartiene alla classe 1.

Esempio

Dati:



M_{Ed} = 210 kNm

N_{Ed} =1500 kN

Sezione	HEB300
A	149 cm^2
W_{pl}	1868 cm^3
Acciaio	S235

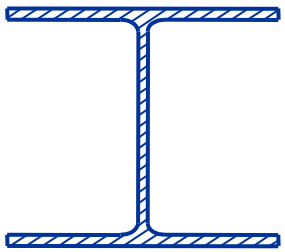
2 - Determinazione di $N_{pl,Rd}$ ed $M_{pl,Rd}$

$$N_{pl,Rd} = \frac{f_y\ A}{\gamma_{M0}} = \frac{235 \times 149}{1.05 \times 10} = 3334.8 \text{ kN}$$

$$M_{pl,Rd} = \frac{f_y\ W_{pl}}{\gamma_{M0}} = \frac{235 \times 1868}{1.05 \times 10^3} = 418.1 \text{ kNm}$$

Esempio

Dati:



M_{Ed} = 210 kNm

N_{Ed} =1500 kN

Sezione	HEB300
A	149 cm^2
W_{pl}	1868 cm^3
Acciaio	S235
b=300 mm	t=19 mm

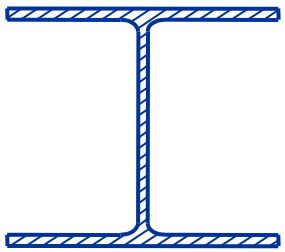
3 - Determinazione di α

$$a = \frac{A - 2\,b\,t_f}{A} = \frac{14900 - 2 \times 300 \times 19}{14900} = 0.235$$

$$\frac{a}{2} N_{pl,Rd} = \frac{0.235}{2} 3334.8 = 391.8 \text{ kN}$$

Esempio

Dati:



M_{Ed} = 210 kNm

N_{Ed} =1500 kN

Sezione	HEB300
Acciaio	S235
$N_{pl,Rd}$	3334.8 kN
$M_{pl,Rd}$	418.1 kNm

4 - Determinazione di $M_{pl,N,Rd}$ e verifica

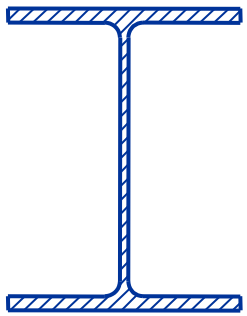
$$N_{Ed} = 1500\ \text{kN} \geq \frac{a}{2} N_{pl,Rd} = 391.8\ \text{kN}$$

$$M_{pl,N,Rd} = M_{pl,Rd}\left(1 - \frac{N_{Ed}}{N_{pl,Rd}}\right)\frac{1}{1 - 0.5 \times 0.235} = 260.7\ \text{kNm} > M_{Ed} = 210.0\ \text{kNm}$$

La sezione è verificata

Domini di resistenza - stato limite ultimo

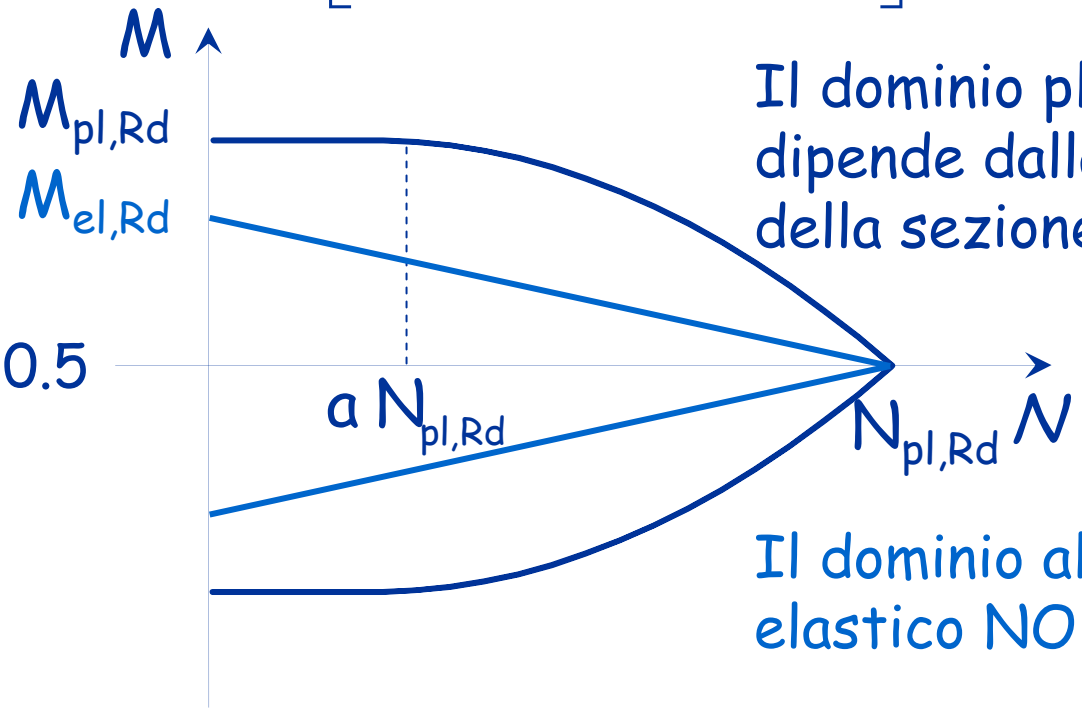
Momento agente attorno all'asse debole



$$M_{pl,N,Rd} = M_{pl,Rd} \qquad N \le a\,N_{pl,Rd}$$

$$M_{pl,N,Rd} = M_{pl,Rd}\left[1-\left(\frac{N/N_{pl,Rd}-a}{1-a}\right)^2\right] \qquad N > a\,N_{pl,Rd}$$

$$a = \frac{A-2\,b\,t_f}{A} \le 0.5$$



Il dominio plastico dipende dalla forma della sezione

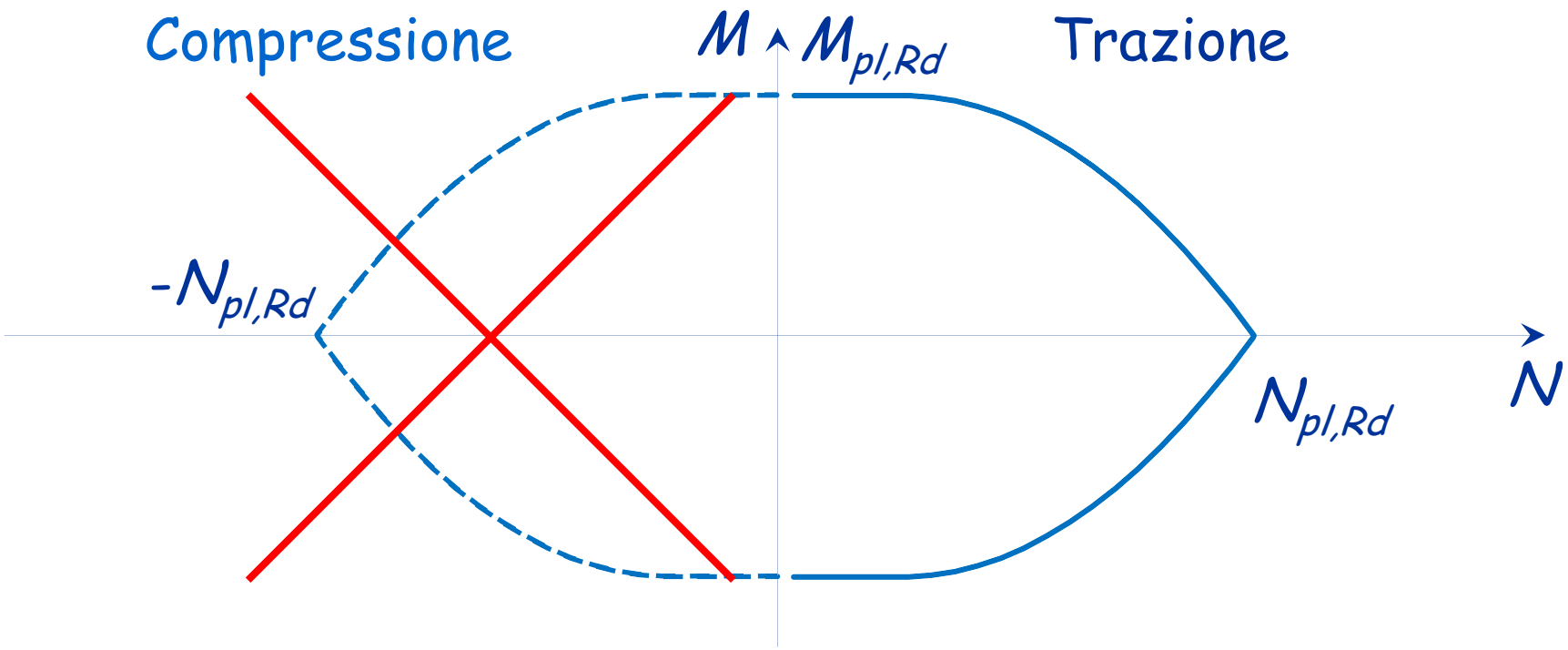
Il dominio al limite elastico NO

Flessione composta
pressoflessione

Domini di resistenza - stato limite ultimo

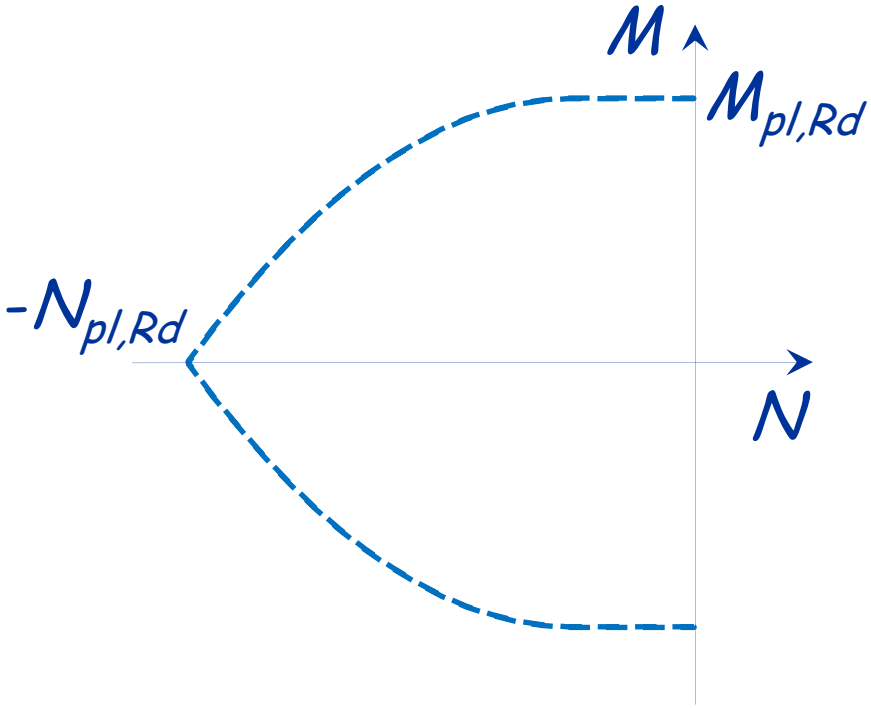
Si possono ottenere semplicemente ribaltando il dominio M-N costruito nel caso di tenso-flessione?

Va bene per la singola sezione, ma per l'asta bisogna tener conto dell'instabilità



