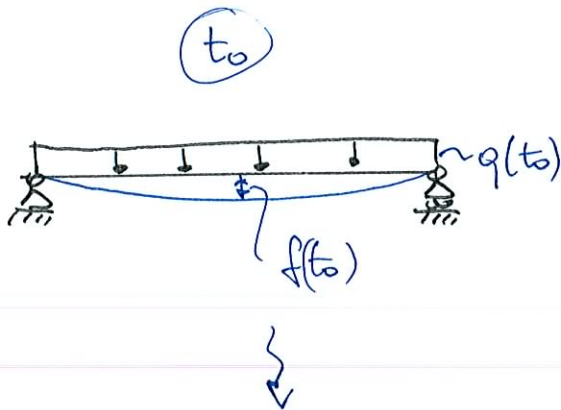
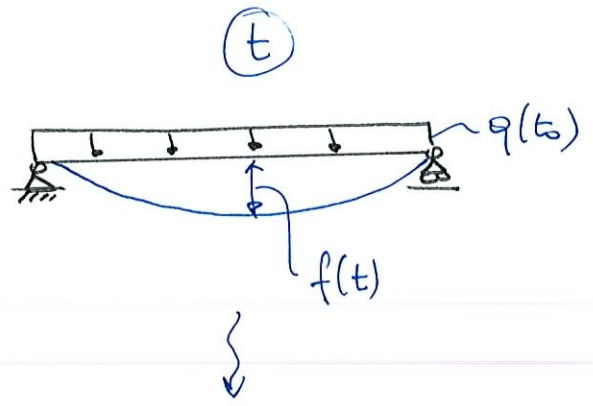


EFFETTO VISCOSITA': E.M.



$$E_c(t_0)$$



$$E_c(t) = \frac{E_c(t_0)}{1 + f(t, t_0)}$$

Effective
modulus E_{ef}

$f(t, t_0)$ = coeff. di viscosità (v. EC2)
└ istante carico

VALE IN AMBITO VISCOELASTICO LINEARE $\rightarrow \sigma_c < 0.4 f_c$

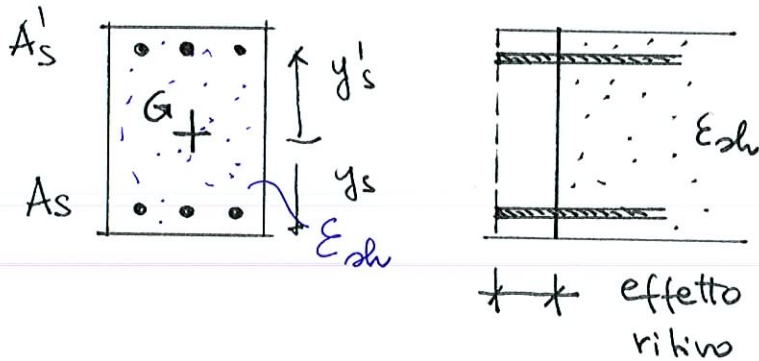
Se la sezione è interamente reagente 

$$f(t) = f(t_0) \cdot (1 + f)$$

es. $f(t_0) = \frac{5}{384} \frac{q L^4}{E(t_0) I}$

$$f(t) = \frac{5}{384} \frac{q L^4}{\frac{E(t_0)}{1+f} \cdot I} = \frac{5}{384} \frac{q L^4}{E(t_0) I} \cdot (1 + f(t, t_0))$$

