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RISPOSTA SISMICA LOCALE

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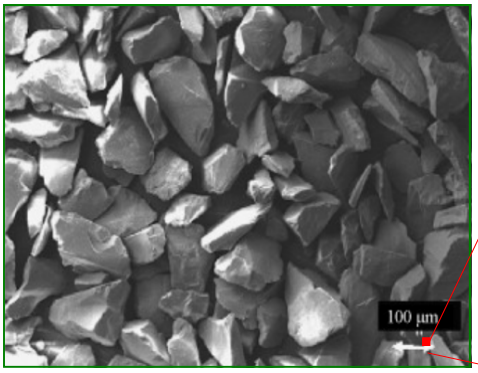
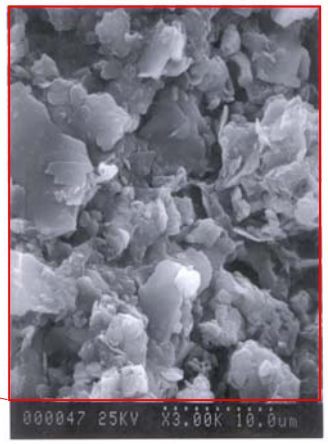
# comportamento meccanico dei terreni per azioni cicliche

**terreno a grana grossa**




Es: sabbia fine (0.1 mm)

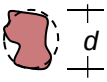
**terreno a grana fine**

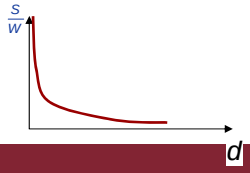



Es: argilla limosa (0.001 mm)

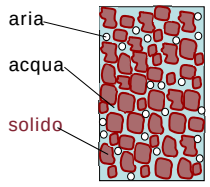
superficie specifica

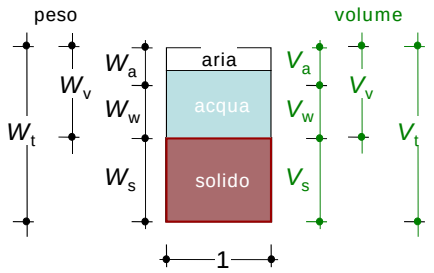
$$\frac{S}{W} \propto \frac{S}{V} \propto \frac{d^2}{d^3} = \frac{1}{d}$$



## rapporti fra le fasi




$n = \frac{V_v}{V_t}$ porosità

$e = \frac{V_v}{V_s}$ = indice dei vuoti

$v = \frac{V_v}{V_t}$ = volume specifico

$S_r = \frac{V_W}{V_v}$ grado di saturazione

$w = \frac{W_w}{W_s}$ $(\times 100)$ contenuto in acqua

$\gamma = \frac{W_t}{V_t}$ peso dell'unità di volume

$\gamma_s = \frac{W_s}{V_s}$ peso specifico delle particelle

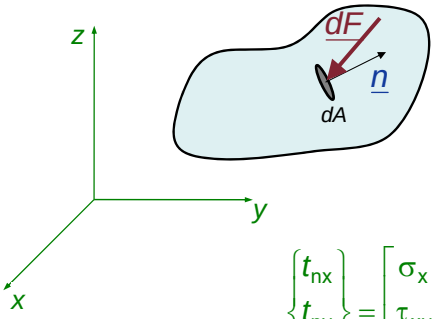
$\gamma_w = \frac{W_w}{V_w}$ peso specifico dell'acqua

$\gamma_d = \frac{W_s}{V_t}$ peso dell'unità di volume del secco

$G_s = \frac{\gamma_s}{\gamma_w}$ = 2.65 - 2.75



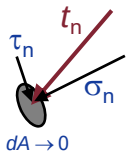
## analisi dello stato tensionale




vettore tensione

$$\underline{t}_n = \lim_{dA \to 0} \frac{dF}{dA} \qquad [FL^{-2}]$$

funzione di $\underline{n}$




$$\begin{Bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \\ n_z \end{Bmatrix}$$

tensore della tensione — coseni direttori

$$\underline{t}_n = \underline{T}\ \underline{n}$$

in geotecnica: compressioni positive

## decomposizione del tensore della tensione

$\sigma$

$$\begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} + \begin{bmatrix} \sigma_x - p & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y - p & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z - p \end{bmatrix}$$

componenti: sferica deviatorica

$\sigma$

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} = \begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} + \begin{bmatrix} \sigma_1 - p & 0 & 0 \\ 0 & \sigma_2 - p & 0 \\ 0 & 0 & \sigma_3 - p \end{bmatrix}$$

invarianti

$$p = \frac{\sigma_x + \sigma_y + \sigma_z}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$ tensione media

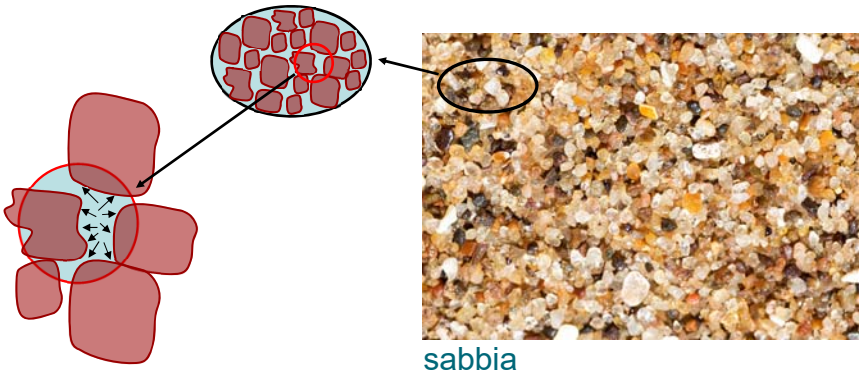
$$q = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$ tensione deviatorica

$$\theta = \arctan \frac{\sqrt{3}(\sigma_2 - \sigma_3)}{2\sigma_1 - \sigma_2 - \sigma_3}$$ angolo di Lode



presenza vuoti

→ effetto della pressione del fluido interstiziale



sabbia

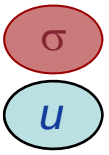
## principio delle tensioni efficaci

$$\underline{t}_n = \lim_{dA \to 0} \frac{dF}{dA}$$

"tensione media"

pressione interstiziale $u$

tensione normale efficace $\sigma' = \sigma - u$




**tutti gli effetti meccanici (deformazione, resistenza) sono legati a variazioni delle tensioni efficaci**

legami costitutivi → tensioni efficaci



## principio delle tensioni efficaci

(generalizzazione)

$$\begin{bmatrix} \sigma'_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma'_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma'_z \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} - \begin{bmatrix} u & 0 & 0 \\ 0 & u & 0 \\ 0 & 0 & u \end{bmatrix}$$