Costruzione del dominio di resistenza

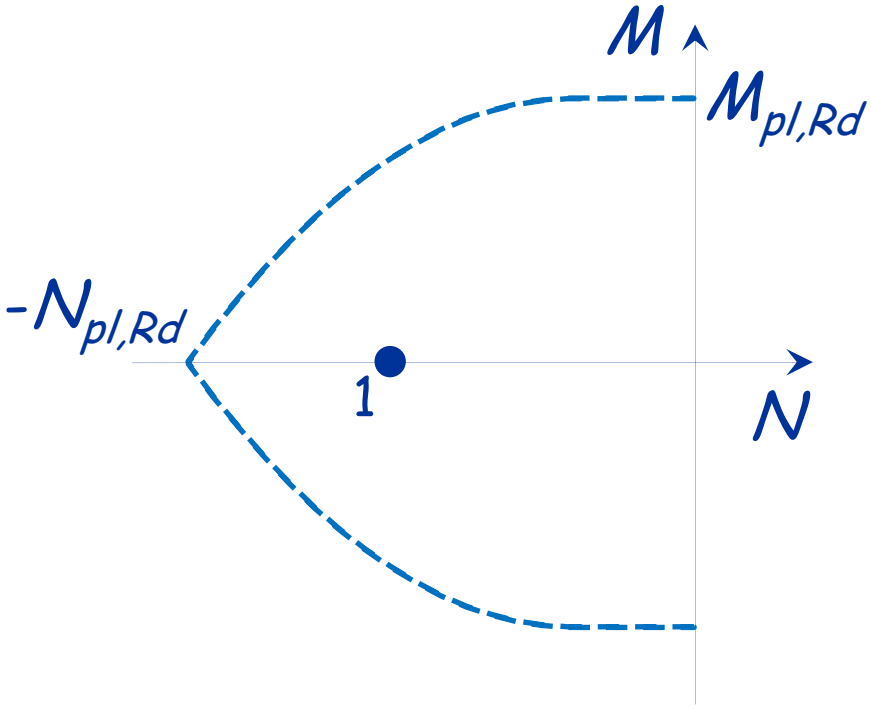
1 -



Costruzione del dominio di resistenza

1 -

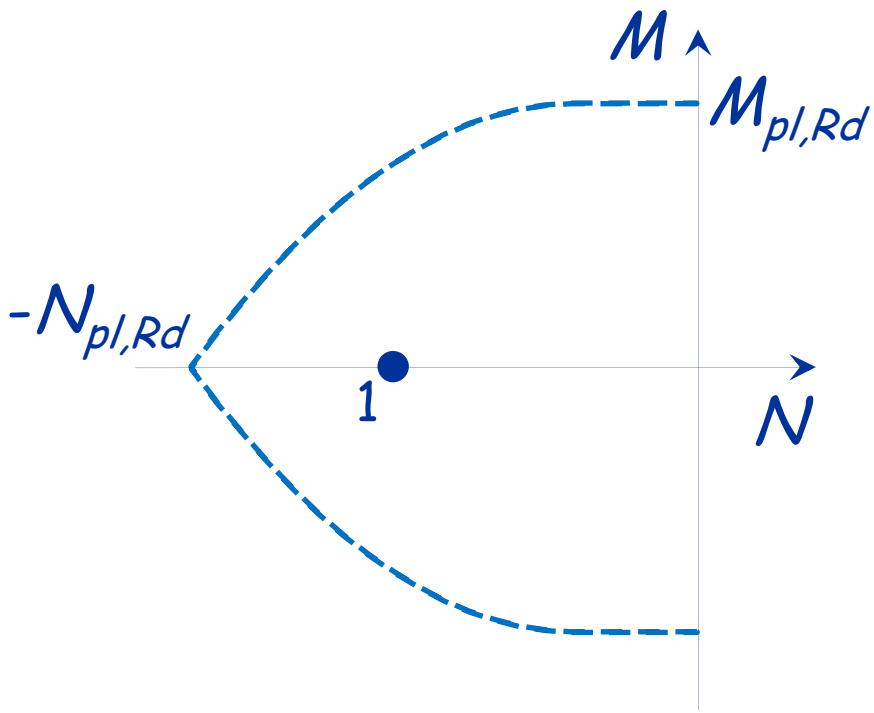
$N_1 = N_{b,Rd} < N_{pl,Rd}$



1 - $N_1 = N_{b,Rd}$, $M_1 = 0$

Costruzione del dominio di resistenza

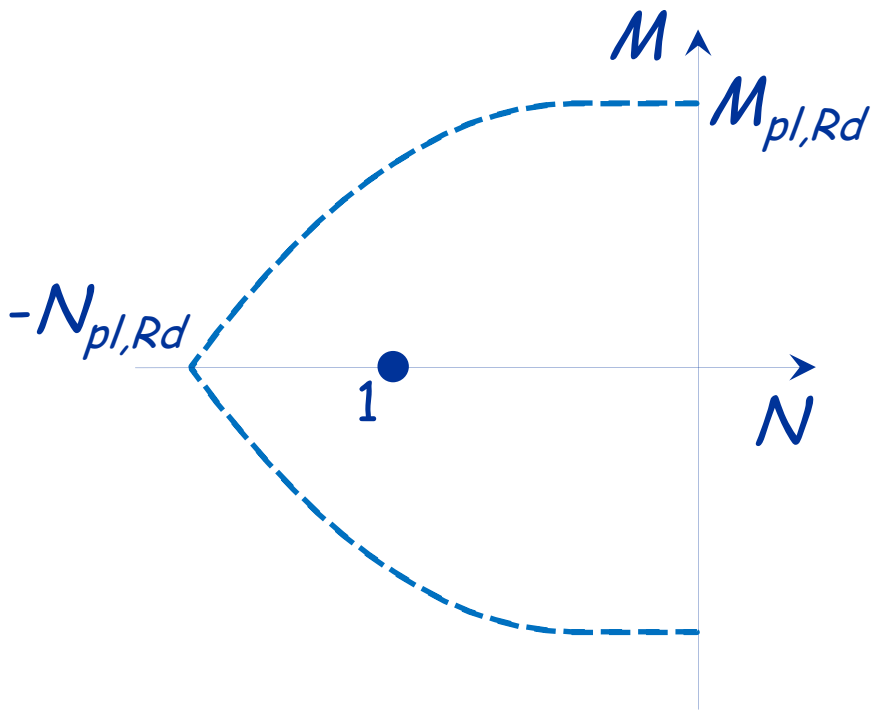
2 -



1 - $N_1 = N_{b,Rd}$, $M_1 = 0$

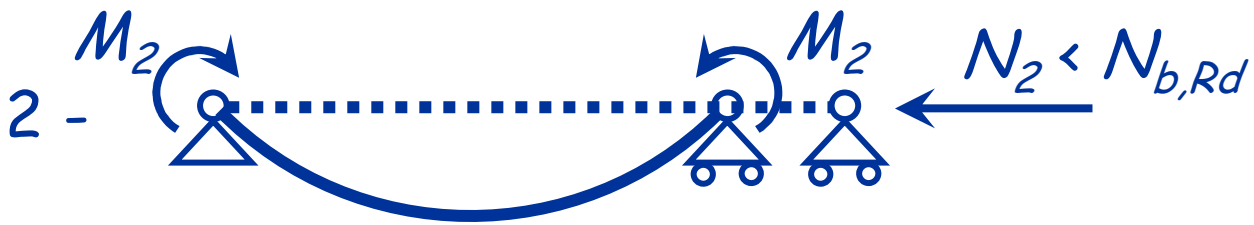
Costruzione del dominio di resistenza

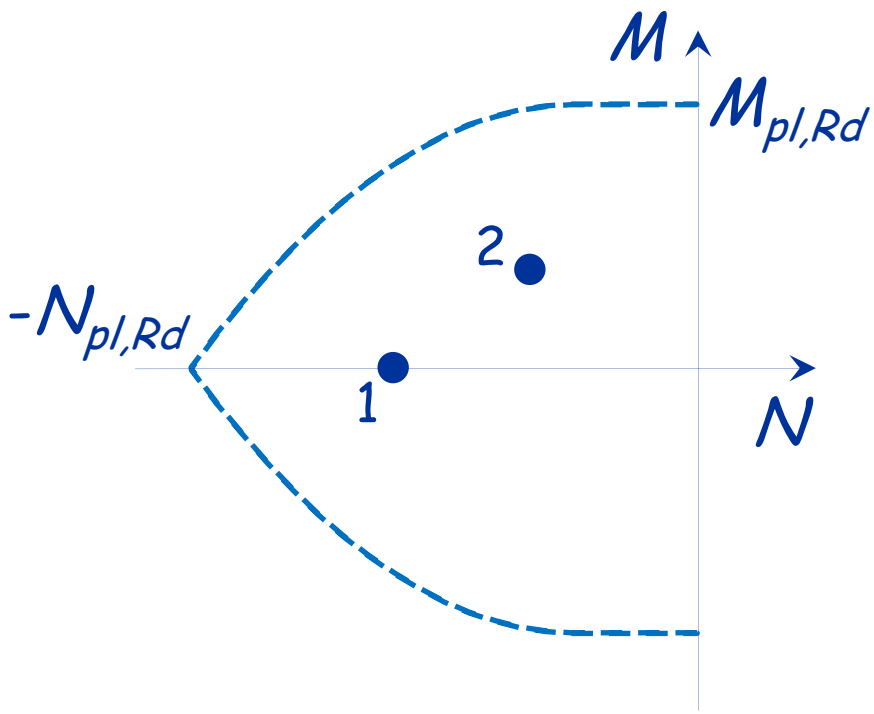
2 -



1 - $N_1 = N_{b,Rd}$, $M_1 = 0$

Costruzione del dominio di resistenza



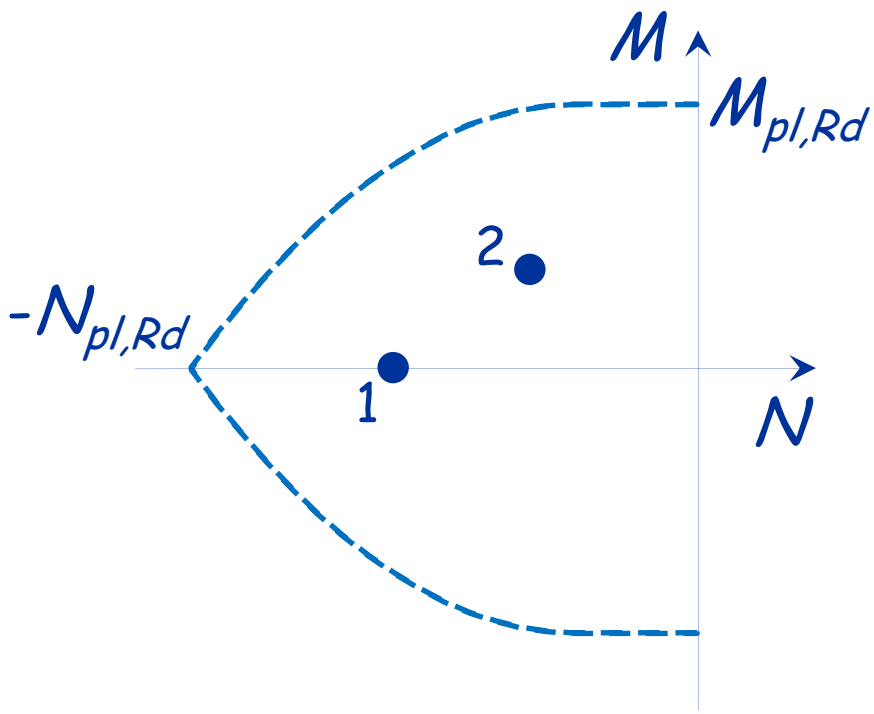


1 - $N_1 = N_{b,Rd}$, $M_1 = 0$

2 - $N_2 < N_{b,Rd}$, $M_2 < M_{pl,Rd}$

Costruzione del dominio di resistenza

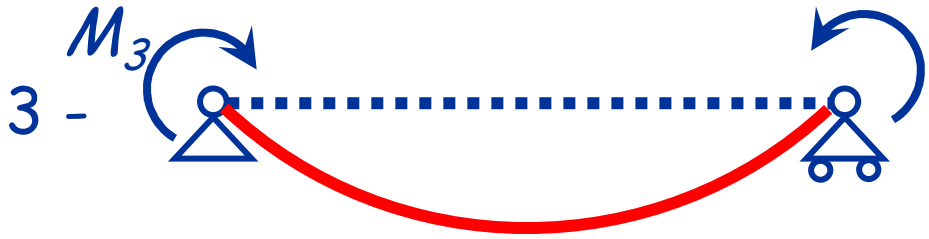
3 -



1 - $N_1 = N_{b,Rd}$, $M_1 = 0$

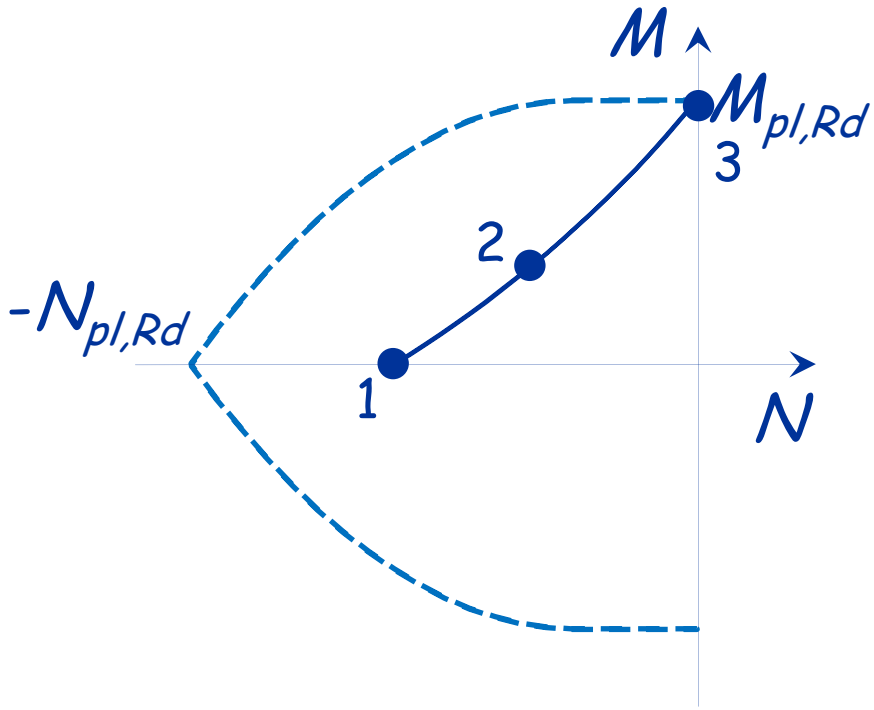
2 - $N_2 < N_{b,Rd}$, $M_2 < M_{pl,Rd}$

Costruzione del dominio di resistenza



$M_3 = M_{pl,Rd}$

L'asta si plasticizza e collassa in assenza di sforzo normale



1 - $N_1 = N_{b,Rd}$, $M_1 = 0$

2 - $N_2 < N_{b,Rd}$, $M_2 < M_{pl,Rd}$

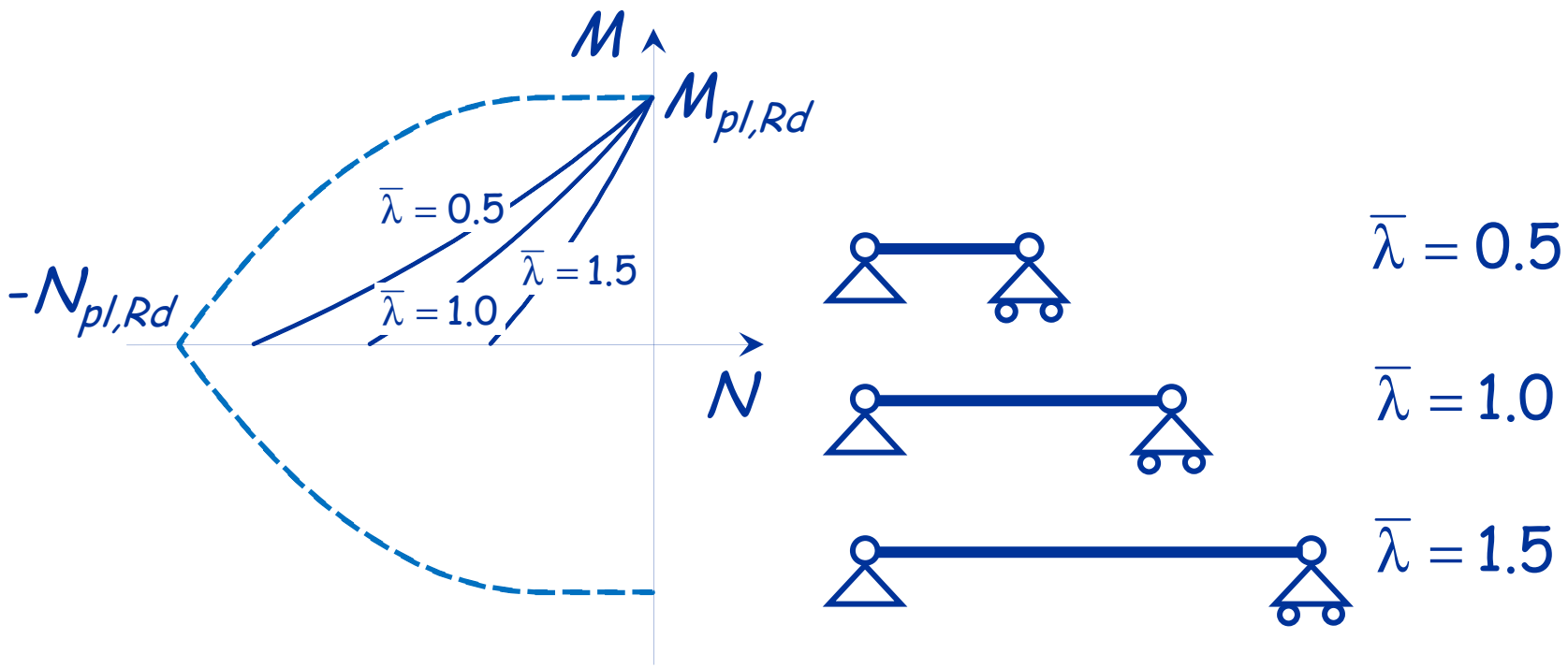
3 - $N_3 = 0$, $M_3 = M_{pl,Rd}$

Collegando i punti si ottiene il dominio

Influenza della snellezza

Il dominio dipende dalla snellezza dell'asta:

