TRAZIONE - COMPRESSIONE

Titolo nota 13/11/2012

$$N_{pl,Rd} = A \frac{f_y}{\gamma_{M0}}$$

IN PRESENZA DI FORI

$$N_{u,Rd} = 0.9 \, A_{net} \frac{f_u}{\gamma_{M2}}$$

INSTABILITA' (locale, dell'asta)

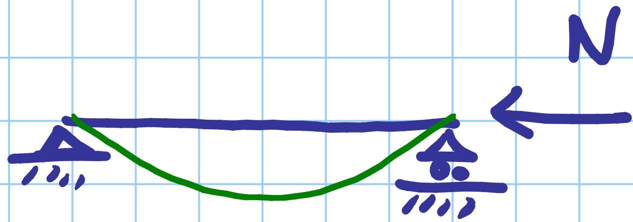
IN ASSENZA DI PROBLEMI DI INSTAB.

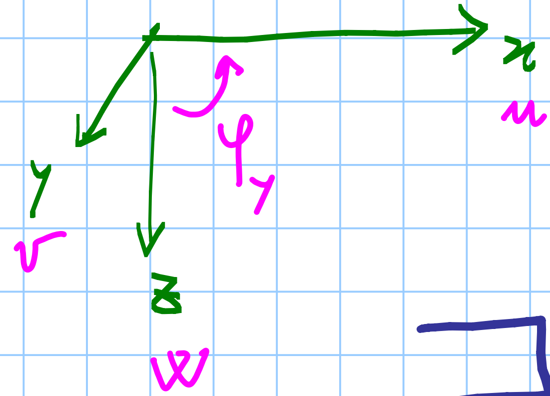
$$N_{pl,Rd} = A \frac{f_y}{\gamma_{M0}}$$

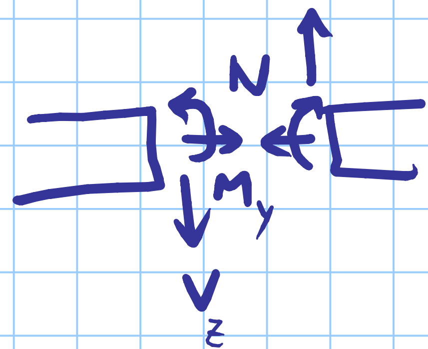
se il foro è con bullone

non occorre verifica

INSTABILITA' DELL'ASTA

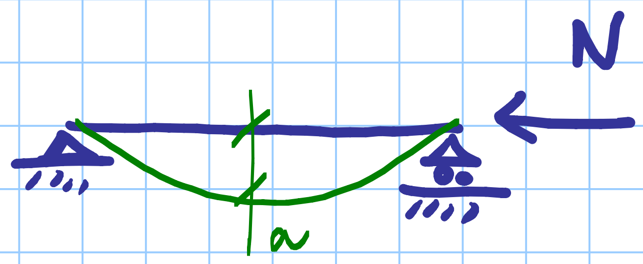






equazioni indefinite di equilibrio

$$\frac{d^2w}{dx^2} = -\frac{M}{EI} \qquad \begin{cases} \dfrac{dw}{dx} = -\varphi & \dfrac{dM}{dx} = V \\[2ex] \dfrac{d\varphi}{dx} = \dfrac{M}{EI} & \dfrac{dV}{dx} = -q \end{cases}$$