σ

u

## principio delle tensioni efficaci

$$\begin{bmatrix} \sigma'_1 & 0 & 0 \\ 0 & \sigma'_2 & 0 \\ 0 & 0 & \sigma'_3 \end{bmatrix} = \begin{bmatrix} \sigma_1 - u & 0 & 0 \\ 0 & \sigma_2 - u & 0 \\ 0 & 0 & \sigma_3 - u \end{bmatrix} =$$

$$= \begin{bmatrix} p-u & 0 & 0 \\ 0 & p-u & 0 \\ 0 & 0 & p-u \end{bmatrix} + \begin{bmatrix} \sigma_1 - p & 0 & 0 \\ 0 & \sigma_2 - p & 0 \\ 0 & 0 & \sigma_3 - p \end{bmatrix}$$

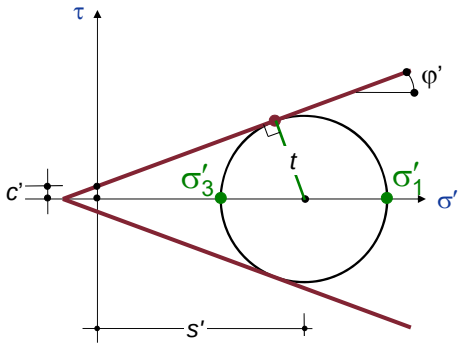
$$p' = p - u \qquad q' = q$$

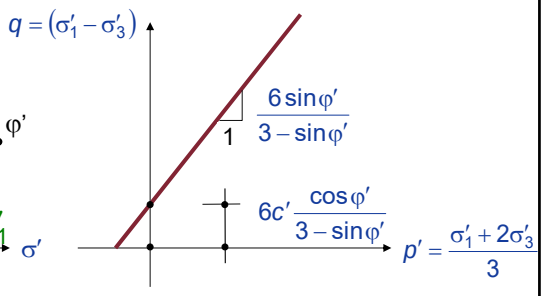


## criterio di resistenza

$$q = 6c' \frac{\cos\varphi'}{3 - \sin\varphi'} + p' \frac{6\sin\varphi'}{3 - \sin\varphi'}$$

$$\tau_f = c' + \sigma'_n \tan\varphi'$$



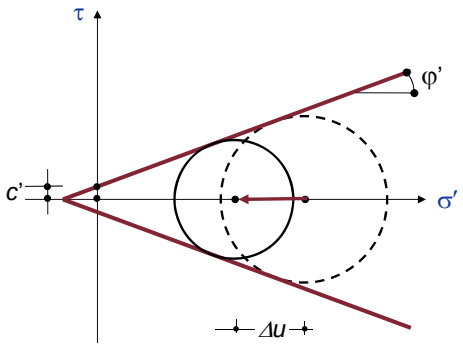

in termini di invarianti

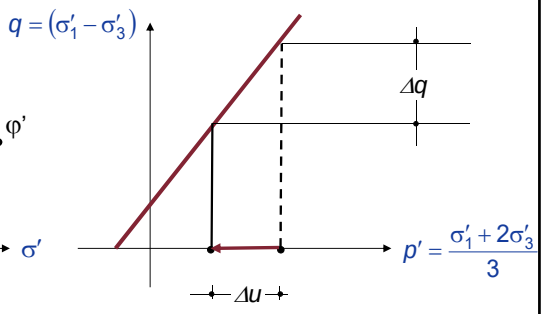
## criterio di resistenza

effetto delle pressioni interstiziali

$$q = 6c'\frac{\cos\varphi'}{3-\sin\varphi'} + p'\frac{6\sin\varphi'}{3-\sin\varphi'}$$

$$\tau_f = c' + \sigma'_n \tan\varphi'$$






## decomposizione del tensore della deformazione

$$\begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} = \begin{bmatrix} \varepsilon_v/3 & 0 & 0 \\ 0 & \varepsilon_v/3 & 0 \\ 0 & 0 & \varepsilon_v/3 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 - \varepsilon_v/3 & 0 & 0 \\ 0 & \varepsilon_2 - \varepsilon_v/3 & 0 \\ 0 & 0 & \varepsilon_3 - \varepsilon_v/3 \end{bmatrix}$$

<u>invarianti</u>

in condizioni "triassiali" ($\varepsilon_2 = \varepsilon_3$)

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \frac{dV}{V}$$

deformazione volumetrica

$$\varepsilon_v = \varepsilon_1 + 2\,\varepsilon_3$$

$$\varepsilon_s = \frac{\sqrt{2}}{3}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]^{1/2}$$

deformazione distorsionale

$$\varepsilon_s = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$$

## elasticità

## relazioni fra invarianti di tensione e di deformazione

$\varepsilon_v = \varepsilon_1 + 2\,\varepsilon_3$ deformazione volumetrica

$\varepsilon_s = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$ deformazione distorsionale

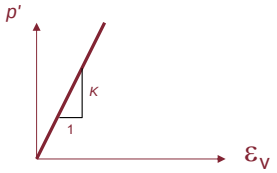
tensione **efficace** media $p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$

tensione deviatorica $q = (\sigma'_1 - \sigma'_3)$

$$\varepsilon_v = \frac{p'}{K}$$

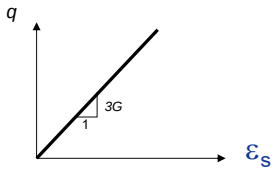
$K = \frac{E}{3(1-2\nu)}$ modulo di elasticità volumetrica

se $\nu \to 0.5$, $K \to \infty$ $\Rightarrow$ mezzo incompressibile




$$\varepsilon_s = \frac{q}{3G}$$

$G = \frac{E}{2(1+\nu)}$ modulo di elasticità tangenziale






mezzo elastico

$$\varepsilon_v = \frac{\Delta p'}{K}$$

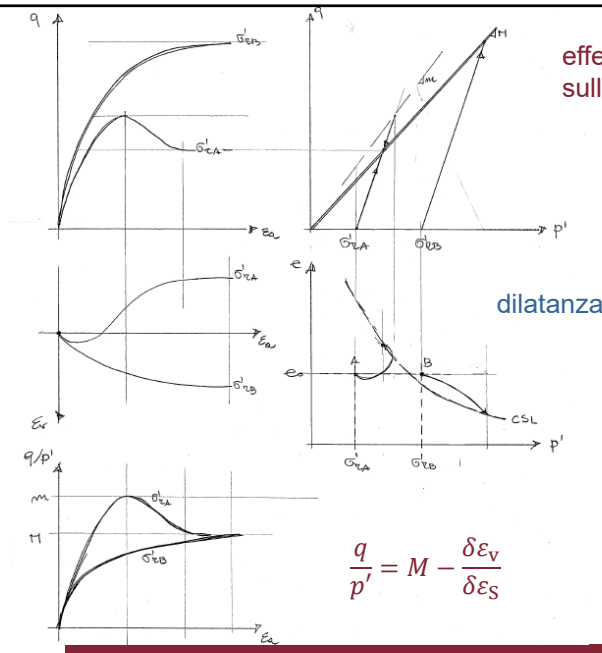
mezzo elasto-plastico

$$\varepsilon_v = C_p \Delta p' + C_q \Delta q$$

accoppiamento deviatorico/volumetrico

$C_q > 0$ comportamento contraente

$C_q < 0$ comportamento dilatante



effetto della dilatanza sulla resistenza

dilatanza → stato: *e*, *p'*

$$\frac{q}{p'} = M - \frac{\delta\varepsilon_v}{\delta\varepsilon_S}$$



condizioni non drenate

$$\varepsilon_v = 0$$

$$\varepsilon_v = C_p \Delta p' + C_q \Delta q = C_p(\Delta p - \Delta u) + C_q \Delta q = 0$$

$$\frac{\Delta p'}{\Delta q} = -\frac{C_q}{C_p} = -a \qquad \Delta u = \Delta p + a\Delta q$$