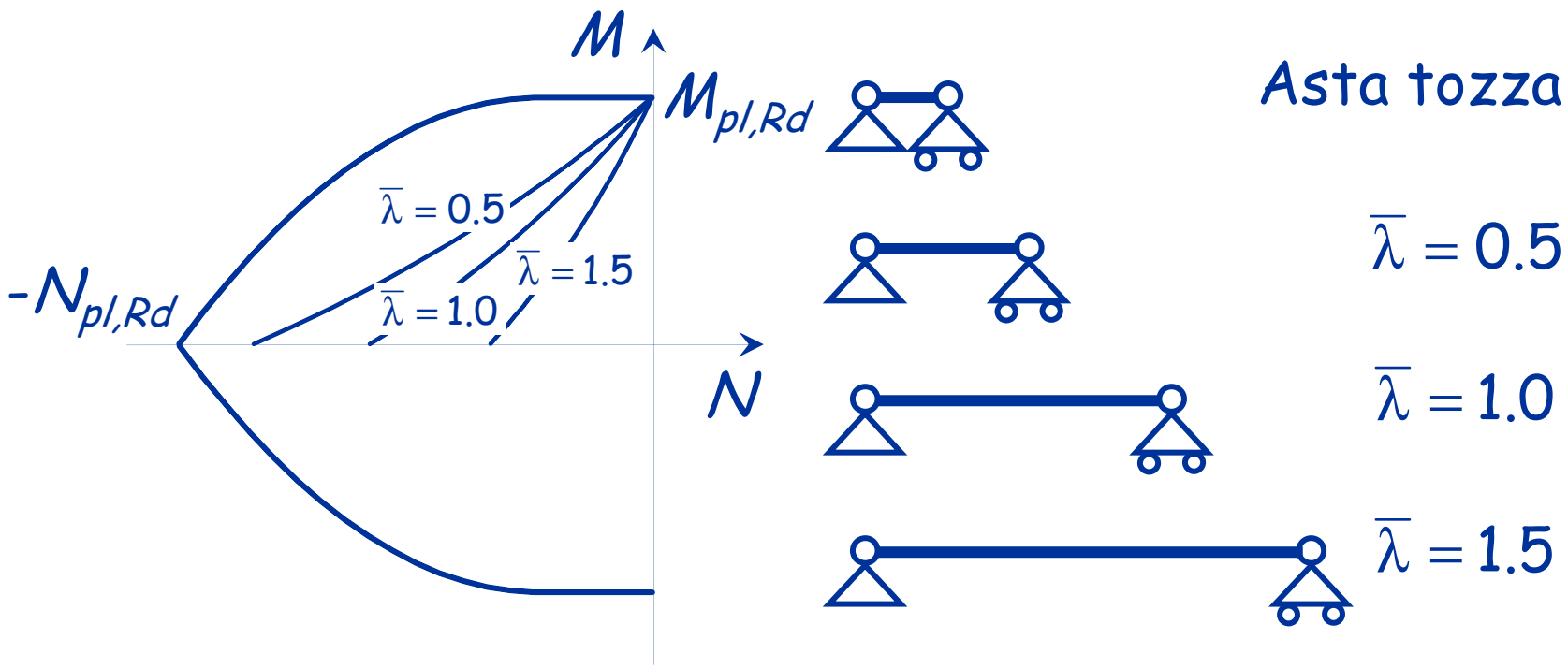
- L'ampiezza del dominio si riduce all'aumentare della snellezza;



Influenza della snellezza

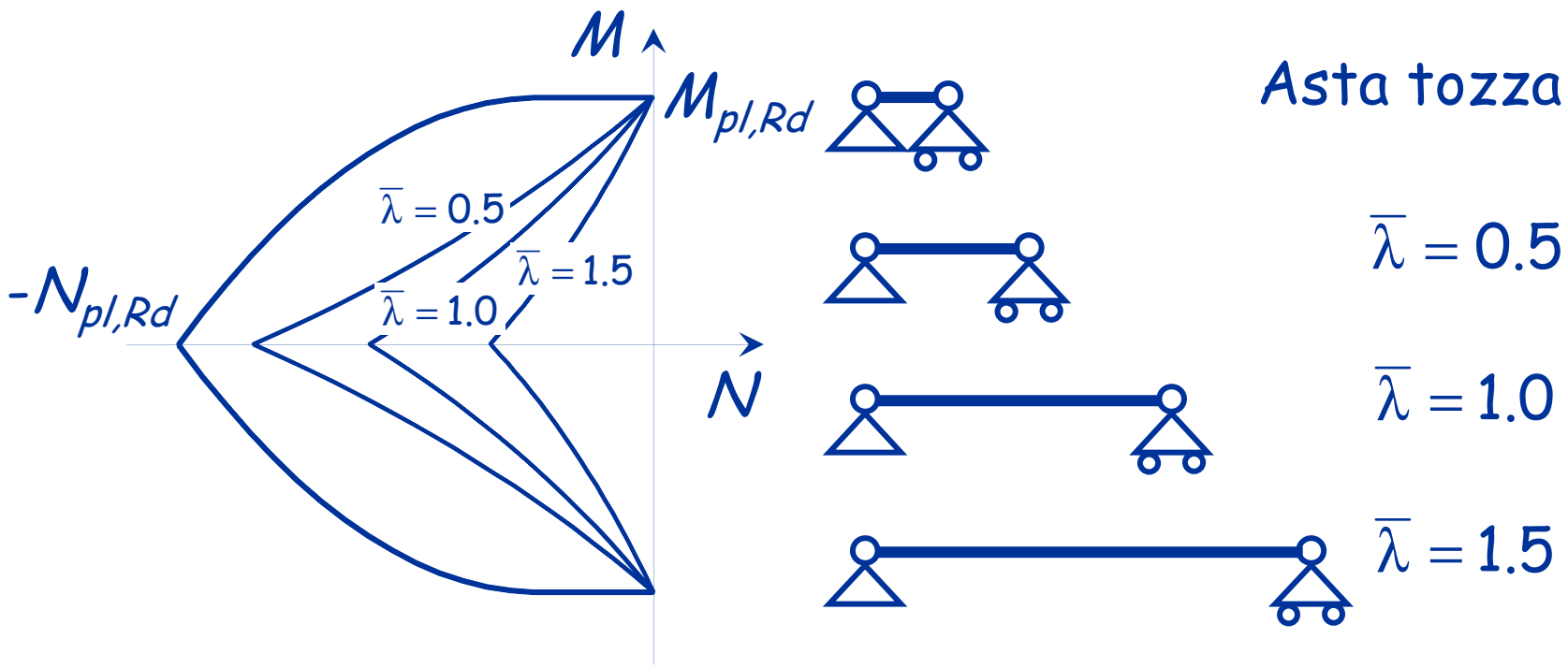
Il dominio dipende dalla snellezza dell'asta:

- Nel caso di aste tozze coincide con quello per presso-flessione della sezione

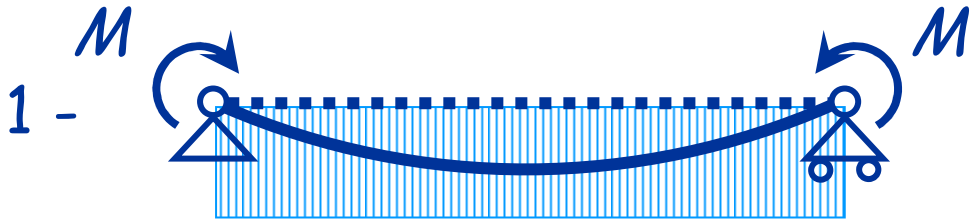


Influenza della snellezza

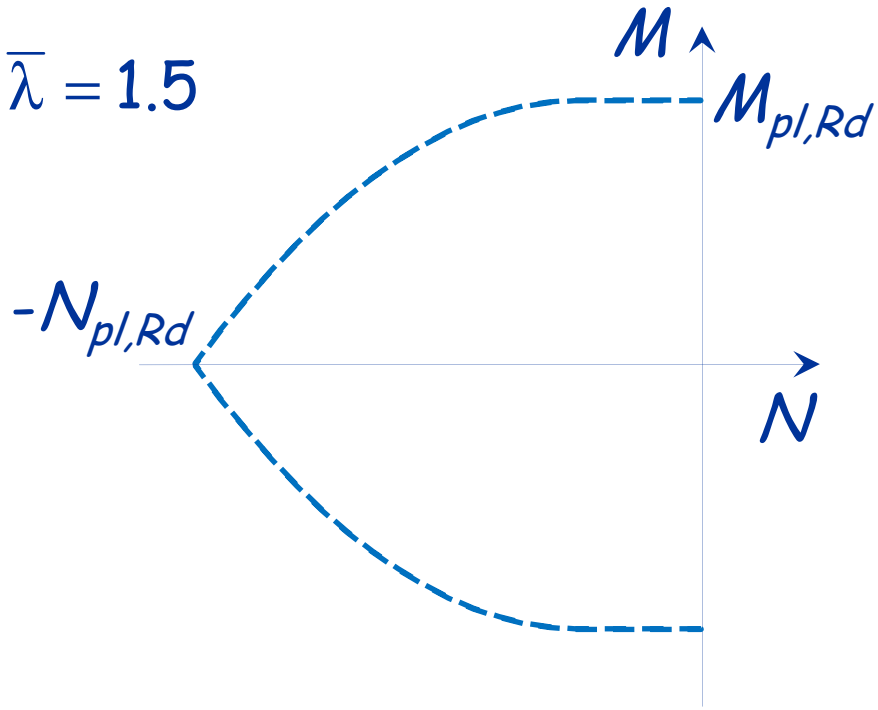
Il dominio è simmetrico per aste con sezione trasversale simmetrica