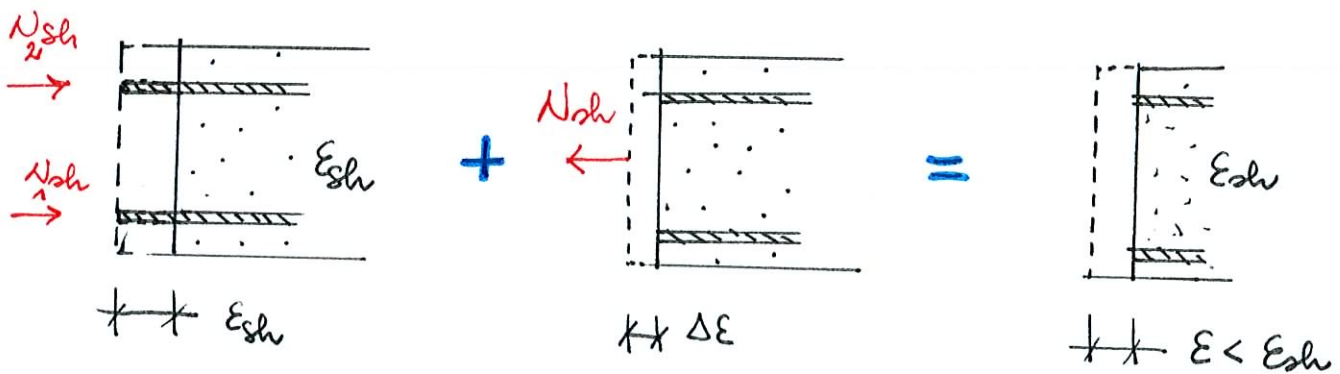
EFFETTO DEL RITIRO ϵ_{sh}



Se fosse permesso lo scorrimento tra acciaio e CLS

ma le sezioni rimangono piane

- Sezione simmetrica ($A_s = A'_s$)



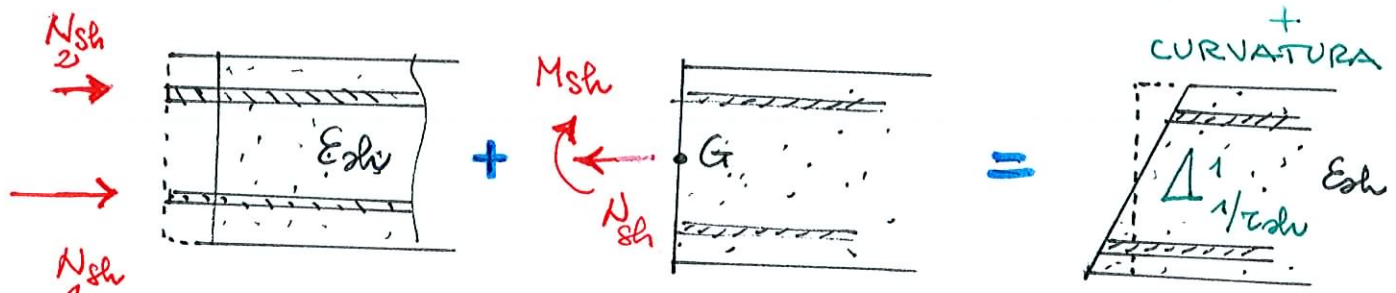
Spingo le barre all'interno della sezione

Applico per equilibrio una forza = e contraria

DEFORMAZIONE ASSIALE

$$N_{sh} = A_s \cdot \epsilon_{sh} E_s$$

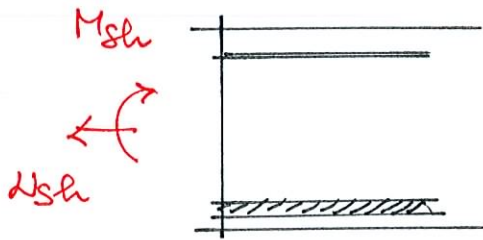
- Sezione non simmetrica ($A_s \neq A'_s$)



DEFORMAZ. ASSIALE + CURVATURA

$$M_{sh} = (E_s \epsilon_{sh} A_s) y_s - (E_s \epsilon_{sh} A'_s) y'_s = \epsilon_{sh} E_s (A_s y_s - A'_s y'_s)$$

$$N_{sh} = N_{1sh} + N_{2sh}$$



$$M_{sh} = \epsilon_{sh} E_s \underbrace{(A_s y_s - A'_s y'_s)}$$

Mom statico armature S

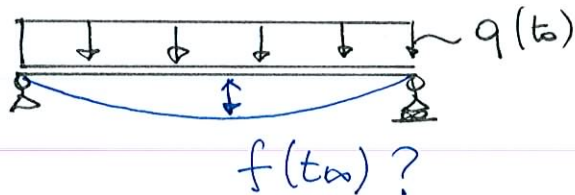
$$M_{sh} = \epsilon_{sh} E_s \cdot S$$

— Curvatura x ritmo

$$\frac{1}{\tau_{sh}} = \frac{M_{sh}}{E_c I} = \frac{\epsilon_{sh} E_s S}{E_c I} = \epsilon_{sh} \underbrace{\frac{E_s}{E_c}}_m \frac{S}{I}$$