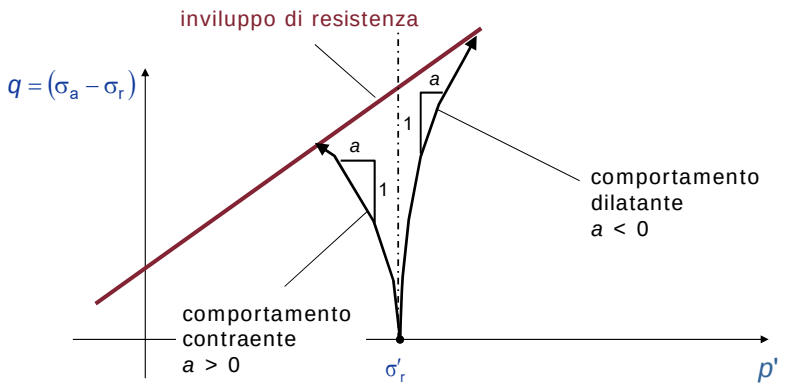
$C_q > 0$ comportamento contraente

$C_q < 0$ comportamento dilatante

**prova triassiale**

**fase deviatorica – condizioni non drenate (CIU)**

risposta in condizioni non drenate

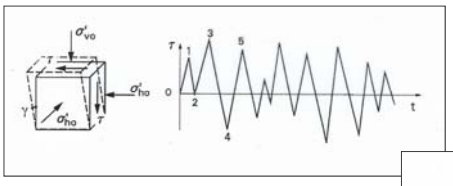





## per effetto di azioni sismiche

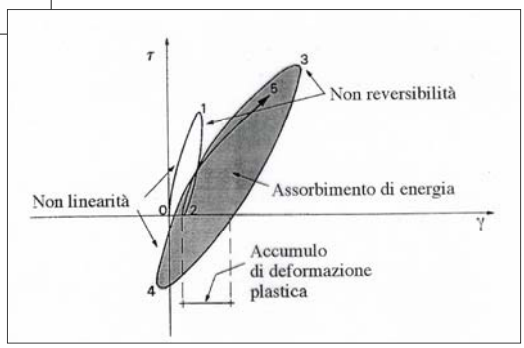
- aspetti fondamentali del comportamento meccanico:
  - rigidezza (iniziale)
  - non linearità (decadimento)
  - capacità di dissipare energia (smorzamento)
- condizioni cicliche:
  - frequenze: 1 – 10 Hz
  - numero di cicli: 5-15 $f(M)$
- modo di deformazione prevalente (ai fini applicativi):
  - taglio semplice (solo onde s)
  - condizioni monodimensionali (propagazione verticale)
- livelli deformativi tipici:
  - $\varepsilon < 10^{-4}$ (0.01 %)

# terreni sotto azioni variabili nel tempo



stati di interesse:

- taglio semplice (per le tensioni)
- distorsionale (per le deformazioni)




comportamento osservato:

- non linearità
- non reversibilità

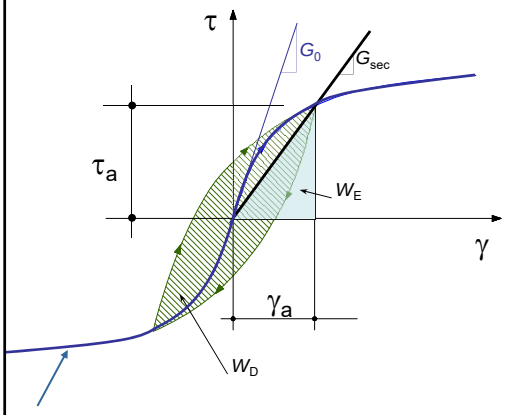
⇓

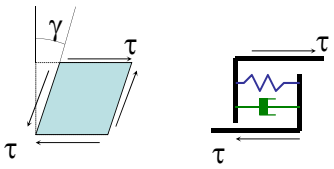
- dissipazione di energia
- deformazioni residue



# comportamento meccanico dei terreni in condizioni cicliche

- non linearità
- dissipazione di energia



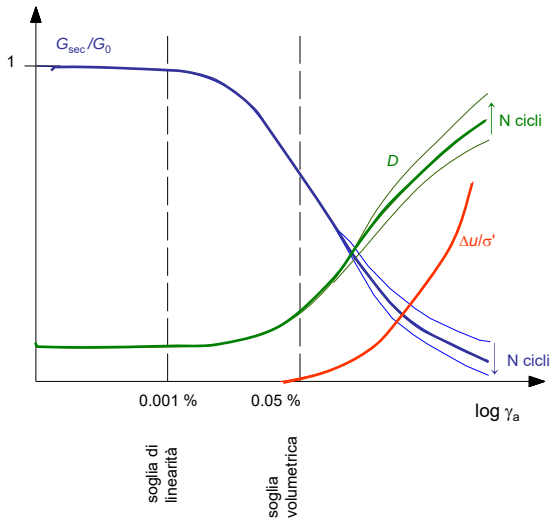



modello visco - elastico non lineare

$$G_{sec} = \frac{\tau_a}{\gamma_a} \qquad D = \frac{W_D}{4\pi W_E}$$

"curva scheletro" (backbone o skeleton curve): variazione di $\tau$ in funzione di $\gamma_a$

## la rigidezza secante e lo smorzamento dipendono dall'ampiezza della deformazione




$\gamma < \gamma_l$
comportamento lineare

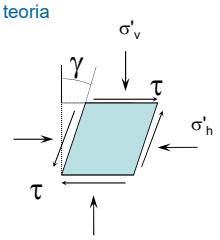
$\gamma_l < \gamma < \gamma_v$
comportamento isteretico

$\gamma > \gamma_v$
degradazione ciclica

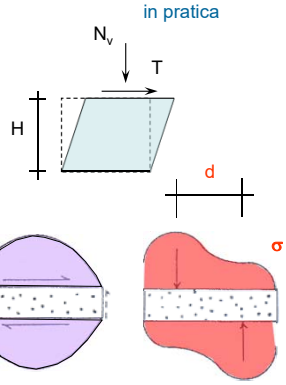


## prova di taglio semplice ciclico (CSS)

in teoria



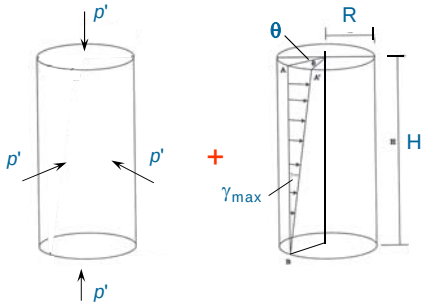

in pratica




- tensioni normali in rapporto $k_0$
  $\sigma'_h = k_0 \cdot \sigma'_v$

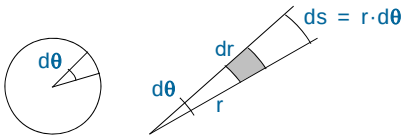
- mancano le $\tau$ coniugate → nasce un momento → $M = T \cdot H = N_v \cdot d$
- distribuzione disuniforme delle $\tau$ e delle $\sigma$ sulle superfici di carico