Influenza del diagramma del momento



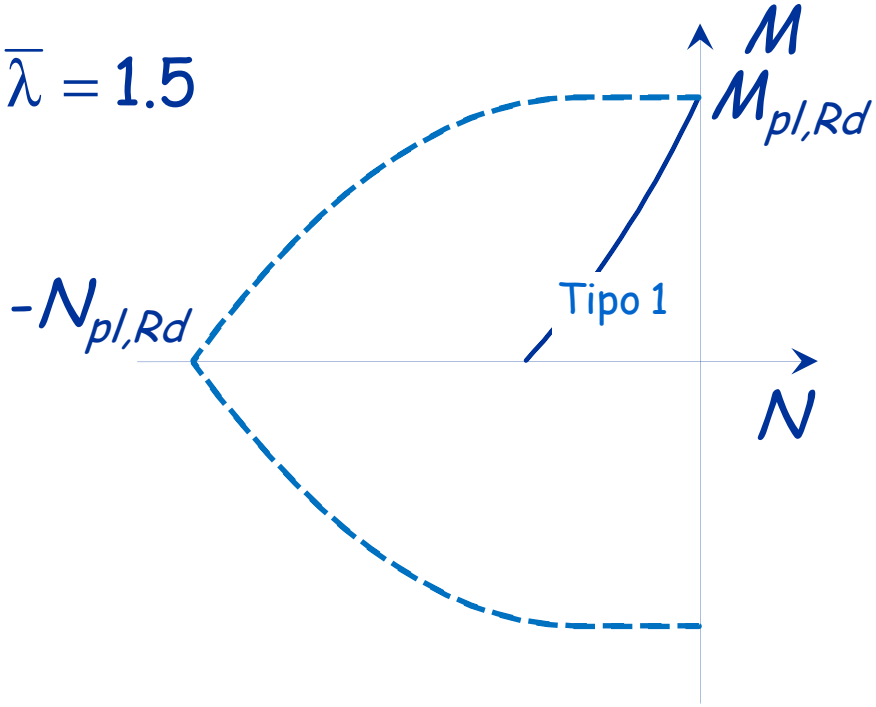
Distribuzione di momenti tipo 1



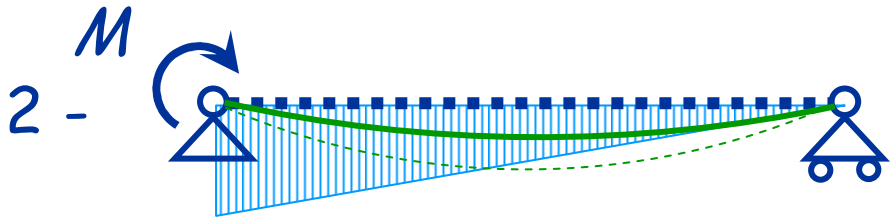
Influenza del diagramma del momento

Distribuzione di momenti tipo 1

$\bar{\lambda} = 1.5$

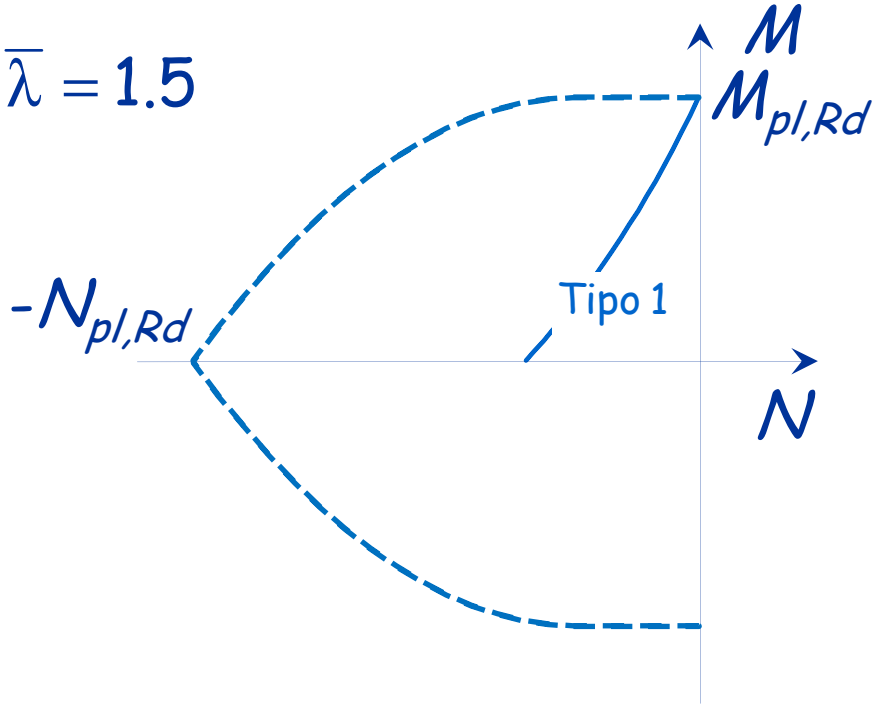


Influenza del diagramma del momento

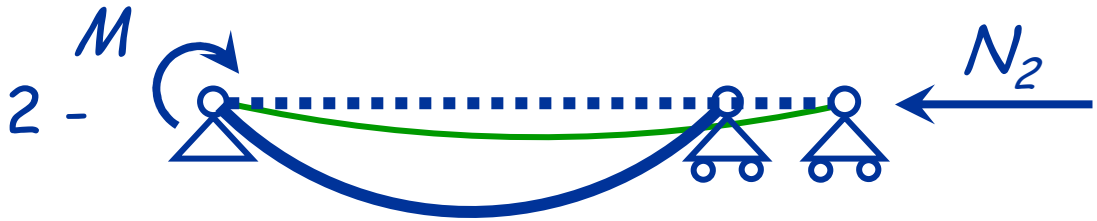


Distribuzione di momenti tipo 2

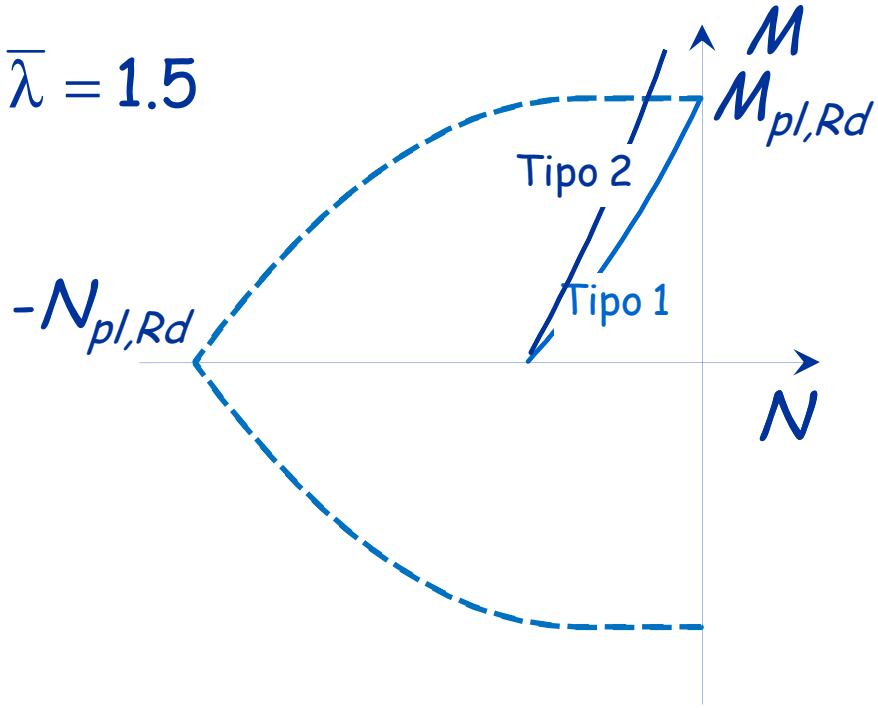
$\bar{\lambda} = 1.5$

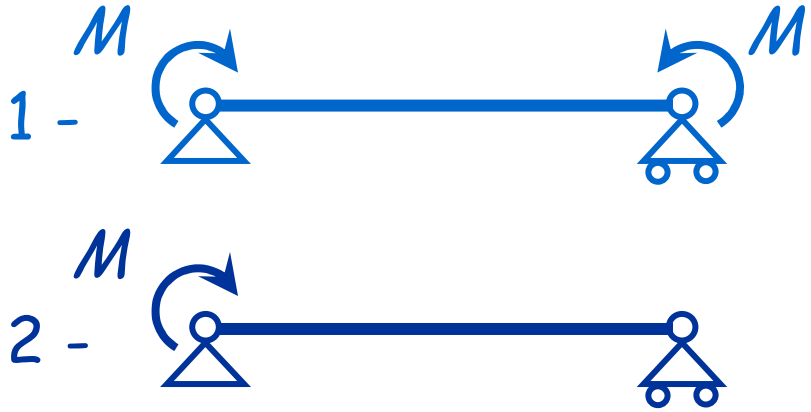


Influenza del diagramma del momento

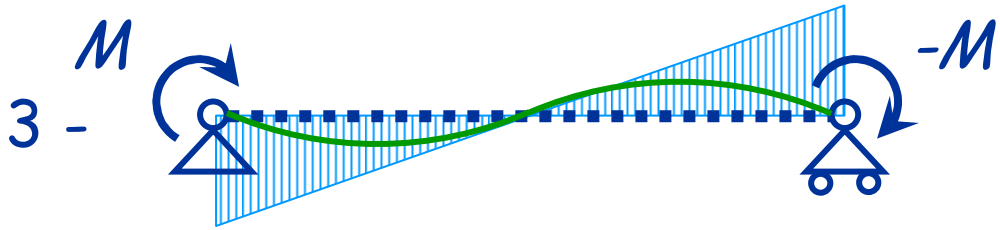


Distribuzione di momenti tipo 2

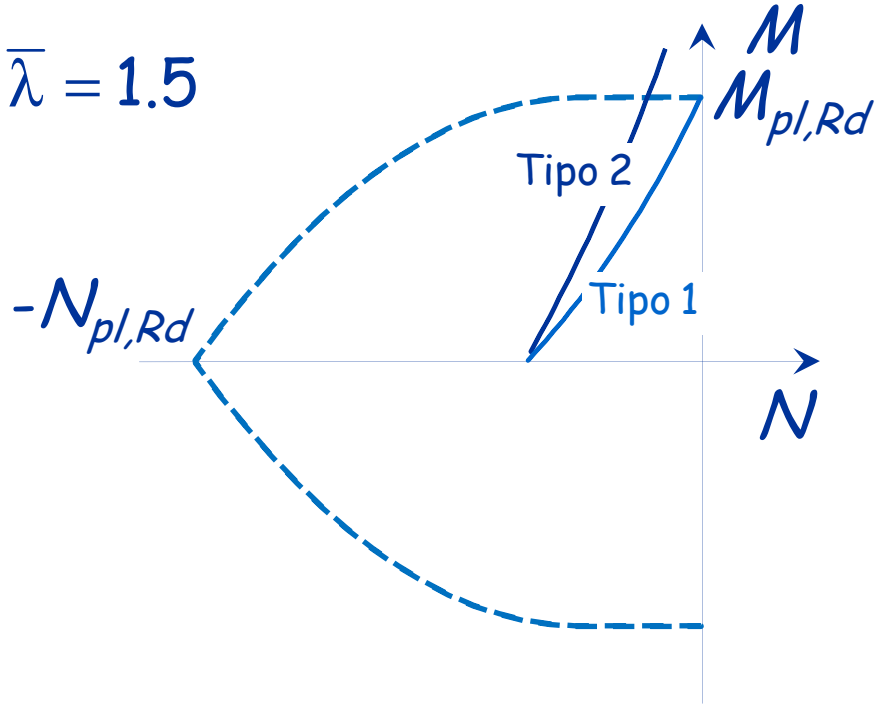


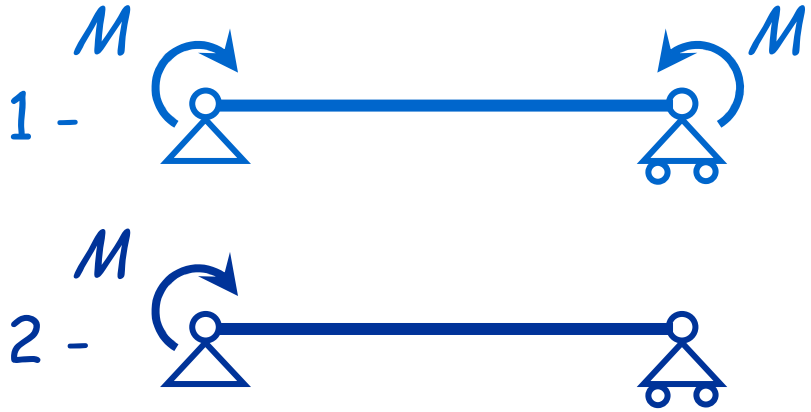


Influenza del diagramma del momento

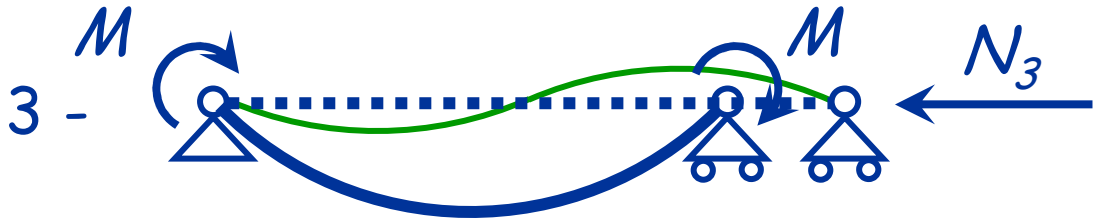


Distribuzione di momenti tipo 3

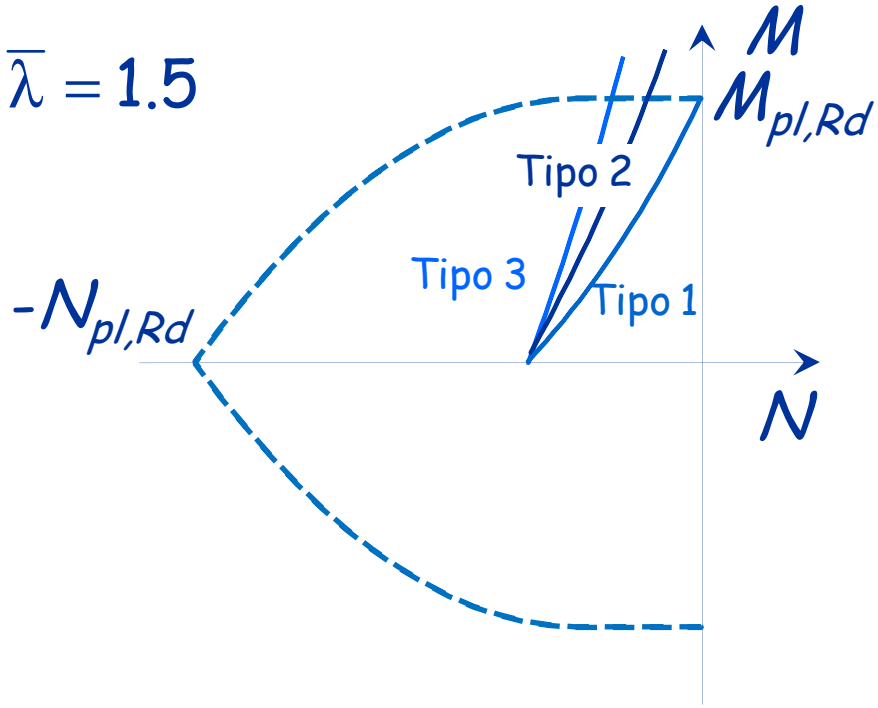


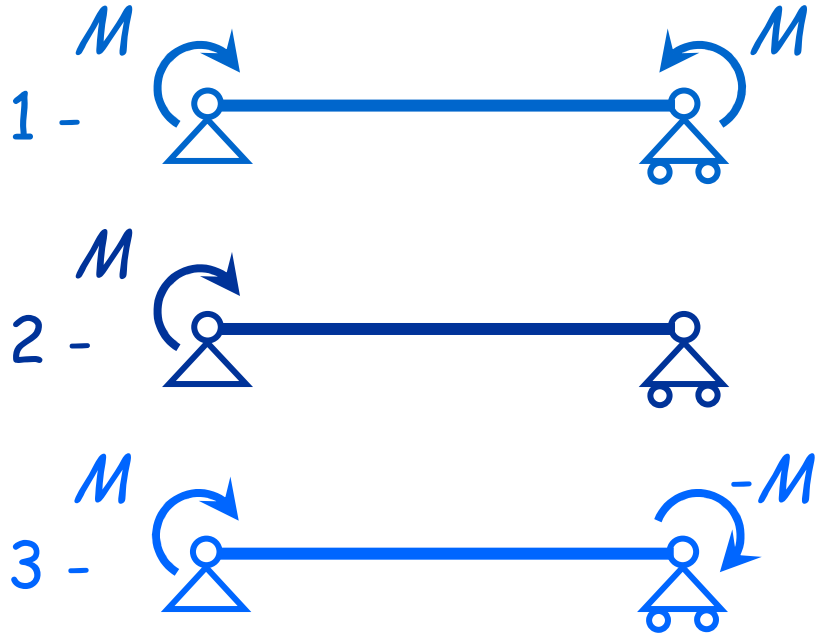


Influenza del diagramma del momento



Distribuzione di momenti tipo 3

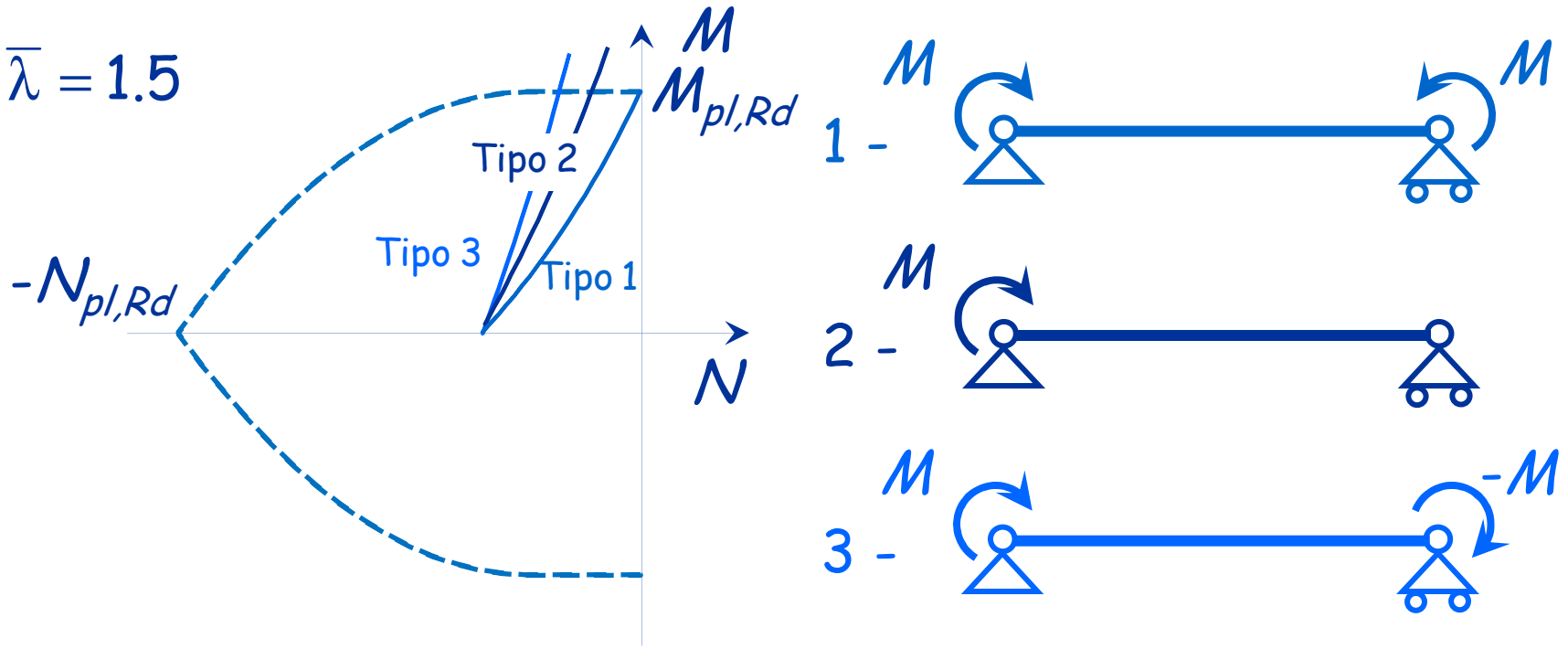




Influenza del diagramma del momento

Il dominio dipende dall'andamento del diagramma del momento flettente:

- L'ampiezza del dominio aumenta passando dal diagramma di tipo 1 a quello di tipo 3.