ESERCIZIO : CALCOLO FRECCIA LUNGO TERMINE

Con riferimento all'esercizio precedente, si calcoli la freccia a tempo t_{∞}



$t_0 = 7 \text{ gg}$ istante messa carico

Cemento 42,5N

RH = 60% umidità relativa ambientale

Imposto calcolo secondo indicazioni
EC2: 2005

— Ritiro per essiccamento ϵ_{cd}

Cemento classe $\begin{cases} R & \text{CEM 42,5R} & \text{CEM 52,5N} & 52,5R \\ N & \text{CEM 32,5R} & \text{42,5N} \\ S & \text{CEM 32,5N} \end{cases}$

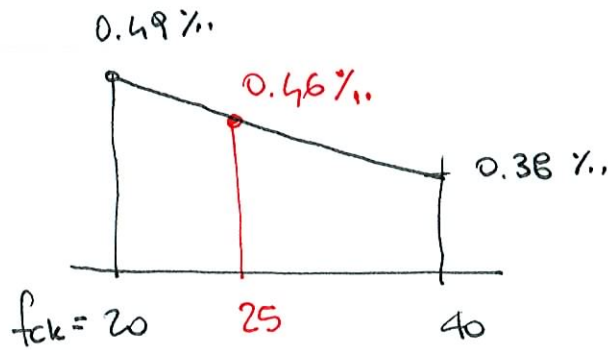
prospetto 3.2 Valori nominali del ritiro (in %) per essiccamento non contrastato $\epsilon_{cd,0}$ del calcestruzzo con cemento CEM classe N

$f_{ck}/f_{ck,cube}$ (MPa)	Umidità relativa (in %)					
	20	40	60	80	90	100
20/25	0,62	0,58	0,49	0,30	0,17	0,00
40/50	0,48	0,46	0,38	0,24	0,13	0,00
60/75	0,38	0,36	0,30	0,19	0,10	0,00
80/95	0,30	0,28	0,24	0,15	0,08	0,00
90/105	0,27	0,25	0,21	0,13	0,07	0,00

C 25/30
→

(EC2: 2005)

Per cementi classe N uso Tabella EC2, altrimenti
lo calcolo con formule riportate appendice B EC2.



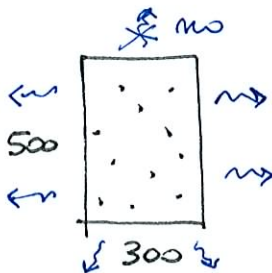
$$\epsilon_{cd} = \cancel{\beta_{ds}(t, t_s)} \cdot \cancel{k_R} \cdot \epsilon_{cd,0}$$

$$\beta_{ds}(t, t_s) = \frac{t - t_s}{(t - t_s) + 0,04 \sqrt[3]{h_o}}$$

$$\lim_{t \rightarrow \infty} \beta_{ds} = 1$$

$$h_o = \text{dimensione convenzionale} = \frac{2 A_c}{u}$$

perimetro
esposto
aria



$$A_c = 300 \times 500 \text{ mm}^2$$

$$u = 500 + 500 + 300 = 1300 \text{ mm}$$

$$h_o = 2 \frac{A_c}{u} = 230,8 \text{ mm}$$

prospetto 3.3 Valori di k_R nell'espressione (3.9)

230,8
→

h_o	k_R
100	1,0
200	0,85
300	0,75
≥500	0,70

→ $k_R =$
0,819

$$\epsilon_{cd} = \cancel{\beta_{ds}} \cdot \cancel{k_R} \cdot \epsilon_{cd,0} = 0,38 \text{ ‰}$$