## prova torsionale ciclica (CTS)



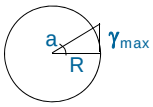

stato tensionale isotropo + coppia torcente variabile con legge armonica

$$M_t = G \cdot I_p \cdot \frac{\partial \vartheta}{\partial z} \qquad I_p = \int_A r^2 dA = \frac{\pi R^4}{2}$$




$$\gamma(r) = \frac{\partial s}{\partial z} = r\frac{\partial \vartheta}{\partial z} = r\frac{M_t}{G \cdot I_p} = \frac{2M_t}{\pi G}\frac{r}{R^4}$$

$$\tau(r) = G \cdot \gamma(r) = \frac{2M_t}{\pi}\frac{r}{R^4} \qquad \gamma(r) = r\frac{\theta}{H}$$



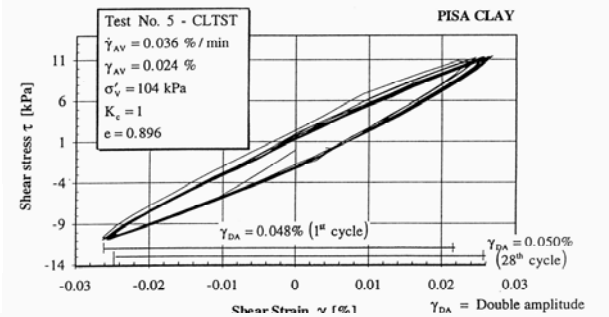

$$\gamma_{max} = R\frac{\vartheta}{H} \qquad \bar{\gamma} = \frac{2}{3}\gamma_{max}$$

$$\tau_{max} = \frac{2M_t}{\pi R^3} \qquad \bar{\tau} = \frac{2}{3}\tau_{max} = \frac{4}{3}\frac{M_t}{\pi R^3}$$

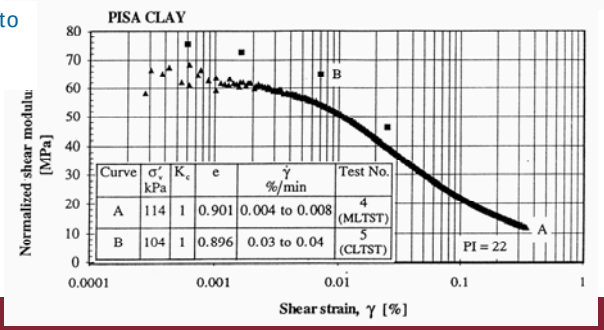


## prova CTS: argilla di Pisa

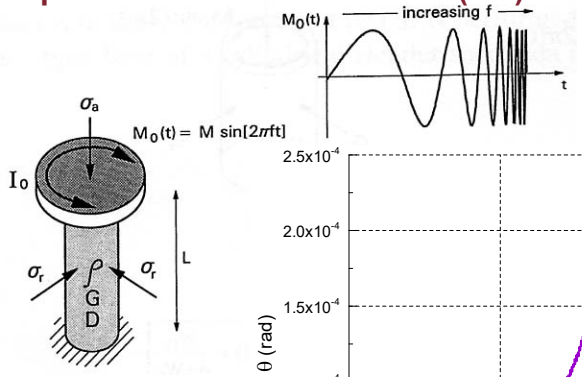
cicli di isteresi τ:γ da misure di coppia M e rotazione θ alla testa del provino




modulo di taglio normalizzato in funzione della deformazione di taglio



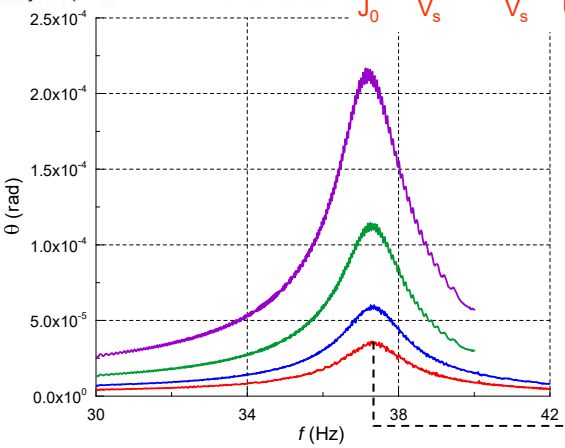
## prova di colonna risonante (RC)







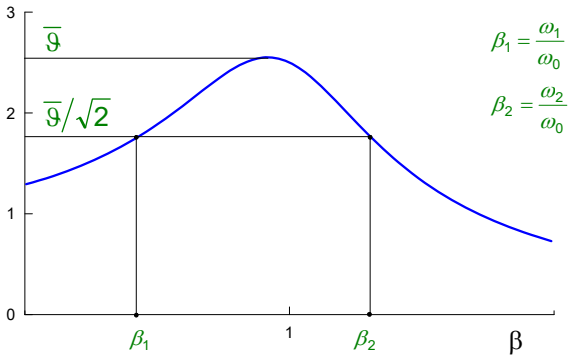
$$\frac{J_p}{J_0} = \frac{\omega H}{V_s} \cdot \tan\frac{\omega H}{V_s} \cong \left(\frac{\omega H}{V_s}\right)^2$$






## prova di colonna risonante (RC)

determinazione del rapporto di smorzamento



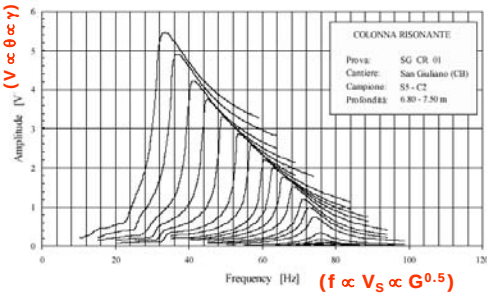

$$\beta_1 = \frac{\omega_1}{\omega_0} \approx 1 - \xi$$

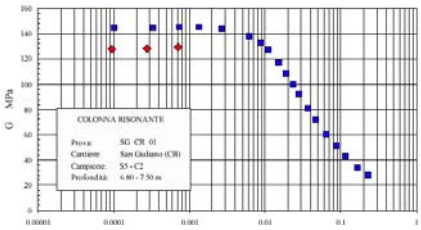
$$\beta_2 = \frac{\omega_2}{\omega_0} \approx 1 + \xi$$