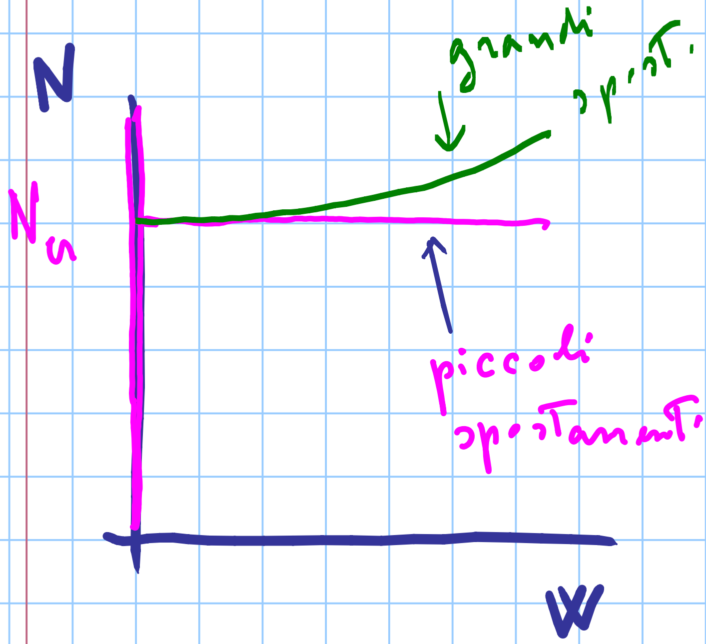
$$w = a \sin \pi \frac{x}{\ell}$$

$$M = N\,w$$

$$\frac{d^2 w}{dx^2} = -\frac{M}{EI} = -\frac{N}{EI}\,w \qquad \frac{d^2 w}{dx^2} = -a\frac{\pi^2}{\ell^2}\sin \pi \frac{x}{\ell}$$

$$-\cancel{a}\,\frac{\pi^2}{\ell^2}\,\cancel{\sin \pi \frac{x}{\ell}} = -\frac{N}{EI}\,\cancel{a}\,\cancel{\sin \pi \frac{x}{\ell}} \Rightarrow \frac{\pi^2}{\ell^2} = \frac{N}{EI}$$

$$N = \frac{\pi^2 EI}{\ell^2} = N_u$$



per $N < N_{cr}$ impossibile deformata con sbandamento flession.

per $N = N_{cr}$ ∞ configurazioni di equilibrio
(eq. INDIFFERENTE)

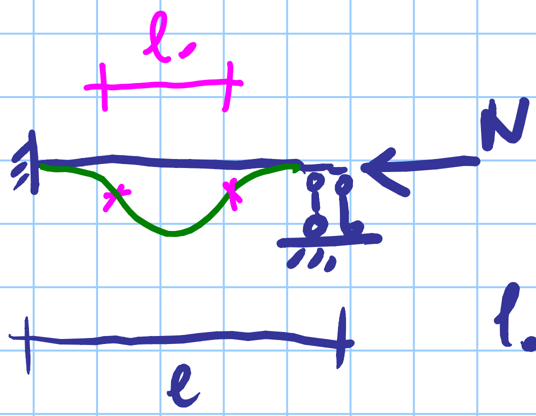
per $N > N_{cr}$ possibili deform. solo assiali
(eq. INSTABILE)

ASTA ideale

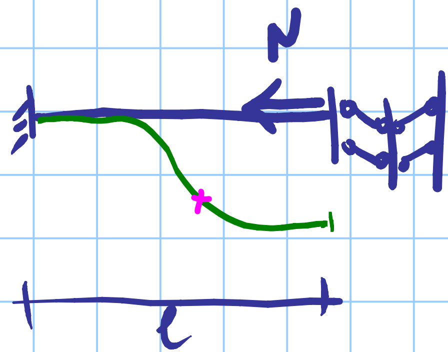
MATERIALE lin. elastico, infinit. resistente