— Ritiro autogeno ϵ_{ca}

$$\epsilon_{ca}(t) = \beta_{es}(t) \epsilon_{ca}(\infty)$$

$$\lim_{t \rightarrow \infty} \beta_{es}(t) = 1$$

$$\begin{aligned} \epsilon_{ca\infty} &= 2.5 (f_{ck} - 10) \cdot 10^{-6} = 2.5 (25 - 10) \cdot 10^{-6} = \\ &= 0.0375 \text{‰} \end{aligned}$$

— Deformazione totale da ritiro ϵ_{cs}

$$\epsilon_{cs} = \epsilon_{cd} + \epsilon_{ca} = 0.42 \text{‰}$$

— Coefficiente di viscosità $f(\infty, t_0)$

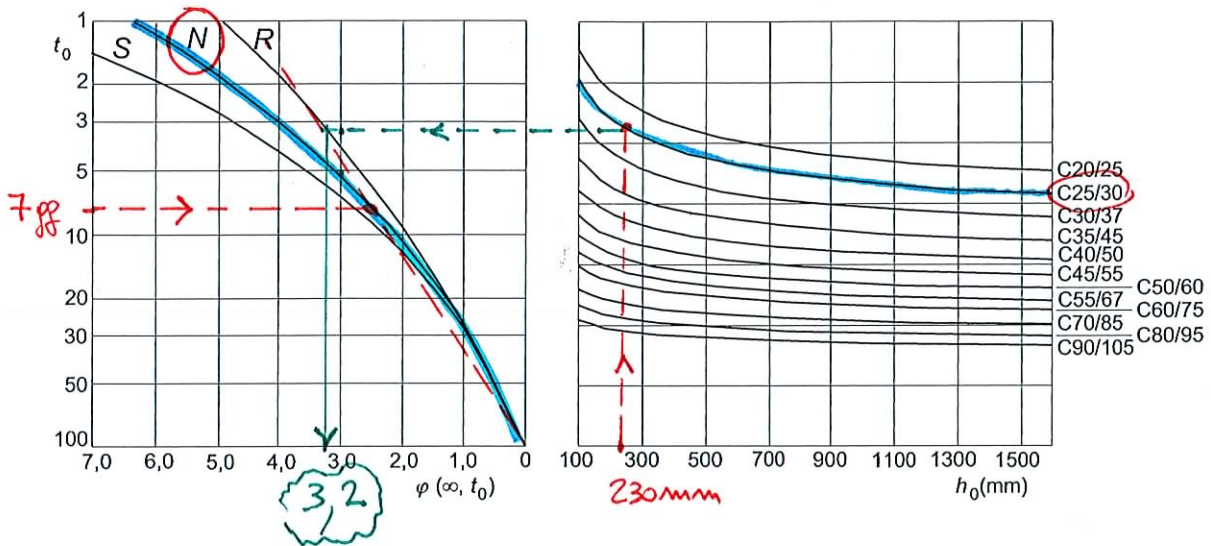
le espressioni sono riportate nell'Appendice B dell'EC2: 2005. Per $RH = 50\%$ ed $RH = 80\%$ sono stati prodotti diagrammi

figura 3.1 Metodo per determinare il coefficiente di viscosità $\varphi(\infty, t_0)$ di calcestruzzi in condizioni ambientali normali

Legenda

- a) Condizioni interne - $RH = 50\%$
- b) Condizioni esterne - $RH = 80\%$

$RH = 50\%$



Usando $RH 50\%$ anziché $RH 60\%$ sovrastimo la viscosità

Svolgendo i calcoli con $RH 60\%$ (v. Appendice B) si ottiene

$$RH 60\% \quad f(\infty, t_0) = 3,$$

$$RH 80\% \quad f(\infty, t_0) = 3,36$$

I valori sono molto "incerti"

$$f = 3,1$$

Scelgo $f = 3,1$

— Calcolo Modulo elastico efficace $E_{c,eff}$

$$E_{c,eff} = \frac{E_c(t_0)}{1 + \rho(a, t_0)} = \frac{31500}{1 + 3,1} = \frac{7677}{7} \cdot \frac{N}{mm^2}$$