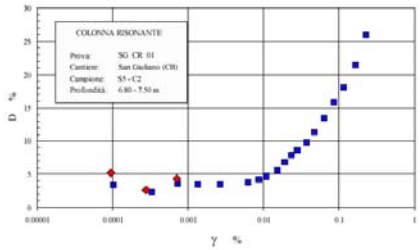
$$\xi = \frac{\omega_2 - \omega_1}{\omega_2 + \omega_1}$$

## prova RC: limo-argilloso di S. Giuliano di Puglia

curve di risposta in frequenza $\gamma$:f
da misure di accelerazione $\ddot{\theta}$
o rotazione $\theta$ alla testa del provino






## risultati sperimentali

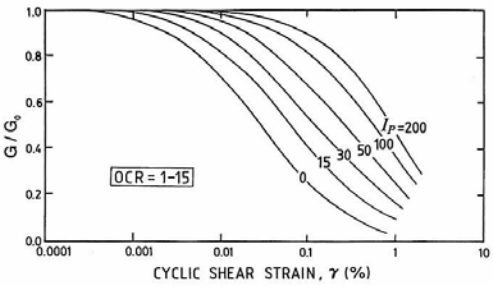
- curve di decadimento della rigidezza normalizzate G/G0 - $\gamma$
- curve di incremento dello smorzamento D - $\gamma$

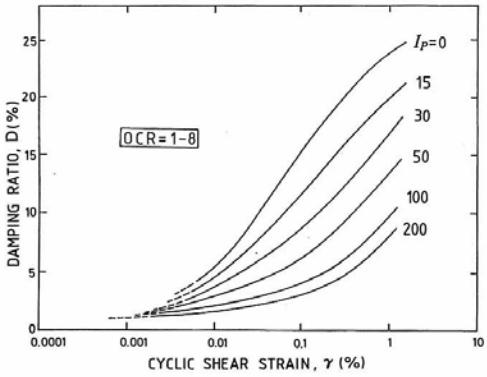
**dipendono da $I_P$**

in subordine:

- dallo stato tensionale efficace di confinamento
- dal numero di cicli
- dal grado di sovraconsolidazione

- per le sabbie si può utilizzare la curva con $I_P$ = 0

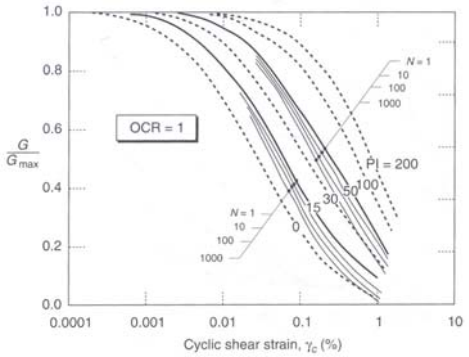


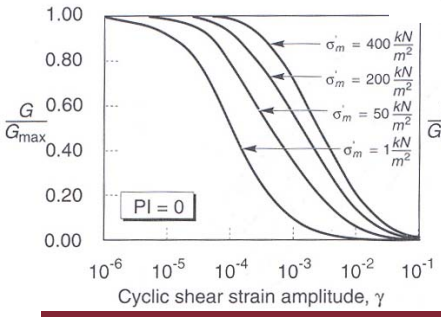



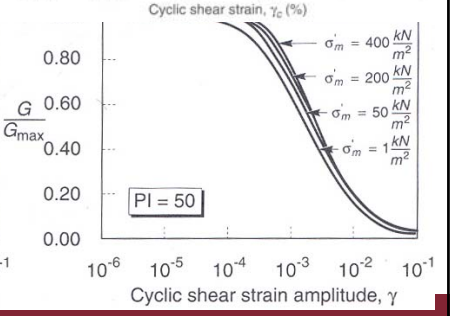
## risultati sperimentali

se c'è degradazione $G/G_0$ diminuisce al crescere del numero di cicli

- decadimento più rapido a basse tensioni efficaci
- effetto più marcato in terreni granulari ($I_P$ =0)






## valutazione di $G_0$

$$f(e) = \frac{(2.973 - e)^2}{1 + e}$$ **(*Hardin & Black, 1968*)**

$G_0$ dipende dallo stato del materiale:
- stato tensionale efficace
- indice dei vuoti

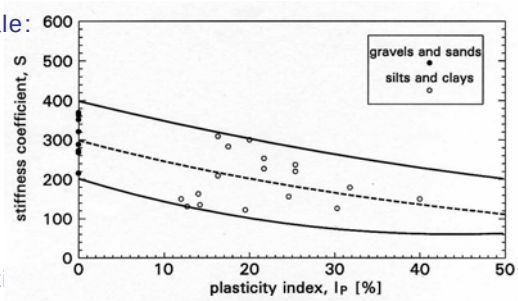
relazioni empiriche nella forma:

$G_0 = A \cdot F(e) \cdot (p')^n$

$F(e)$: funzione dell'indice dei vuot

non è adimensionale
($G_0$ e p' in kPa)

***A* ed n dipendono da *F*(e):**
- ***A* diminuisce con $I_P$**
- **n aumenta con $I_P$**



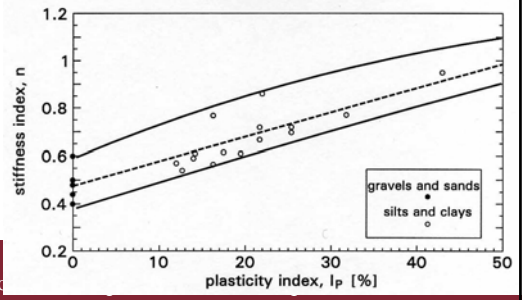



**Table 6.1** Constants in proposed empirical equations on small strain modulus:* $G_0 = A \cdot F(e)(\sigma'_0)^n$ (Kokusho, 1987)

| | References | $A$ | $F(e)$ | $n$ | Soil material | Test method |
|---|---|---|---|---|---|---|
| Sand | Hardin–Richart (1963) | 7000 | $(2.17-e)^2/(1+e)$ | 0.5 | Round grained Ottawa sand | Resonant column |
| | | 3300 | $(2.97-e)^2/(1+e)$ | 0.5 | Angular grained crushed quartz | Resonant column |
| | Shibata–Soelarno (1975) | 42000 | $0.67-e/(1+e)$ | 0.5 | Three kinds of clean sand | Ultrasonic pulse |
| | Iwasaki *et al.* (1978) | 9000 | $(2.17-e)^2/(1+e)$ | 0.38 | Eleven kinds of clean sand | Resonant column |
| | Kokusho (1980) | 8400 | $(2.17-e)^2/(1+e)$ | 0.5 | Toyoura sand | Cyclic triaxial |
| | Yu–Richart (1984) | 7000 | $(2.17-e)^2/(1+e)$ | 0.5 | Three kinds of clean sand | Resonant column |
| Clay | Hardin–Black (1968) | 3300 | $(2.97-e)^2/(1+e)$ | 0.5 | Kaolinite, etc. | Resonant column |
| | Marcuson–Wahls (1972) | 4500 | $(2.97-e)^2/(1+e)$ | 0.5 | Kaolinite, $I_p^{**} = 35$ | Resonant column |
| | | 450 | $(4.4-e)^2/(1+e)$ | 0.5 | Bentonite, $I_p = 60$ | Resonant column |
| | Zen–Umehara (1978) | 2000 ~ 4000 | $(2.97-e)^2/(1+e)$ | 0.5 | Remolded clay, $I_p = 0 \sim 50$ | Resonant column |
| | Kokusho *et al.* (1982) | 141 | $(7.32-e)^2/(1+e)$ | 0.6 | Undisturbed clays, $I_p = 40 \sim 85$ | Cyclic triaxial |