$$N_{cr} = \frac{\pi^2 EI}{l_0^2}$$

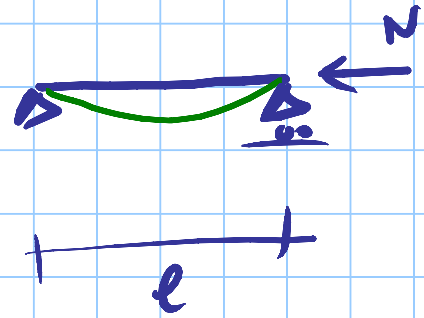
l_0 = lunghezza libera di inflessione



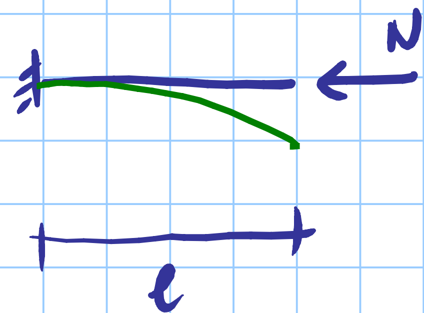
$l_0 = \frac{l}{2}$



$l_0 = l$



$l_0 = l$



$l_0 = 2l$

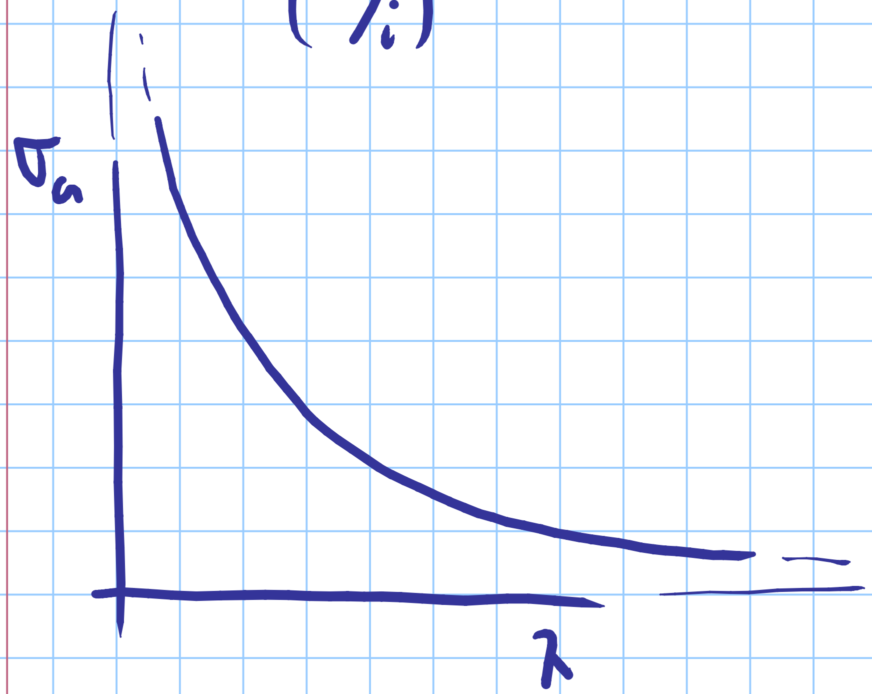
$$\sigma_u = \frac{N_u}{A} = \frac{\pi^2 E I}{l_0^2 A}$$

$$\frac{I}{A} = i^2$$

raggio d'inerzia (ex ρ)