— Curvatura impressa per ritiro $\frac{1}{r_{cs}}$

$$\frac{1}{r_{cs}} = \epsilon_{cs} \cdot de \cdot \frac{S}{I}$$

$$\epsilon_{cs} = 0,42\%$$

$$de = E_s / E_{c,eff} = \frac{200000}{7677} = 26,05 !!$$

$$\frac{S}{I} = \begin{cases} S_1 / I_1 = 2,87 \cdot 10^{-5} 1/mm & \text{non fessurata} \\ S_2 / I_2 = 7,96 \cdot 10^{-5} 1/mm & \text{stadio 2} \end{cases}$$

$$S_1 = A_s (y_{G1} - c) - A'_s (h - y_{G1} - c)$$

$$S_2 = A_s (h - x - c) - A'_s (x - c)$$

CALCOLO FRECCIA TRAVE APPOGGIATA

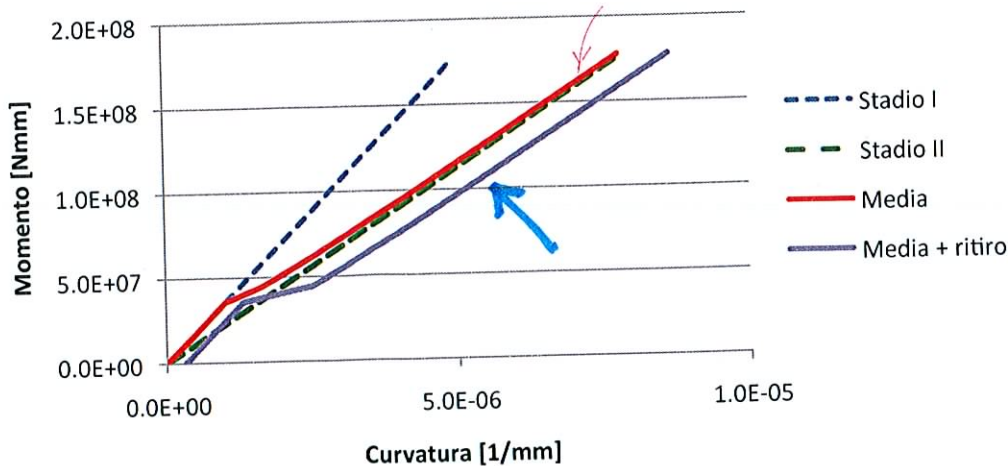
Secondo EC2:2005

DATI

I1 = 4.64E+09 momento inerzia stadio 1 [mm^4]
 I2 = 2.94E+09 momento inerzia stadio 2 [mm^4]
 Mcr = 4.37E+07 momento di fessurazione [Nmm]
 Mrd = 1.78E+08 momento ultimo [Nmm]
 Ec = 7.68E+03 modulo Young [N/mm^2]
 β = 0.50
 S1/I1 = 2.87E-05 [1/mm] S = mom statico armature rispetto baricentro sezione reagente
 S2/I2 = 7.96E-05 [1/mm] I = momento d' inerzia della sezione reagente
 εCS = 4.20E-04 [-]