Verifica - stato limite ultimo

- E' possibile usare un approccio semplificato.
  Se non vi è rischio di instabilità flesso-torsionale, occorre controllare che:

$$\frac{N_{Ed} \cdot \gamma_{M1}}{\chi_{min} \cdot f_{yk} \cdot A} + \frac{M_{x,eq,Ed} \cdot \gamma_{M1}}{f_{yk} \cdot W_x \cdot \left(1 - \frac{N_{Ed}}{N_{cr,x}}\right)} + \frac{M_{y,eq,Ed} \cdot \gamma_{M1}}{f_{yk} \cdot W_y \cdot \left(1 - \frac{N_{Ed}}{N_{cr,y}}\right)} \leq 1$$

o, in sostanza:

$$\frac{N_{Ed}}{N_{bRd}} + \frac{M_{x,eq,Ed}}{M_{x,Rd}\left(1 - \frac{N_{Ed}}{N_{cr,x}}\right)} + \frac{M_{y,eq,Ed}}{M_{y,Rd}\left(1 - \frac{N_{Ed}}{N_{cr,y}}\right)} \leq 1$$

N_{cr} è il carico critico Euleriano

Circolare, punto 4.2.4.1.3.3.1 - metodo A

Verifica - stato limite ultimo

- Il momento equivalente $M_{eq,Ed}$ tiene conto della variazione del momento nell'asta e può essere preso pari a

$$M_{eq,Ed} = 1.3\ M_{m,Ed}$$

assumendo comunque $0.75\ M_{max,Ed} \leq M_{eq,Ed} \leq M_{max,Ed}$

$M_{m,Ed}$ è il valore medio del momento nell'asta

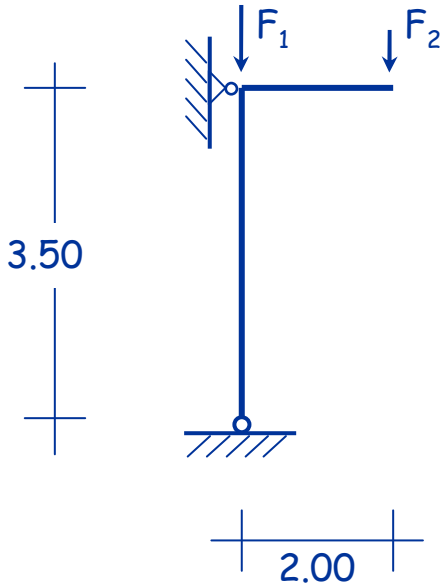
- Per asta vincolata agli estremi con momento variabile linearmente si può assumere

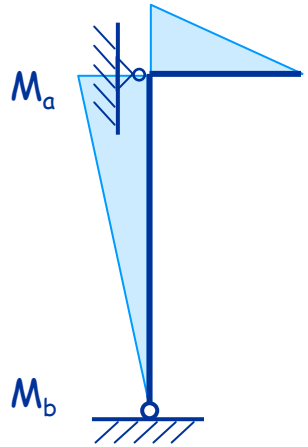
$$M_{eq,Ed} = 0.6\ M_a - 0.4\ M_b$$

assumendo comunque $M_{eq,Ed} \geq 0.4\ M_a$

M_a è il massimo tra i due ed il segno si riferisce al verso della coppia M (se M_a=-M_b il diagramma di M è costante e $M_{eq,Ed}$=M_a)

Esempio





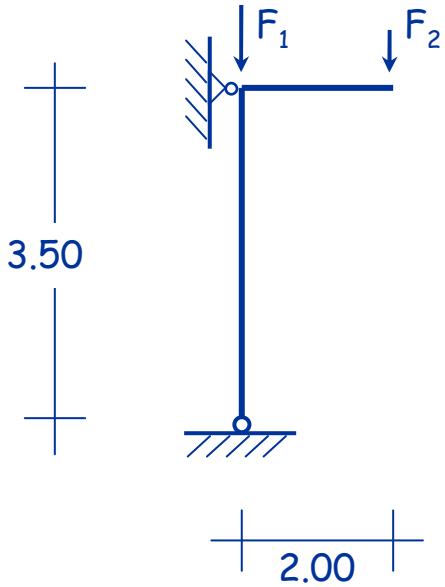
F_1 = 800 kN
F_2 = 60 kN

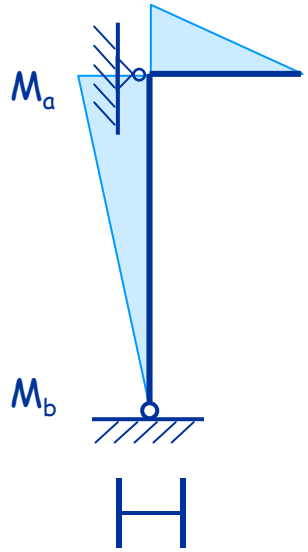
M_a = 120 kNm
M_b = 0 kNm

N_{Ed}= 860 kN

$$M_{eq,Ed} = 0.6 \times 120 - 0.4 \times 0 = 72 \text{ kNm}$$

Esempio





Sezione	HEB300
A	149 cm^2
W_{pl}	1868 cm^3
Acciaio	S235

$M_{pl,Rd}$ = 418.1 kNm

N_{bRd} = 2826.8 kN

⇩

instabilità intorno a y

l_0 = 3.50 m

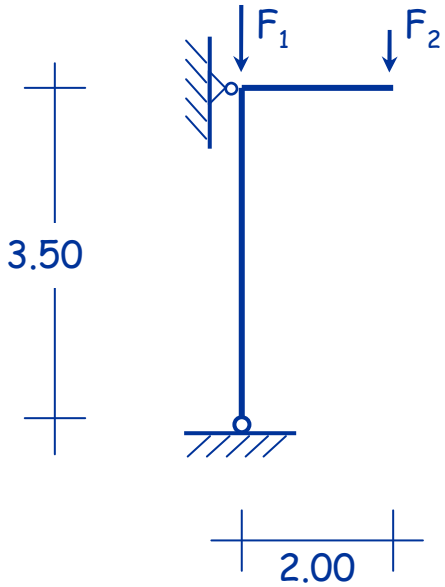
$$N_{cr} = \frac{\pi^2 E I_x}{l_0^2}$$

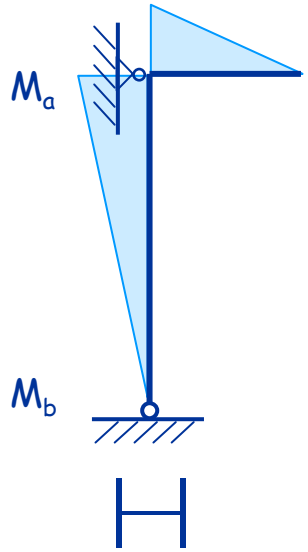
⇩

instabilità intorno a x

N_{cr} = 42579 kN

Esempio





Sezione	HEB300
A	149 cm^2
W_{pl}	1868 cm^3
Acciaio	S235

$M_{pl,Rd}$ = 418.1 kNm

N_{bRd} = 2826.8 kN

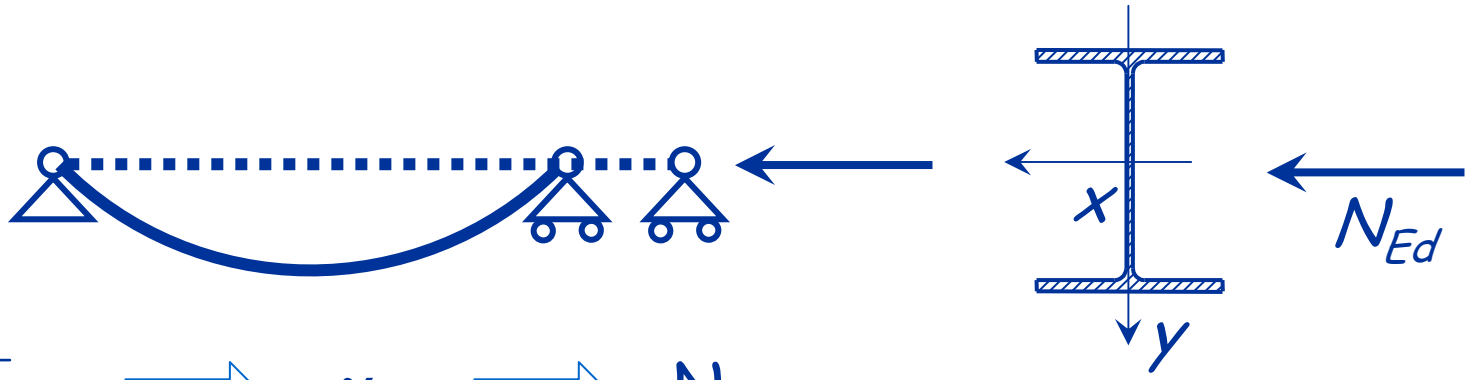
$N_{bRd,x}$ = 3231.2 kN

$$\frac{N_{Ed}}{N_{bRd}} + \frac{M_{x,eq,Ed}}{M_{x,Rd}\left(1 - \frac{N_{Ed}}{N_{cr,x}}\right)} = \frac{860}{2826.8} + \frac{72}{418.1\left(1 - \frac{860}{42579}\right)} =$$

$$= 0.304 + 0.176 = 0.480 \leq 1$$

Considerazioni su domini di resistenza

Un'asta in acciaio soggetta a compressione si instabilizza nel piano con minor resistenza



$$\bar{\lambda}_x \Rightarrow \chi_x \Rightarrow N_{b,Rd,x}$$

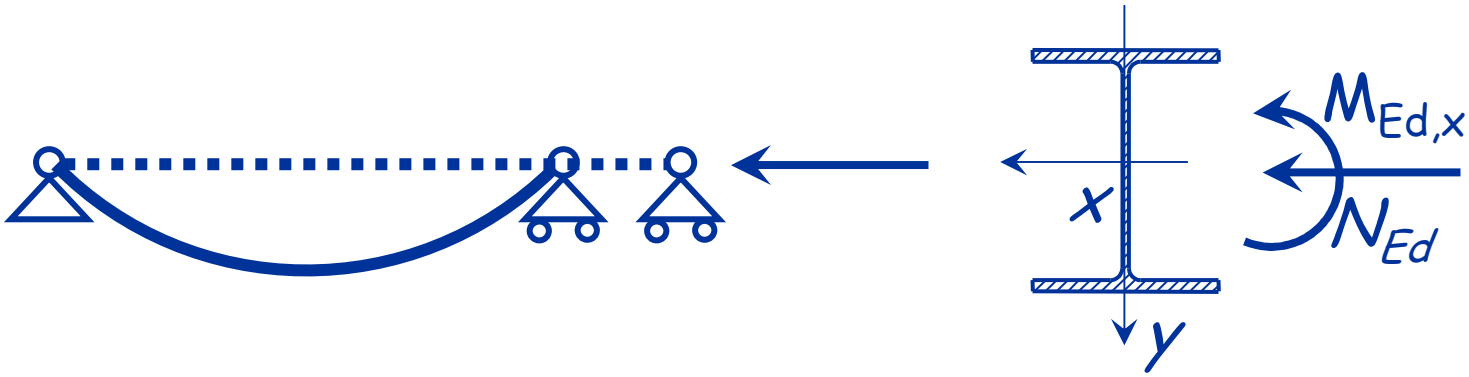
$$\bar{\lambda}_y \Rightarrow \chi_y \Rightarrow N_{b,Rd,y}$$

$$\text{Se } \bar{\lambda}_y > \bar{\lambda}_x \Rightarrow N_{b,Rd,y} < N_{b,Rd,x} \Rightarrow N_{b,Rd} = N_{b,Rd,y}$$

L'instabilità si verifica nel piano ortogonale all'asse y

Considerazioni su domini di resistenza

Un'asta in acciaio soggetta a compressione si instabilizza nel piano con minor resistenza



$$\bar{\lambda}_y > \bar{\lambda}_x \quad \Rightarrow \quad N_{b,Rd,y} < N_{b,Rd,x} \quad \Rightarrow \quad N_{b,Rd} = N_{b,Rd,y}$$

L'instabilità si verifica nel piano ortogonale all'asse y

La presenza di un momento M_{Edx} favorisce l'instabilità dell'asta quanto un momento M_{Edy}? Probabilmente NO

Verifica - stato limite ultimo

- In alternativa, occorre controllare che:

$$\frac{N_{Ed}\ \gamma_{M1}}{\chi_x\ A\ f_{yk}} + k_{xx}\frac{M_{Ed,x}\ \gamma_{M1}}{W_x\ f_{yk}} + k_{xy}\frac{M_{Ed,y}\ \gamma_{M1}}{W_y\ f_{yk}} \leq 1$$

$$\frac{N_{Ed}\ \gamma_{M1}}{\chi_y\ A\ f_{yk}} + k_{yx}\frac{M_{Ed,x}\ \gamma_{M1}}{W_x\ f_{yk}} + k_{yy}\frac{M_{Ed,y}\ \gamma_{M1}}{W_y\ f_{yk}} \leq 1$$