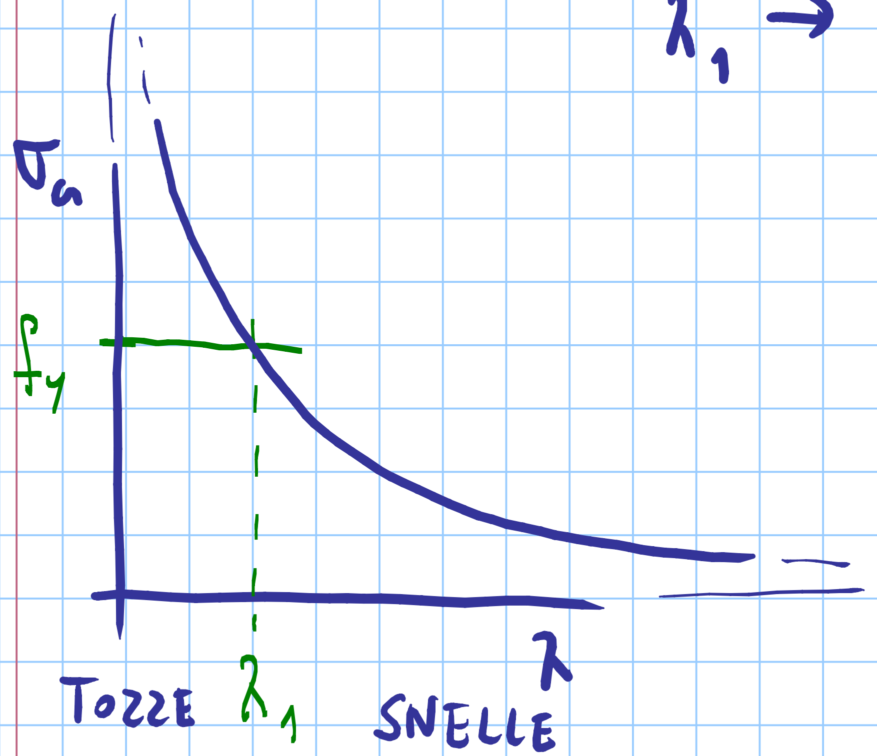
$$\sigma_u = \frac{\pi^2 E}{\left(l_0/i\right)^2} = \frac{\pi^2 E}{\lambda^2}$$

$$\lambda = \frac{l_0}{i}$$ SNELLEZZA



MA il materiale NON è infinitamente resistente

snervamento per $\sigma = f_y$



$$\lambda_1 \rightarrow \sigma_a = f_y$$

$$\frac{\pi^2 E}{\lambda_1^2} = f_y$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

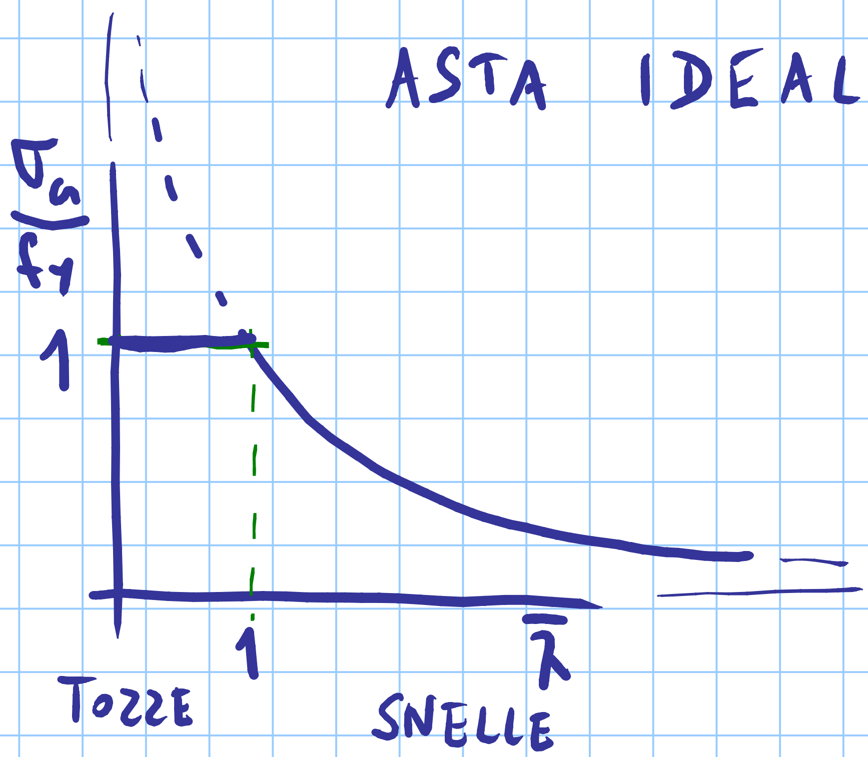
$f_y = 235$ MPa $\rightarrow$ $\lambda_1 = 93.9$