*$\sigma'_0$ : kPa, $G_0$ : kPa, **$I_p$ : Plasticity Index



## valori tipici di $V_S$ (→ $G_0$) per terreni naturali

### Densità ρ e Velocità $V_S$ → Rigidezza $G_0$

(correlazioni in genere riferite a $V_S$)

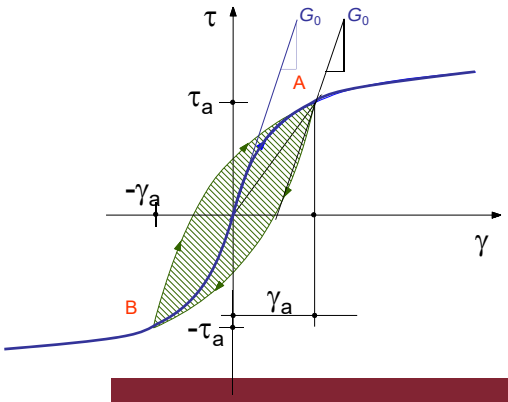
| Terreni | Valori indicativi ρ (t/m³) | $V_S$ (m/s) |
|---|---|---|
| terreni organici, torbe<br>argille molto molli | 1.0÷1.3<br>1.4÷1.7 | <100 |
| terreni piroclastici sciolti<br>argille tenere, sabbie sciolte | 1.0÷1.7<br>1.6÷1.9 | 100÷200 |
| terreni piroclastici densi/cementati<br>argille consistenti, sabbie dense | 1.2÷1.9<br>1.8÷2.1 | 200÷400 |
| argilliti, sabbie e ghiaie cementate<br>roccia tenera alterata o fratturata | 1.9÷2.2<br>1.8÷2.2 | 400÷800 |
| roccia tenera intatta, roccia lapidea | 2.2÷2.4 | >800 |

## modelli non lineari

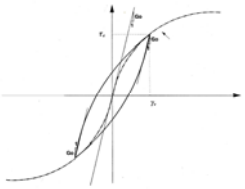
curva scheletro: $\tau = f(\gamma) \rightarrow f(-\gamma) = -f(\gamma)$

condizioni: $\left.\frac{df(\gamma)}{d\gamma}\right|_{\gamma=0} = G_0$ $\quad \lim_{\gamma\to\infty} f(\gamma) = \tau_f = c' + \sigma' \tan\varphi'$

scarico – ricarico: criteri di Masing

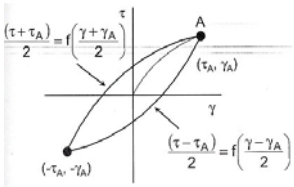



**1° criterio di Masing**



il modulo iniziale di ogni scarico-ricarico è uguale al valore di $G_0$ al primo carico

**2° criterio di Masing**




la curva τ:γ di ogni scarico-ricarico si ottiene per omotetia di τ:γ al primo carico



## modello iperbolico

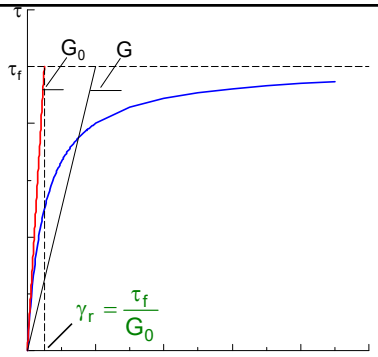
$$\tau = \frac{G_0 \cdot \gamma}{1 + \frac{G_0}{\tau_f} \cdot \gamma} = \frac{G_0 \cdot \gamma}{1 + \frac{\gamma}{\gamma_r}} = \frac{a \cdot \gamma}{1 + b \cdot \gamma} = \frac{\gamma}{\frac{1}{a} + \frac{b}{a} \cdot \gamma}$$

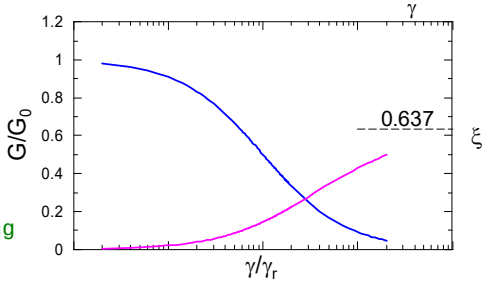
$$\lim_{\gamma\to\infty} \tau = \tau_f = \frac{a}{b} \qquad \left.\frac{d\tau}{d\gamma}\right|_{\gamma=0} = G_0 = a$$

$$G = \frac{\tau}{\gamma} = \frac{G_0}{1 + \frac{\gamma}{\gamma_r}}$$

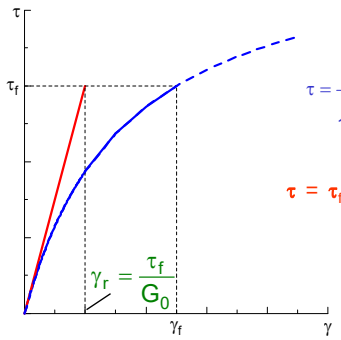
$$\xi = \frac{W_E}{4\pi W_D} = \frac{4}{\pi} \cdot \left(1 + \frac{\gamma_r}{\gamma}\right) \cdot \left[1 - \frac{\ln\left(1 + \frac{\gamma_a}{\gamma_r}\right)}{\frac{\gamma_a}{\gamma_r}}\right] - \frac{2}{\pi}$$

ξ valutato applicando le regole di Masing ad un ciclo di isteresi di ampiezza γ



## modello di Ramberg - Osgood




$$\gamma_r = \frac{\tau_f}{G_0}$$

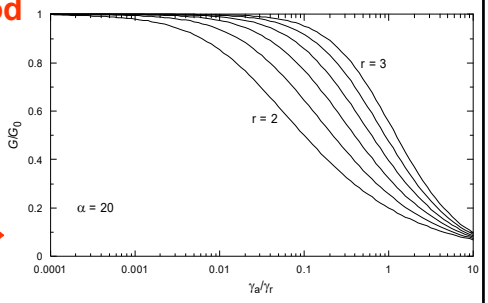
$$\tau = \frac{G_0 \cdot \gamma}{1+\alpha\left(\frac{\tau}{\tau_f}\right)^{r-1}}$$