i	M	1/r1	1/r2	ξ	1/rm	1/rcs	1/r
[-]	[Nmm]	[1/mm]	[1/mm]	[-]	[1/mm]	[1/mm]	[1/mm]
0	0.00E+00	0.00E+00	0.00E+00		0	0.00E+00	3.14028E-07
1	8.88E+06	2.49E-07	3.94E-07		0	2.49E-07	3.14028E-07
2	1.78E+07	4.99E-07	7.88E-07		0	4.99E-07	3.14028E-07
3	2.66E+07	7.48E-07	1.18E-06		0	7.48E-07	3.14028E-07
4	3.55E+07	9.97E-07	1.58E-06		0	9.97E-07	3.14028E-07
5	4.44E+07	1.25E-06	1.97E-06	0.5160847	1.62E-06	8.70962E-07	2.49E-06
6	5.33E+07	1.50E-06	2.36E-06	0.6639477	2.07E-06	8.70962E-07	2.94E-06
7	6.22E+07	1.75E-06	2.76E-06	0.7531045	2.51E-06	8.70962E-07	3.38E-06
8	7.10E+07	1.99E-06	3.15E-06	0.8109706	2.93E-06	8.70962E-07	3.80E-06
9	7.99E+07	2.24E-06	3.55E-06	0.8506434	3.35E-06	8.70962E-07	4.22E-06
10	8.88E+07	2.49E-06	3.94E-06	0.8790212	3.76E-06	8.70962E-07	4.64E-06
11	9.77E+07	2.74E-06	4.33E-06	0.9000175	4.17E-06	8.70962E-07	5.05E-06
12	1.07E+08	2.99E-06	4.73E-06	0.9159869	4.58E-06	8.70962E-07	5.45E-06
13	1.15E+08	3.24E-06	5.12E-06	0.9284149	4.99E-06	8.70962E-07	5.86E-06
14	1.24E+08	3.49E-06	5.52E-06	0.9382761	5.39E-06	8.70962E-07	6.26E-06
15	1.33E+08	3.74E-06	5.91E-06	0.9462316	5.79E-06	8.70962E-07	6.66E-06
16	1.42E+08	3.99E-06	6.30E-06	0.9537126	6.19E-06	8.70962E-07	7.07E-06

momento-curvatura

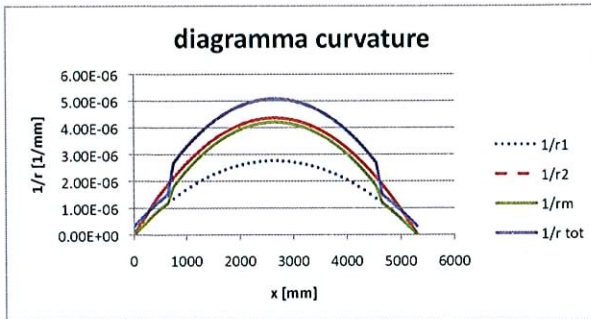
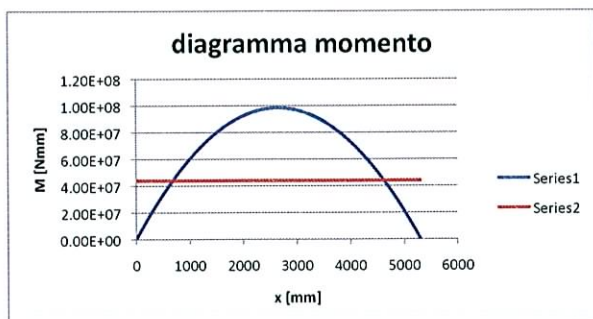


7.47E-06
 7.87E-06
 8.26E-06
 8.66E-06

q = 28 carico (N/mm)
L = 5300 luce (mm)