Circolare, punto 4.2.4.1.3.3.1 - metodo B

Eurocodice 3, punto 6.3.3 e annesso B

Verifica - stato limite ultimo

- In alternativa, occorre controllare che:

$$\frac{N_{Ed}}{N_{b,Rd,x}} + k_{xx}\frac{M_{Ed,x}}{M_{Rd,x}} + k_{xy}\frac{M_{Ed,y}}{M_{Rd,y}} \leq 1$$

$$\frac{N_{Ed}}{N_{b,Rd,y}} + k_{yx}\frac{M_{Ed,x}}{M_{Rd,x}} + k_{yy}\frac{M_{Ed,y}}{M_{Rd,y}} \leq 1$$

espressioni analoghe a quella del Metodo A se

$$\frac{M_{eq,Ed}}{\left(1 - \frac{N_{Ed}}{N_{cr}}\right)} = k\, M_{Ed}$$

Circolare, punto 4.2.4.1.3.3.1 - metodo B

Eurocodice 3, punto 6.3.3 e annesso B

Verifica - stato limite ultimo

- In alternativa, occorre controllare che:

$$\frac{N_{Ed}}{N_{b,Rd,x}} + k_{xx}\frac{M_{Ed,x}}{M_{Rd,x}} + k_{xy}\frac{M_{Ed,y}}{M_{Rd,y}} \leq 1$$

$$\frac{N_{Ed}}{N_{b,Rd,y}} + k_{yx}\frac{M_{Ed,x}}{M_{Rd,x}} + k_{yy}\frac{M_{Ed,y}}{M_{Rd,y}} \leq 1$$

Si noti che:

1. N_{bRd} è calcolato separatamente nelle due direzione
2. i coefficienti k sono diversi per le due direzioni

Circolare, punto 4.2.4.1.3.3.1 - metodo B

Eurocodice 3, punto 6.3.3 e annesso B

Verifica - stato limite ultimo

- In alternativa, occorre controllare che:

$$\frac{N_{Ed}}{N_{b,Rd,x}} + k_{xx}\frac{M_{Ed,x}}{M_{Rd,x}} + k_{xy}\frac{M_{Ed,y}}{M_{Rd,y}} \leq 1$$

$$\frac{N_{Ed}}{N_{b,Rd,y}} + k_{yx}\frac{M_{Ed,x}}{M_{Rd,x}} + k_{yy}\frac{M_{Ed,y}}{M_{Rd,y}} \leq 1$$

Le espressioni per il calcolo dei k sono complesse

$k_{xx}, k_{yy}, k_{xy}, k_{yx}$ dipendono da:
- snellezza dell'asta
- diagramma del momento
- sforzo normale

Circolare, punto 4.2.4.1.3.3.1 - metodo B

Eurocodice 3, punto 6.3.3 e annesso B

Fattori d'interazione k_{xx}, k_{yy} k_{xy} k_{yx} per sezioni a doppio T di classe 1 e 2

$$k_{xx} = C_{mx}\left(1 + \left(\bar{\lambda}_x - 0.2\right)\frac{N_{Ed}}{N_{b,Rd,x}}\right) \leq C_{mx}\left(1 + 0.8\frac{N_{Ed}}{N_{b,Rd,x}}\right)$$

$$k_{xy} = 0.6\, k_{yy}$$

$$k_{yx} = 0.6\, k_{xx}$$

$$k_{yy} = C_{my}\left(1 + \left(2\bar{\lambda}_y - 0.6\right)\frac{N_{Ed}}{N_{b,Rd,y}}\right) \leq C_{my}\left(1 + 1.4\frac{N_{Ed}}{N_{b,Rd,y}}\right)$$

Per presso-flessione retta con $M_{Ed,x} \neq 0$ si assume $k_{yx} = 0$

Eurocodice 3, annesso B

Table B.1: Interaction factors k_{ij} for members not susceptible to torsional deformations

Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{xx}	I-sections RHS-sections	$C_{mx}\left(1+0{,}6\bar{\lambda}_x \frac{N_{Ed}}{\chi_x N_{Rk}/\gamma_{M1}}\right) \le C_{mx}\left(1+0{,}6\frac{N_{Ed}}{\chi_x N_{Rk}/\gamma_{M1}}\right)$	$C_{mx}\left(1+\left(\bar{\lambda}_x-0{,}2\right)\frac{N_{Ed}}{\chi_x N_{Rk}/\gamma_{M1}}\right) \le C_{mx}\left(1+0{,}8\frac{N_{Ed}}{\chi_x N_{Rk}/\gamma_{M1}}\right)$
k_{xy}	I-sections RHS-sections	k_{yy}	$0{,}6\ k_{yy}$
k_{yx}	I-sections RHS-sections	$0{,}8\ k_{xx}$	$0{,}6\ k_{xx}$
k_{yy}	I-sections	$C_{my}\left(1+0{,}6\bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+0{,}6\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+\left(2\bar{\lambda}_y-0{,}6\right)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+1{,}4\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
	RHS-sections		$C_{my}\left(1+\left(\bar{\lambda}_y-0{,}2\right)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+0{,}8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{x,Ed}$ the coefficient k_{yx} may be $k_{yx} = 0$.			

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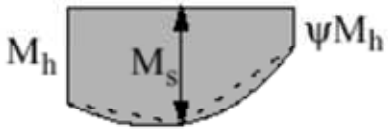
k_{xx}	I-sections RHS-sections	$C_{mx}\left(1+0{,}6\bar{\lambda}_x \frac{N_{Ed}}{\chi_x N_{Rk}/\gamma_{M1}}\right) \le C_{mx}\left(1+0{,}6\frac{N_{Ed}}{\chi_x N_{Rk}/\gamma_{M1}}\right)$	$C_{mx}\left(1+(\bar{\lambda}_x-0{,}2)\frac{N_{Ed}}{\chi_x N_{Rk}/\gamma_{M1}}\right) \le C_{mx}\left(1+0{,}8\frac{N_{Ed}}{\chi_x N_{Rk}/\gamma_{M1}}\right)$
k_{xy}	I-sections RHS-sections	k_{yy}	$0{,}6\ k_{yy}$
k_{yx}	I-sections RHS-sections		$0{,}6\ k_{xx}$
k_{yy}	I-sections	$C_{my}\left(1+0{,}6\bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+0{,}6\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+(\bar{\lambda}_y-0{,}6)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+1{,}4\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
	RHS-sections		$C_{my}\left(1+(\bar{\lambda}_y-0{,}2)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+0{,}8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$

For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{x,Ed}$ the coefficient k_{yx} may be $k_{yx} = 0$.

Nota: nella Circolare è indicato con α_m quello che qui (sull'EC3) è indicato con C_m

Fattori di momento uniforme equivalente C_{mx} e C_{my}

Moment diagram	range		C_{mx} and C_{my} and C_{mLT}	
			uniform loading	concentrated load
	$-1 \le \psi \le 1$		$0{,}6 + 0{,}4\psi \ge 0{,}4$	
$\alpha_s = M_s/M_h$	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0{,}2 + 0{,}8\alpha_s \ge 0{,}4$	$0{,}2 + 0{,}8\alpha_s \ge 0{,}4$
	$-1 \le \alpha_s < 0$	$0 \le \psi \le 1$	$0{,}1 - 0{,}8\alpha_s \ge 0{,}4$	$-0{,}8\alpha_s \ge 0{,}4$
		$-1 \le \psi < 0$	$0{,}1(1-\psi) - 0{,}8\alpha_s \ge 0{,}4$	$0{,}2(-\psi) - 0{,}8\alpha_s \ge 0{,}4$
 $\alpha_h = M_h/M_s$	$0 \le \alpha_h \le 1$	$-1 \le \psi \le 1$	$0{,}95 + 0{,}05\alpha_h$	$0{,}90 + 0{,}10\alpha_h$
	$-1 \le \alpha_h < 0$	$0 \le \psi \le 1$	$0{,}95 + 0{,}05\alpha_h$	$0{,}90 + 0{,}10\alpha_h$
		$-1 \le \psi < 0$	$0{,}95 + 0{,}05\alpha_h(1+2\psi)$	$0{,}90 - 0{,}10\alpha_h(1+2\psi)$

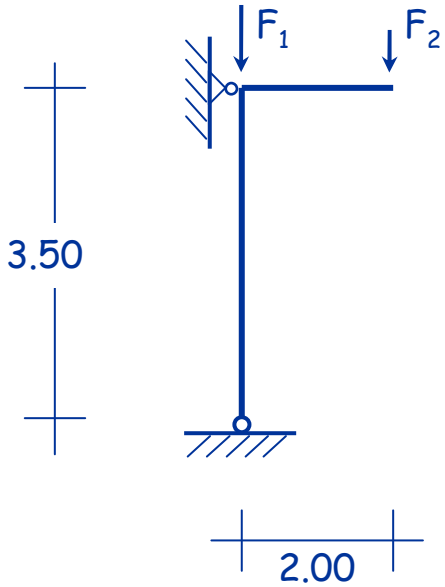
For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{mx} = 0{,}9$ or $C_{my} = 0{,}9$ respectively.

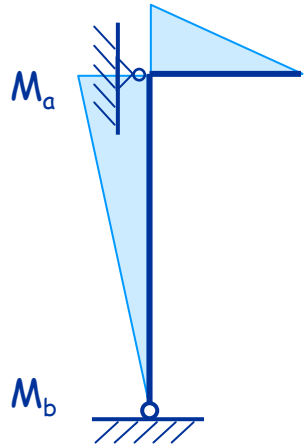
C_{mx}, C_{my} and C_{mLT} should be obtained according to the bending moment diagram between the relevant braced points as follows:

moment factor	bending axis	points braced in direction
C_{mx}	x-x	y-y
C_{my}	y-y	x-x
C_{mLT}	x-x	x-x

Eurocodice 3, annesso B

Esempio





$\psi = 0$

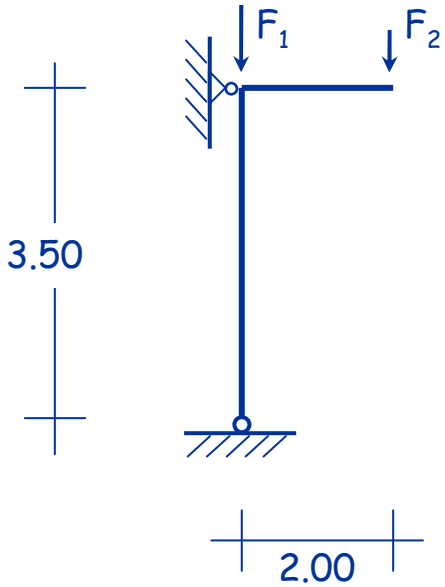
F_1 = 800 kN
F_2 = 60 kN

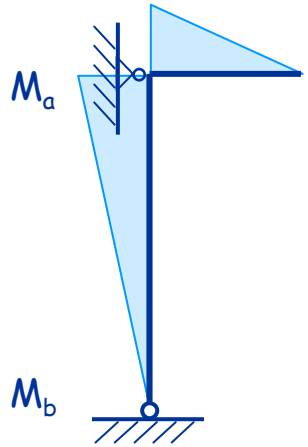
M_a = 120 kNm
M_b = 0 kNm

N_{Ed}= 860 kN

$$C_{mx} = 0.6 + 0.4\,\psi = 0.6$$

Esempio





$$\bar{\lambda}_x = 0.287$$

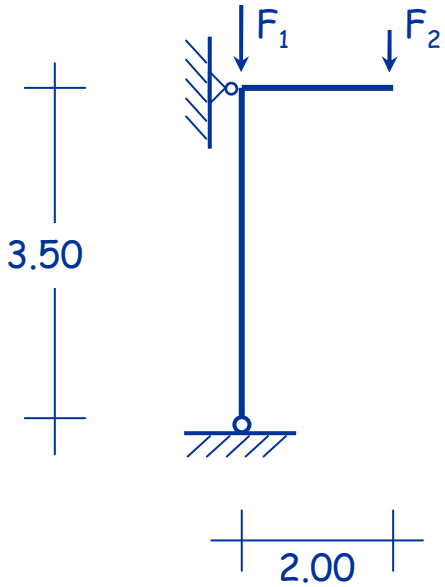
$$\chi_x = 0.9689$$

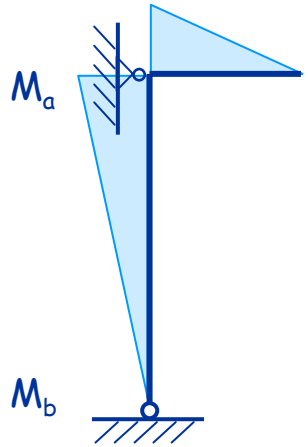
$$\bar{\lambda}_y = 0.492$$

$$\chi_y = 0.8477$$

$$k_{xx} = C_{mx}\left[1 + (\bar{\lambda}_x - 0.2)\underbrace{\frac{N_{Ed}\,\gamma_{M1}}{\chi_x\, A\, f_x}}_{N_{bRd,x}}\right] = 0.6\left[1 + (0.287 - 0.2)\frac{860}{3231.2}\right] = 0.614$$

Esempio





$$\bar{\lambda}_x = 0.287$$

$$\chi_x = 0.9689$$

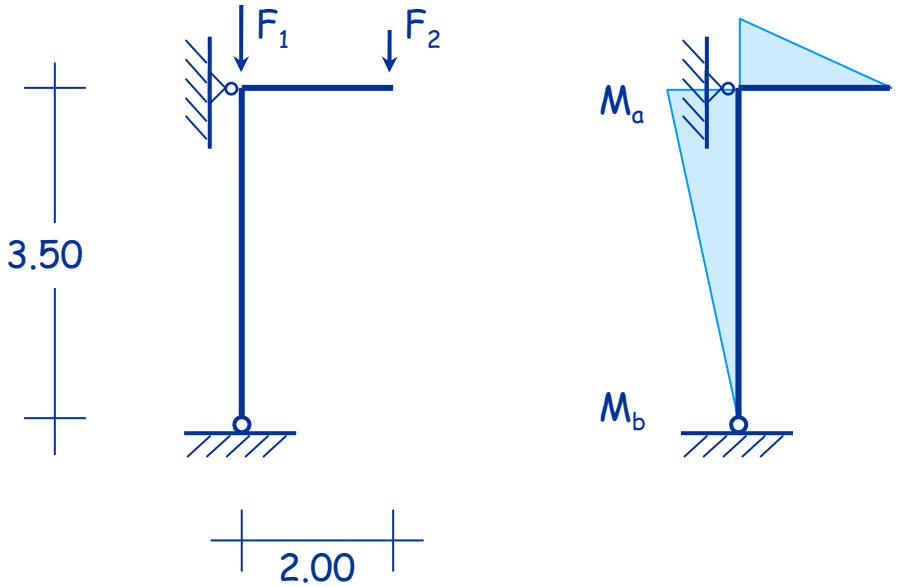
$$\bar{\lambda}_y = 0.492$$

$$\chi_y = 0.8477$$

$$k_{xx} \le C_{mx}\left[1 + 0.8\frac{N_{Ed}\,\gamma_{M1}}{\chi_x\,A\,f_y}\right] = 0.6\left[1 + 0.8\frac{860}{3231.2}\right] = 0.728$$