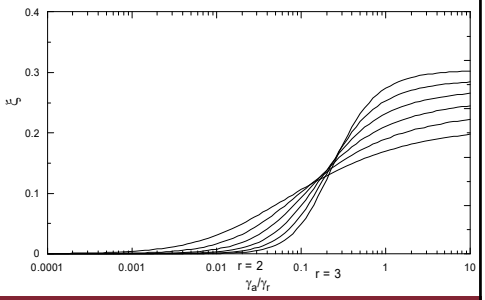
$$\tau = \tau_f \rightarrow \gamma = \gamma_f \rightarrow$$

$$\alpha = \frac{\gamma_f}{\gamma_r} - 1$$

$$\frac{G}{G_0} = \frac{1}{1+\alpha\left(\frac{\tau}{\tau_f}\right)^{r-1}} = \frac{1}{1+\alpha\left(\frac{\gamma}{\gamma_r}\right)^{r-1}}$$

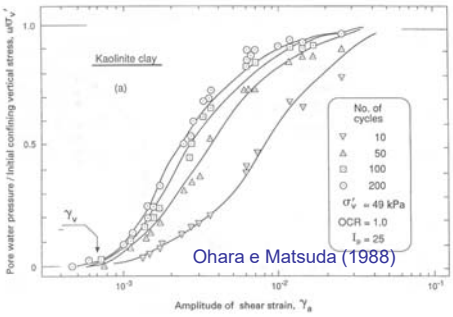
$$\xi = \frac{2}{\pi}\cdot\frac{r-1}{r+1}\cdot\alpha\cdot\frac{\left(\frac{G}{G_0}\cdot\frac{\gamma}{\gamma_r}\right)^{r-1}}{1+\alpha\left(\frac{G}{G_0}\cdot\frac{\gamma}{\gamma_r}\right)^{r-1}}$$



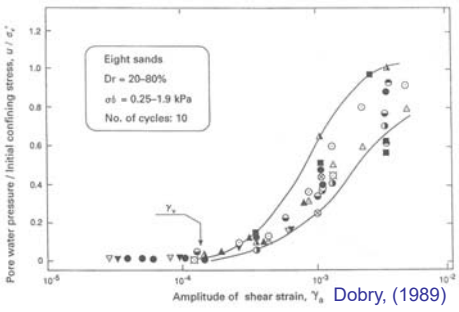






## misure di Δu indotte in condizioni cicliche




Ohara e Matsuda (1988)



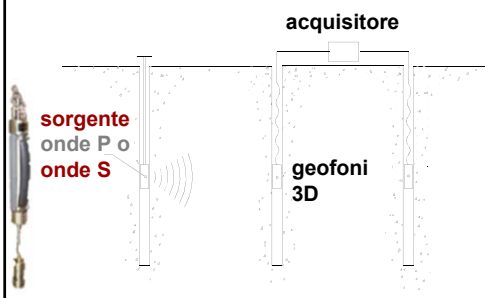

Dobry, (1989)

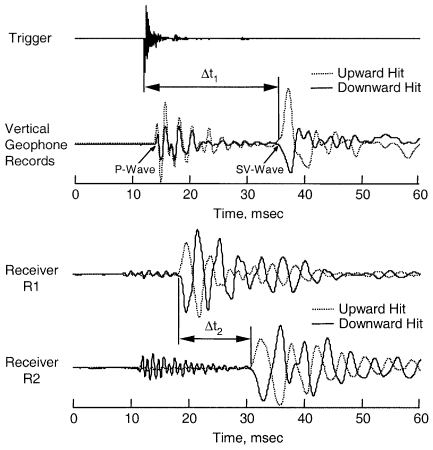
# prove dinamiche in sito

- metodi Invasivi
  - prove cross-hole
  - prove down-hole
    - in foro
    - SCPT-SDMT
- metodi non-invasivi
  - prove sismica a rifrazione (onde SH)
  - analisi delle onde superficiali
    - metodi attivi (SASW, MASW)
    - metodi passivi (fk, SPAC, ReMi, H/V)



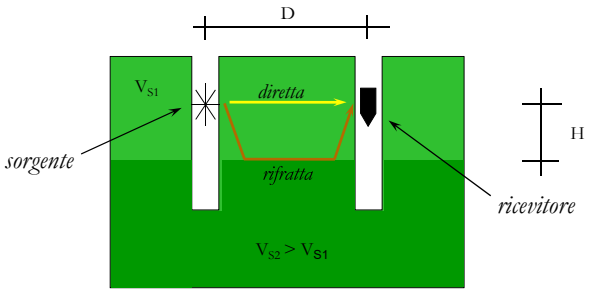
## cross-hole
(standard ASTM D-4428-M)




- 2 o 3 fori rivestiti e cementati con controllo verticalità (costi ↑)
- tempi di arrivo → Vp & Vs
- onde P Sv Sh

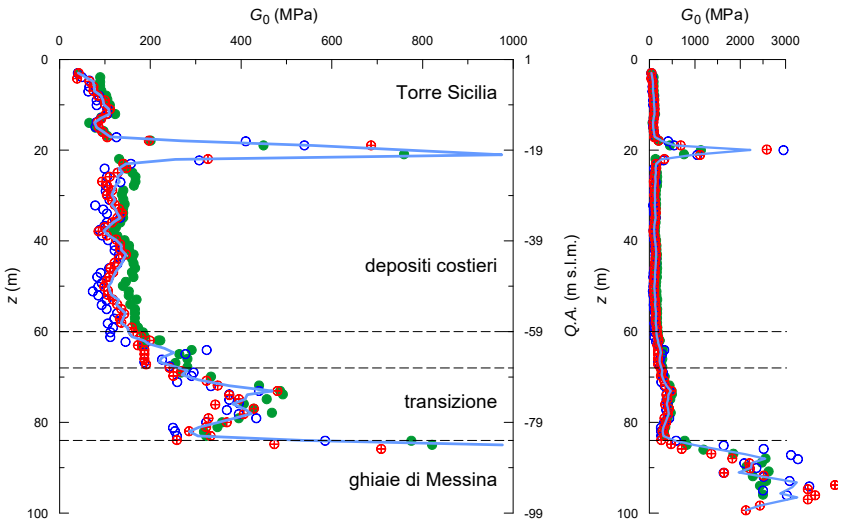
cross-hole: rifrazioni da bruschi contrasti d'impedenza




usualmente D=3-5m

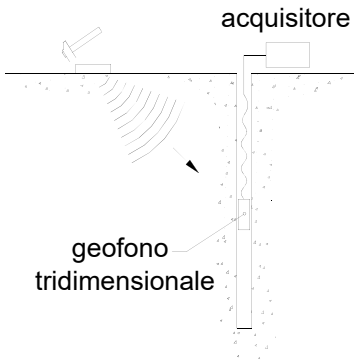


## prova Cross-Hole (CH): profili $G_0$



## down-Hole

un solo foro; minore sensibilità
*verticalità* foro di sondaggio

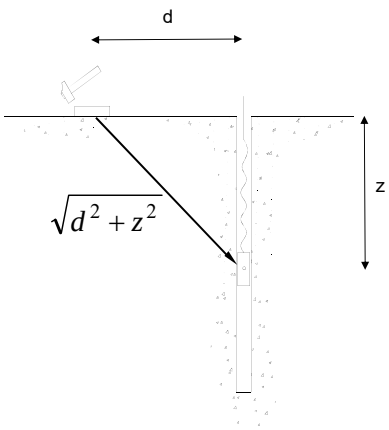



limitazione a profondità
di 50-60m

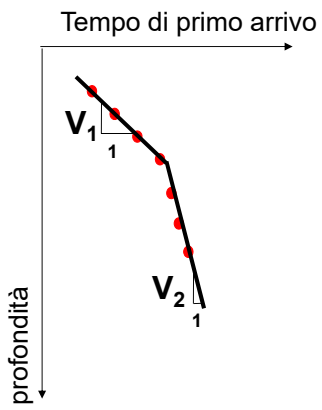
metodi di interpretazione
- metodo tempi intercetti
- metodo true interval