i	x	M	Mcr	ζ	1/r1	1/r2	1/rm	S/I	1/rcs	1/rtot	M1	M1*1/rm*Δ
[-]	[mm]	[Nmm]	[Nmm]		[1/mm]	[1/mm]	[1/mm]				[Nmm]	
1	0	0	4.37E+07	0	0.00E+00	0.00E+00	0.00E+00	0.0000287	3.14E-07	3.14E-07	0	0
2	108	7861924	4.37E+07	0	2.21E-07	3.49E-07	2.21E-07	0.0000287	3.14E-07	5.35E-07	54.08163	0.0015641
3	216	15396268	4.37E+07	0	4.32E-07	6.83E-07	4.32E-07	0.0000287	3.14E-07	7.46E-07	108.1633	0.00593
4	324	22603032	4.37E+07	0	6.35E-07	1.00E-06	6.35E-07	0.0000287	3.14E-07	9.49E-07	162.2449	0.0126902
5	433	29482216	4.37E+07	0	8.28E-07	1.31E-06	8.28E-07	0.0000287	3.14E-07	1.14E-06	216.3265	0.0216833
6	541	36033819	4.37E+07	0	1.01E-06	1.60E-06	1.01E-06	0.0000287	3.14E-07	1.33E-06	270.4082	0.0327479
7	649	42257843	4.37E+07	0	1.19E-06	1.87E-06	1.19E-06	0.0000287	3.14E-07	1.50E-06	324.4898	0.0457227
8	757	48154286	4.37E+07	0.588599	1.35E-06	2.14E-06	1.81E-06	0.0000796	8.71E-07	2.68E-06	378.5714	0.0813
9	865	53723149	4.37E+07	0.669469	1.51E-06	2.38E-06	2.09E-06	0.0000796	8.71E-07	2.97E-06	432.6531	0.1243486
10	973	58964431	4.37E+07	0.725618	1.66E-06	2.62E-06	2.35E-06	0.0000796	8.71E-07	3.22E-06	486.7347	0.1542352
11	1082	63878134	4.37E+07	0.766207	1.79E-06	2.83E-06	2.59E-06	0.0000796	8.71E-07	3.46E-06	540.8163	0.1861029
12	1190	68464257	4.37E+07	0.79648	1.92E-06	3.04E-06	2.81E-06	0.0000796	8.71E-07	3.68E-06	594.898	0.2196954
13	1298	72722799	4.37E+07	0.819618	2.04E-06	3.23E-06	3.01E-06	0.0000796	8.71E-07	3.88E-06	648.9796	0.2547562
14	1406	76653761	4.37E+07	0.837644	2.15E-06	3.40E-06	3.20E-06	0.0000796	8.71E-07	4.07E-06	703.0612	0.2910286
15	1514	80257143	4.37E+07	0.851896	2.25E-06	3.56E-06	3.37E-06	0.0000796	8.71E-07	4.24E-06	757.1429	0.3282559
16	1622	83532945	4.37E+07	0.863284	2.35E-06	3.71E-06	3.52E-06	0.0000796	8.71E-07	4.39E-06	811.2245	0.3661809
17	1731	86481166	4.37E+07	0.872447	2.43E-06	3.84E-06	3.66E-06	0.0000796	8.71E-07	4.53E-06	865.3061	0.4045464
18	1839	89101808	4.37E+07	0.879839	2.50E-06	3.95E-06	3.78E-06	0.0000796	8.71E-07	4.65E-06	919.3878	0.4430949
19	1947	91394869	4.37E+07	0.885793	2.57E-06	4.05E-06	3.88E-06	0.0000796	8.71E-07	4.76E-06	973.4694	0.4815684
20	2055	93360350	4.37E+07	0.890551	2.62E-06	4.14E-06	3.98E-06	0.0000796	8.71E-07	4.85E-06	1027.551	0.5197088
21	2163	94998251	4.37E+07	0.894293	2.67E-06	4.21E-06	4.05E-06	0.0000796	8.71E-07	4.92E-06	1081.633	0.5572574
22	2271	96308571	4.37E+07	0.89715	2.70E-06	4.27E-06	4.11E-06	0.0000796	8.71E-07	4.98E-06	1135.714	0.593955
23	2380	97291312	4.37E+07	0.899217	2.73E-06	4.32E-06	4.16E-06	0.0000796	8.71E-07	5.03E-06	1189.796	0.6295418
24	2488	97946472	4.37E+07	0.900561	2.75E-06	4.35E-06	4.19E-06	0.0000796	8.71E-07	5.06E-06	1243.878	0.6637573
25	2596	98274052	4.37E+07	0.901223	2.76E-06	4.36E-06	4.20E-06	0.0000796	8.71E-07	5.07E-06	1297.959	0.6963402