Ok, quindi $k_{xx} = 0.614$

Esempio



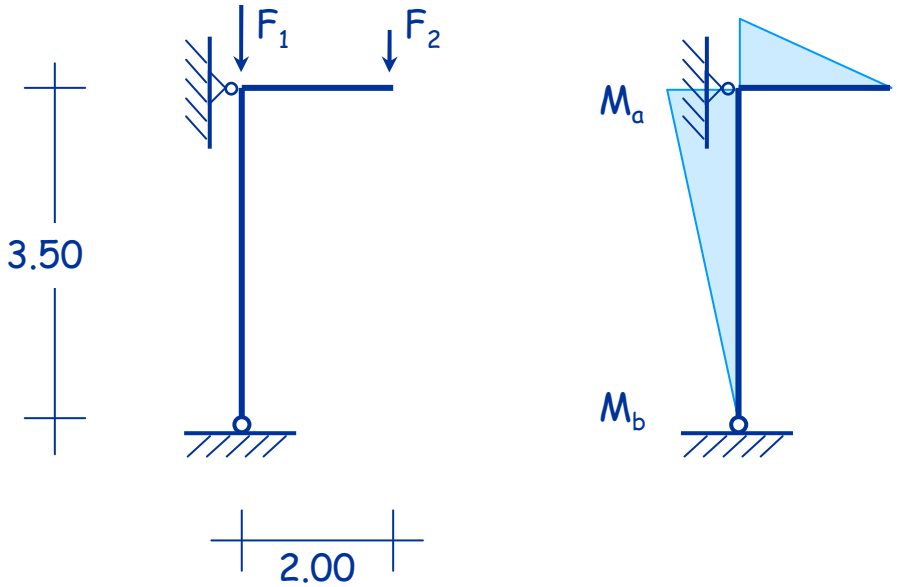
$$\frac{N_{Ed}\,\gamma_{M1}}{\chi_x\,A\,f_y}+\frac{k_{xx}\,M_{x,Ed}\,\gamma_{M1}}{W_{pl,x}\,f_y}=\frac{860}{3233.2}+\frac{0.614\times 120}{418.1}=$$

$N_{bRd,x}$ $\qquad$ $M_{pl,Rd,x}$

$$=0.266+0.176=0.442<1$$

Con il Metodo A era 0.480

Esempio

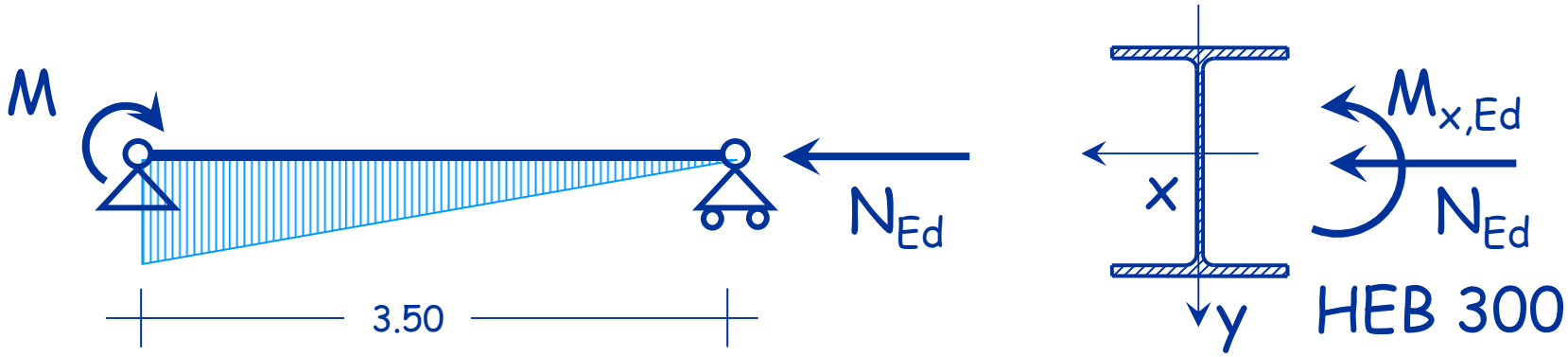


Perché $k_{yx} = 0$

$$\underbrace{\frac{N_{Ed}\,\gamma_{M1}}{\chi_y\,A\,f_y}}_{N_{bRd,y}} + \cancel{\frac{k_{yx}\,M_{x,Ed}\,\gamma_{M1}}{W_{pl,x}\,f_y}} = \frac{860}{2828.6} = 0.304$$

Verifica di stabilità con sforzo normale centrato

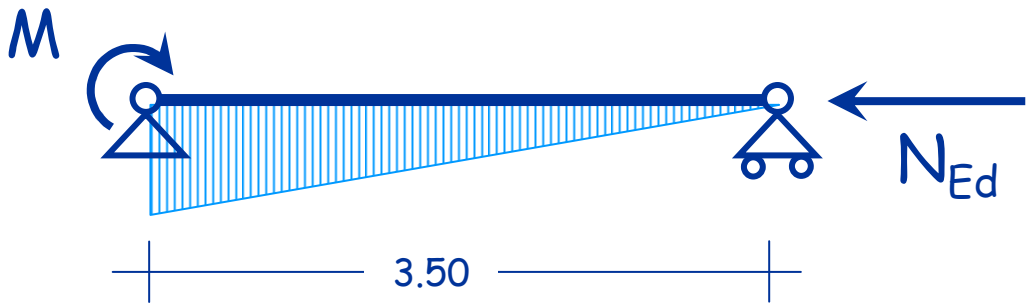
Domini di resistenza - presso-flessione retta (Metodo A)

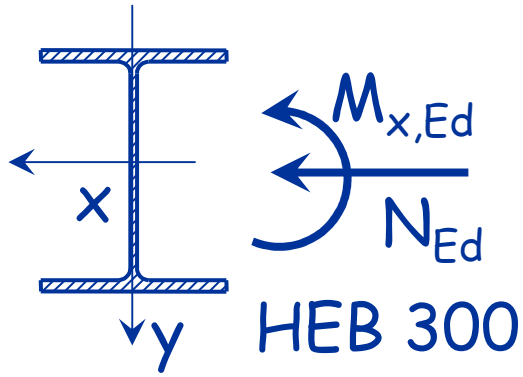


$$\frac{N_{Ed}}{N_{bRd}} + \frac{M_{x,eq,Ed}}{M_{x,Rd}\left(1 - \frac{N_{Ed}}{N_{cr,x}}\right)} \leq 1$$

$$M_{x,eq,Ed} = 0.6\, M_a - 0.4\, M_b = \overset{M_{x,Ed}}{M_a} \underbrace{\left(0.6 - 0.4 \overset{\psi}{\frac{M_b}{M_a}}\right)}_{C_{mx}}$$

Domini di resistenza - presso-flessione retta
(Metodo A)

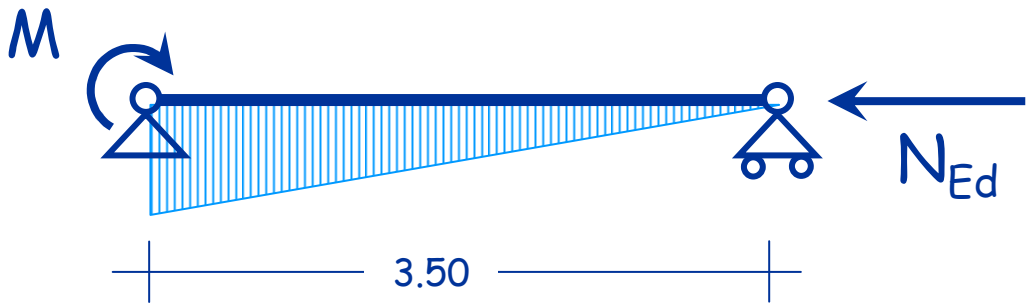


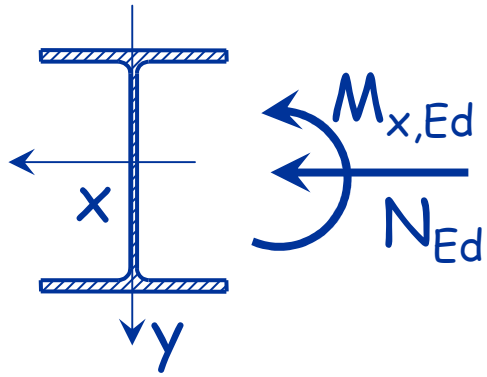


$$\frac{N_{Ed}}{N_{bRd}} + \frac{C_{mx}\, M_{x,Ed}}{M_{x,Rd}\left(1 - \frac{N_{Ed}}{N_{cr,x}}\right)} = 1$$

k^*_{xx}

Domini di resistenza - presso-flessione retta
(Metodo A)





HEB 300

$M_{pl,x,Rd}$ = 418.1 kNm

N_{bRd} = 2826.7 kN

$N_{cr,x}$ = 42579.2 kN

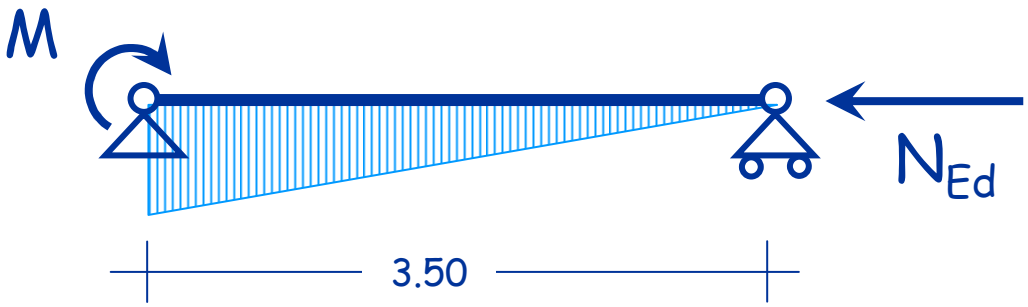
$$\frac{N_{Ed}}{N_{bRd}} + \frac{k^*_{xx}\, M_{Ed,x}}{M_{Rd,x}} \leq 1$$

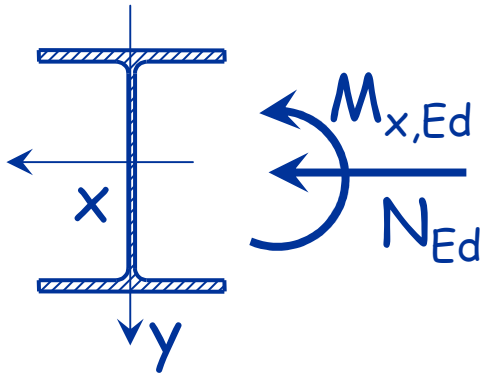
$$k^*_{xx} = \frac{C_{mx}}{1 - \dfrac{N_{Ed}}{N_{cr,x}}}$$

Nel caso in questione

$$C_{mx} = (0.6 + 0.4\,\psi) = 0.6 \qquad \psi = \frac{M_b}{M_a} = 0$$

Domini di resistenza - presso-flessione retta
(Metodo A)





HEB 300

$M_{pl,x,Rd}$ = 418.1 kNm

N_{bRd} = 2826.7 kN

$N_{cr,x}$ = 42579.2 kN

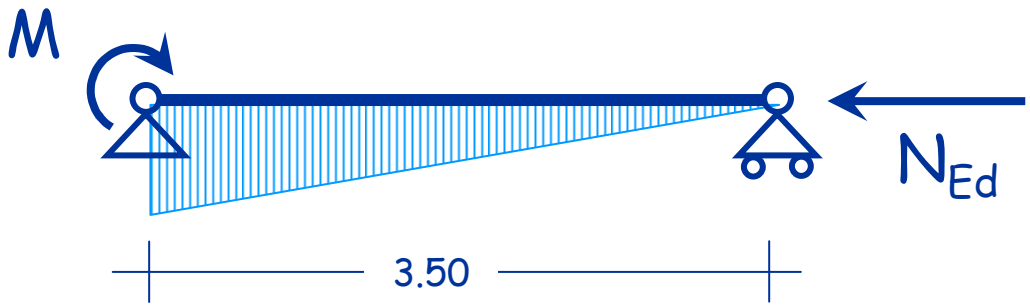
$$\frac{N_{Ed}}{N_{bRd}} + \frac{k^*_{xx}\, M_{Ed,x}}{M_{Rd,x}} \leq 1$$

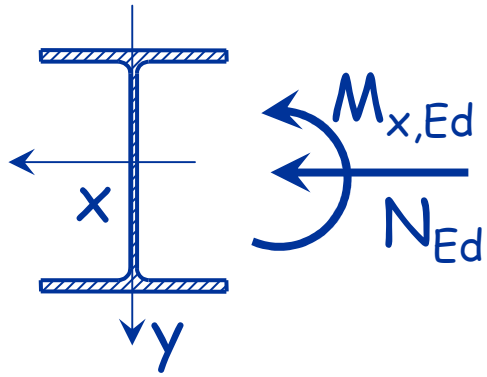
Costruisco il dominio per punti

Fisso N_{Ed} ⇨ k^*_{xx} ⇨ $M_{Ed,x} = \dfrac{M_{Rd,x}}{k^*_{xx}}\left(1 - \dfrac{N_{Ed}}{N_{bRd}}\right)$

Vedere anche foglio Excel Presso-flessione-DomX

Domini di resistenza - presso-flessione retta
(Metodo B)