## down-Hole

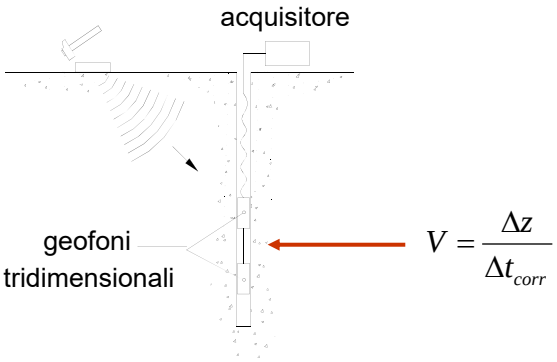



$$t_{corr} = t_{mis} \frac{z}{\sqrt{d^2+z^2}}$$



down-Hole

due ricevitori (true interval)

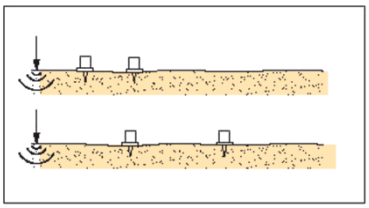



$$V = \frac{\Delta z}{\Delta t_{corr}}$$



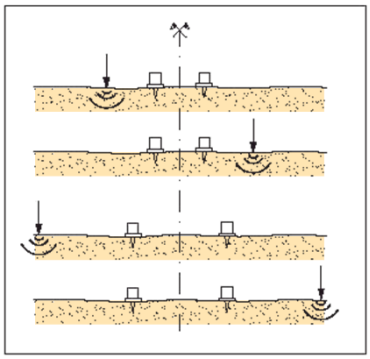
## prove SASW

**allineamento con sorgente fissa**

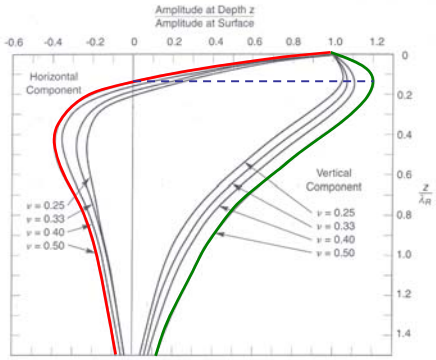


impulsi verticali

**allineamento con asse mediano fisso**



## onde di Rayleigh




la porzione di terreno influenzata dalla propagazione delle onde R è proporzionale alla lunghezza d'onda $\lambda_R$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi \cdot V}{\omega} = \frac{V}{f}$$

$\lambda_R = V_R/f_R$

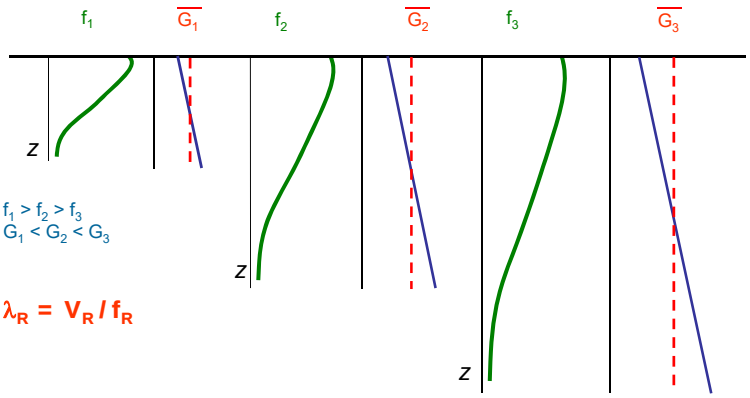
lunghezza d'onda: periodo della forma d'onda nello spazio

la max profondità di terreno interessata è $z_{max} = 1.0 - 1.5\ \lambda$

nella prova i segnali registrati vengono trasformati nel dominio delle frequenze
⇓
somma di armoniche di ampiezza e fase diverse



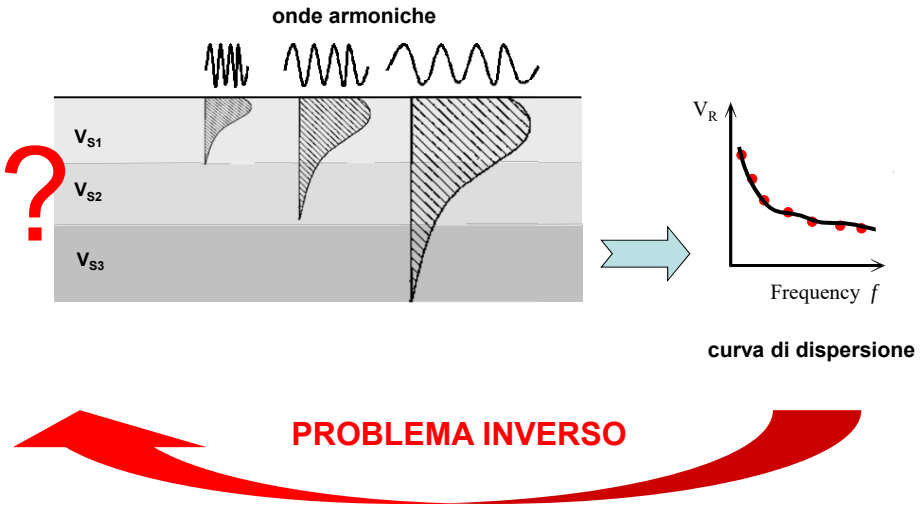
## dispersività onde di Rayleigh in mezzi eterogenei




$f_1 > f_2 > f_3$
$G_1 < G_2 < G_3$

$\lambda_R = V_R / f_R$

armoniche di diversa frequenza viaggiano con velocità diversa → dispersività






acquisizione

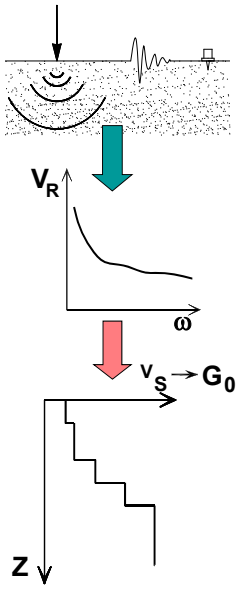
campo di spostamenti in superficie

analisi dei segnali

curva di dispersione delle onde di Rayleigh: velocità di fase vs frequenza

processo di inversione

profilo di velocità delle onde di taglio

$$G_0 = \rho \cdot V_S^2$$

modulo di rigidezza ($G_0$ vs profondità)