26	2704	98274052	4.37E+07	0.901223	2.76E-06	4.36E-06	4.20E-06	0.0000796	8.71E-07	5.07E-06	1297.959	0.7121908
27	2812	97946472	4.37E+07	0.900561	2.75E-06	4.35E-06	4.19E-06	0.0000796	8.71E-07	5.06E-06	1243.878	0.6963402
28	2920	97291312	4.37E+07	0.899217	2.73E-06	4.32E-06	4.16E-06	0.0000796	8.71E-07	5.03E-06	1189.796	0.6637573
29	3029	96308571	4.37E+07	0.89715	2.70E-06	4.27E-06	4.11E-06	0.0000796	8.71E-07	4.98E-06	1135.714	0.6295418
30	3137	94998251	4.37E+07	0.894293	2.67E-06	4.21E-06	4.05E-06	0.0000796	8.71E-07	4.92E-06	1081.633	0.593955
31	3245	93360350	4.37E+07	0.890551	2.62E-06	4.14E-06	3.98E-06	0.0000796	8.71E-07	4.85E-06	1027.551	0.5572574
32	3353	91394869	4.37E+07	0.885793	2.57E-06	4.05E-06	3.88E-06	0.0000796	8.71E-07	4.76E-06	973.4694	0.5197088
33	3461	89101808	4.37E+07	0.879839	2.50E-06	3.95E-06	3.78E-06	0.0000796	8.71E-07	4.65E-06	919.3878	0.4815684
34	3569	86481166	4.37E+07	0.872447	2.43E-06	3.84E-06	3.66E-06	0.0000796	8.71E-07	4.53E-06	865.3061	0.4430949
35	3678	83532945	4.37E+07	0.863284	2.35E-06	3.71E-06	3.52E-06	0.0000796	8.71E-07	4.39E-06	811.2245	0.4045464
36	3786	80257143	4.37E+07	0.851896	2.25E-06	3.56E-06	3.37E-06	0.0000796	8.71E-07	4.24E-06	757.1429	0.3661809
37	3894	76653761	4.37E+07	0.837644	2.15E-06	3.40E-06	3.20E-06	0.0000796	8.71E-07	4.07E-06	703.0612	0.3282559
38	4002	72722799	4.37E+07	0.819618	2.04E-06	3.23E-06	3.01E-06	0.0000796	8.71E-07	3.88E-06	648.9796	0.2910286
39	4110	68464257	4.37E+07	0.79648	1.92E-06	3.04E-06	2.81E-06	0.0000796	8.71E-07	3.68E-06	594.898	0.2547562
40	4218	63878134	4.37E+07	0.766207	1.79E-06	2.83E-06	2.59E-06	0.0000796	8.71E-07	3.46E-06	540.8163	0.2196954
41	4327	58964431	4.37E+07	0.725618	1.66E-06	2.62E-06	2.35E-06	0.0000796	8.71E-07	3.22E-06	486.7347	0.1861029
42	4435	53723149	4.37E+07	0.669469	1.51E-06	2.38E-06	2.09E-06	0.0000796	8.71E-07	2.97E-06	432.6531	0.1542352
43	4543	48154286	4.37E+07	0.588599	1.35E-06	2.14E-06	1.81E-06	0.0000796	8.71E-07	2.68E-06	378.5714	0.1243486
44	4651	42257843	4.37E+07	0	1.19E-06	1.87E-06	1.19E-06	0.0000287	3.14E-07	1.50E-06	324.4898	0.0813
45	4759	36033819	4.37E+07	0	1.01E-06	1.60E-06	1.01E-06	0.0000287	3.14E-07	1.33E-06	270.4082	0.0457227
46	4867	29482216	4.37E+07	0	8.28E-07	1.31E-06	8.28E-07	0.0000287	3.14E-07	1.14E-06	216.3265	0.0327479
47	4976	22603032	4.37E+07	0	6.35E-07	1.00E-06	6.35E-07	0.0000287	3.14E-07	9.49E-07	162.2449	0.0216833
48	5084	15396268	4.37E+07	0	4.32E-07	6.83E-07	4.32E-07	0.0000287	3.14E-07	7.46E-07	108.1633	0.0126902
49	5192	7861924	4.37E+07	0	2.21E-07	3.49E-07	2.21E-07	0.0000287	3.14E-07	5.35E-07	54.08163	0.00593
50	5300	0	4.37E+07	0	0.00E+00	0.00E+00	0.00E+00	0.0000287	3.14E-07	3.14E-07	0	0.0015641

f = 14.944



$$f(t_0) = 4,44 \text{ mm}$$

$$f(t_{\infty}) = 14,94 \text{ mm}$$

$$\frac{f(t_{\infty})}{f(t_0)} = 3,3$$