HEB 300

$M_{pl,x,Rd}$ = 418.1 kNm

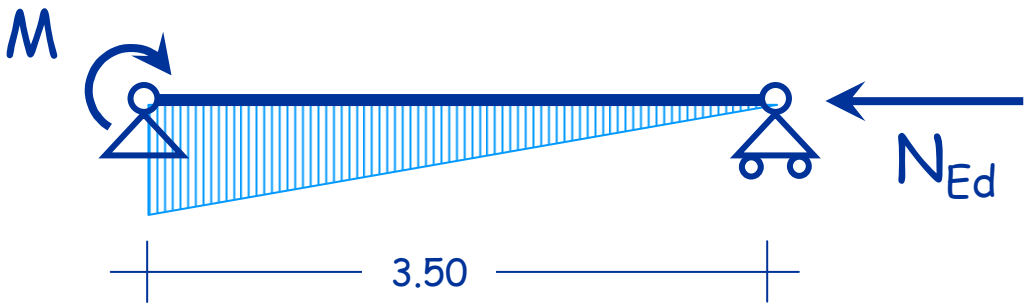
$N_{bRd,x}$ = 3231.2 kN

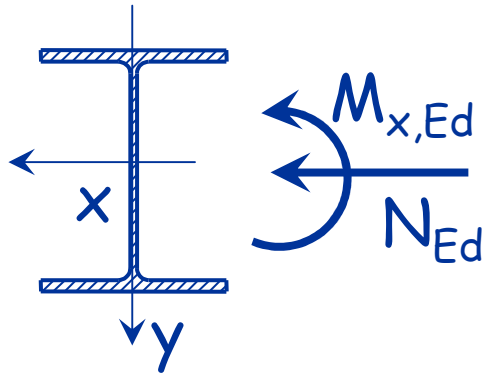
$$\frac{N_{Ed}}{N_{bRd,x}} + \frac{k_{xx}\, M_{Ed,x}}{M_{Rd,x}} \leq 1$$

k_{xx} = vedi EC3

$$C_{mx} = (0.6 + 0.4\,\psi) = 0.6 \qquad \psi = \frac{M_b}{M_a} = 0$$

Domini di resistenza - presso-flessione retta
(Metodo B)





$$\frac{N_{Ed}}{N_{bRd,x}} + \frac{k_{xx}\, M_{Ed,x}}{M_{Rd,x}} \leq 1$$

HEB 300

$M_{pl,x,Rd}$ = 418.1 kNm

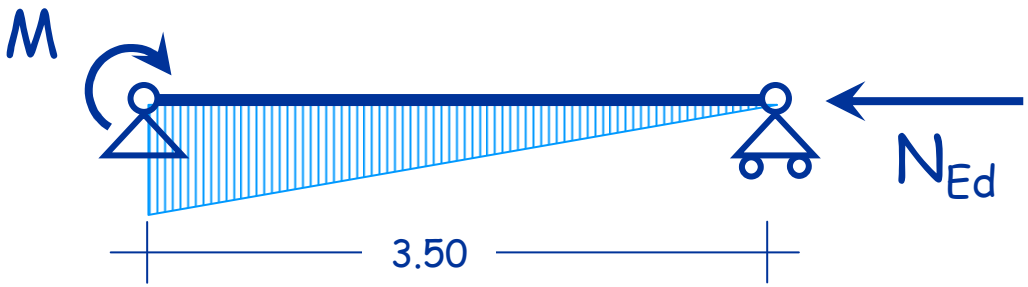
$N_{bRd,x}$ = 2961.0 kN

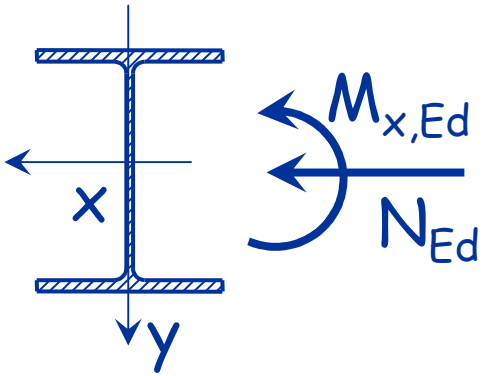
Costruisco il dominio per punti

Fisso N_{Ed} ⇨ k_{xx} ⇨ $M_{Ed,x} = \frac{M_{Rd,x}}{k_{xx}}\left(1 - \frac{N_{Ed}}{N_{bRd,x}}\right)$

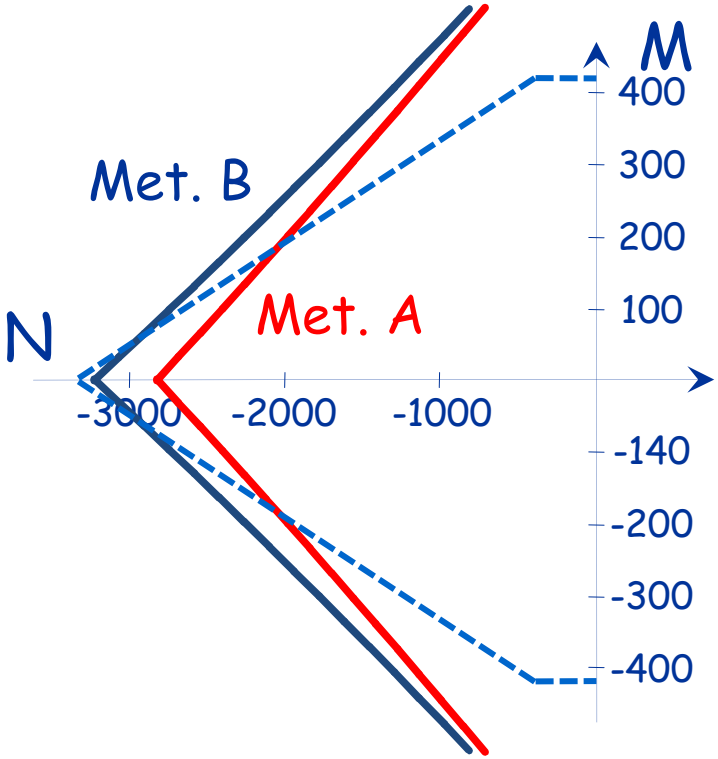
Vedere anche foglio Excel Presso-flessione-DomX

Domini di resistenza - presso-flessione retta
(Metodo A vs Metodo B)





HEB 300 - S235



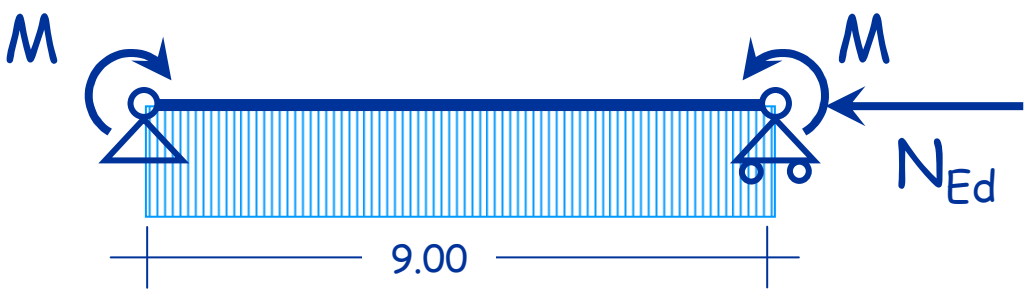
Il dominio costruito con il Metodo A è più conservativo

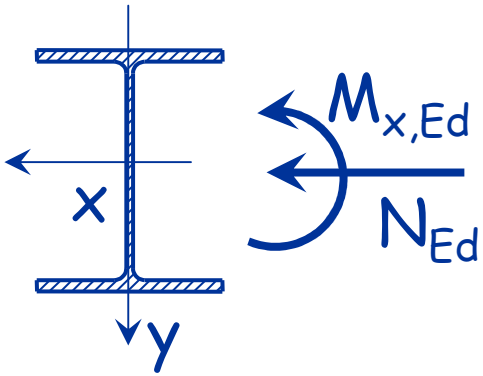
Il confronto è confermato anche cambiando:

- Diagramma Momento
- Snellezza

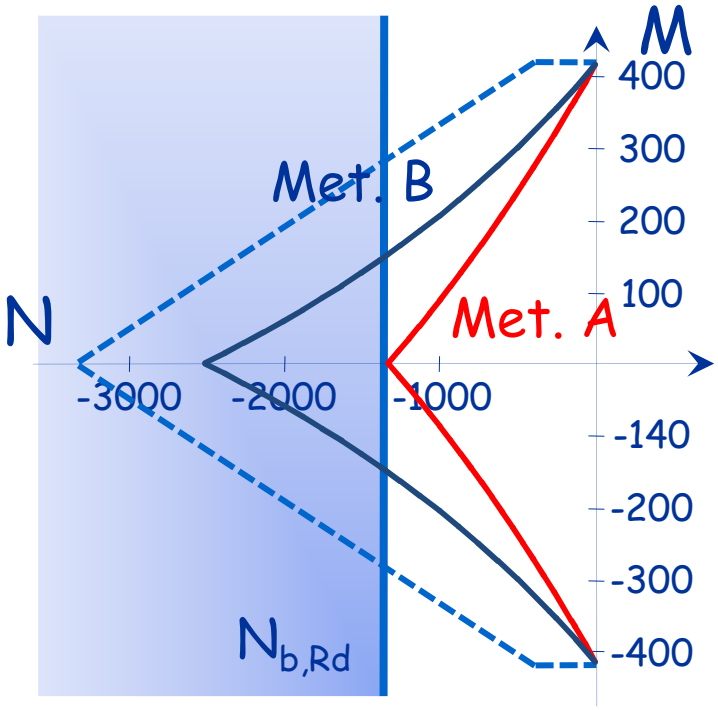
Vedere anche foglio Excel Presso-flessione-DomX

Domini di resistenza - presso-flessione retta
(Metodo A vs Metodo B)





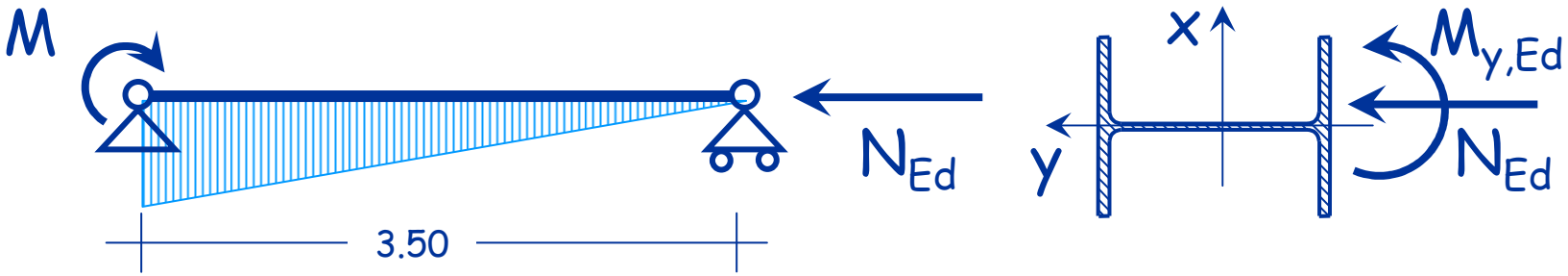
HEB 300 - S235

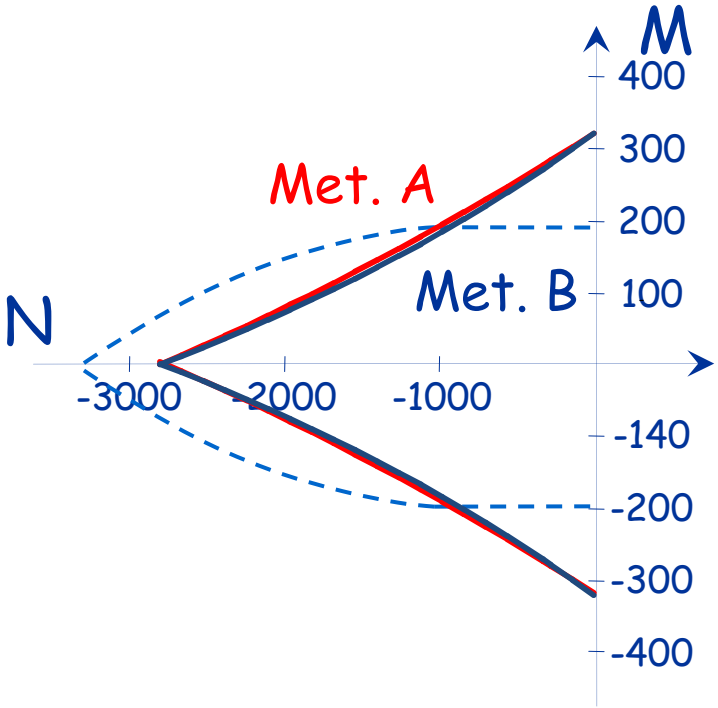


Le differenze crescono con la snellezza

Bisogna tenere conto che non si può comunque superare $N_{b,Rd}$

Domini di resistenza - presso-flessione retta
(Metodo A vs Metodo B)





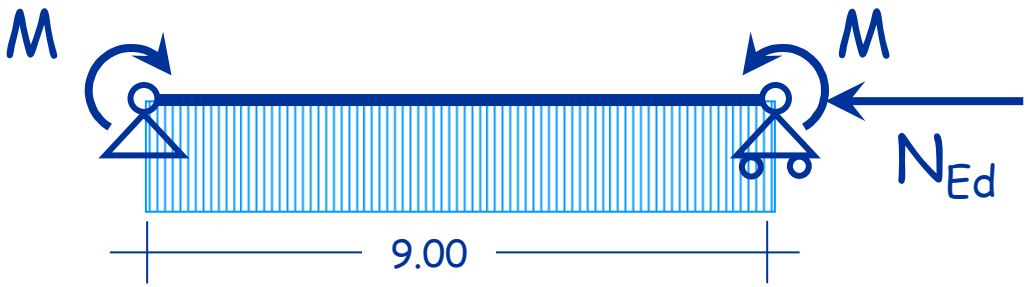
HEB 300 – S235

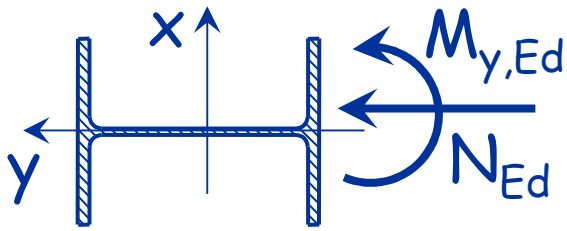
Il domini si costruiscono allo stesso modo

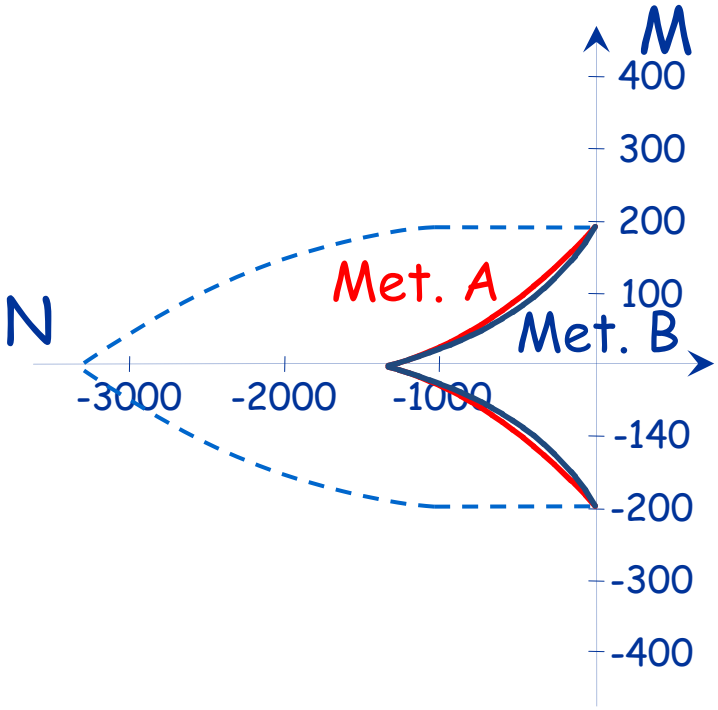
I due domini sono quasi coincidenti

Vedere anche foglio Excel Presso-flessione-DomY

Domini di resistenza - presso-flessione retta
(Metodo A vs Metodo B)







HEB 300 – S235

I due domini sono quasi coincidenti anche in altre condizioni.