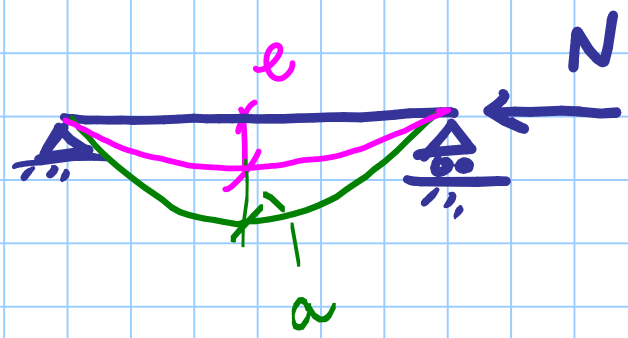
snellezza normalizzata $$\bar{\lambda} = \frac{\lambda}{\lambda_1}$$

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \qquad f_y = \frac{\pi^2 E}{\lambda_1^2} \qquad \frac{f_y}{\sigma_{cr}} = \frac{\dfrac{\cancel{\pi^2} \cancel{E}}{\lambda_1^2}}{\dfrac{\cancel{\pi^2} \cancel{E}}{\lambda^2}} = \frac{\lambda^2}{\lambda_1^2}$$

$$\bar{\lambda} = \sqrt{\frac{f_y}{\sigma_{cr}}} = \sqrt{\frac{N_y}{N_{cr}}}$$

ASTA IDEALE





$$w_0 = e \sin \pi \frac{x}{\ell}$$

$$\Delta w = a \sin \pi \frac{x}{\ell}$$

$$w = w_0 + \Delta w = (e+a) \sin \pi \frac{x}{\ell}$$

$$M = N\,w$$

$$\frac{d^2 \Delta w}{dx^2} = -\frac{M}{EI} = -\frac{N}{EI}\,w$$

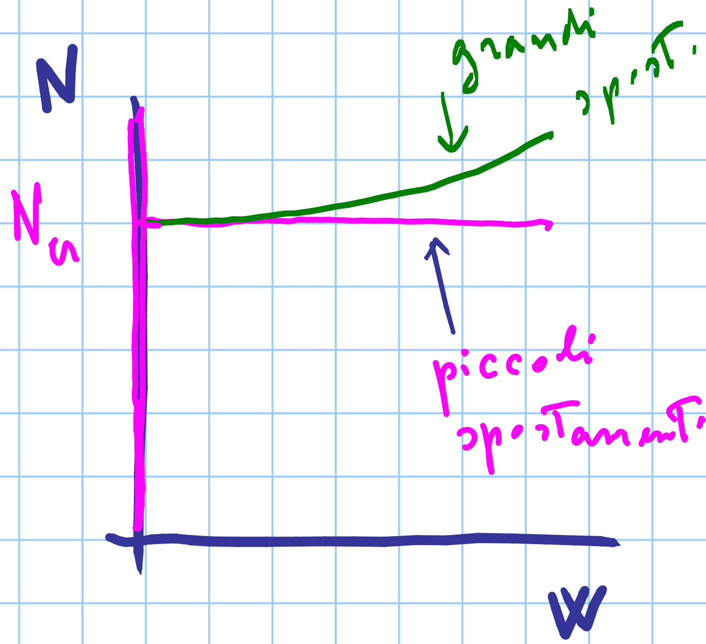
$$-a\frac{\pi^2}{\ell^2}\sin\cancel{\pi\frac{x}{\ell}} = -\frac{N}{EI}(e+a)\sin\cancel{\pi\frac{x}{\ell}}$$

$$a\frac{\pi^2}{\ell^2} = (e+a)\frac{N}{EI} \quad\Rightarrow\quad a\,N_u = (e+a)\,N$$

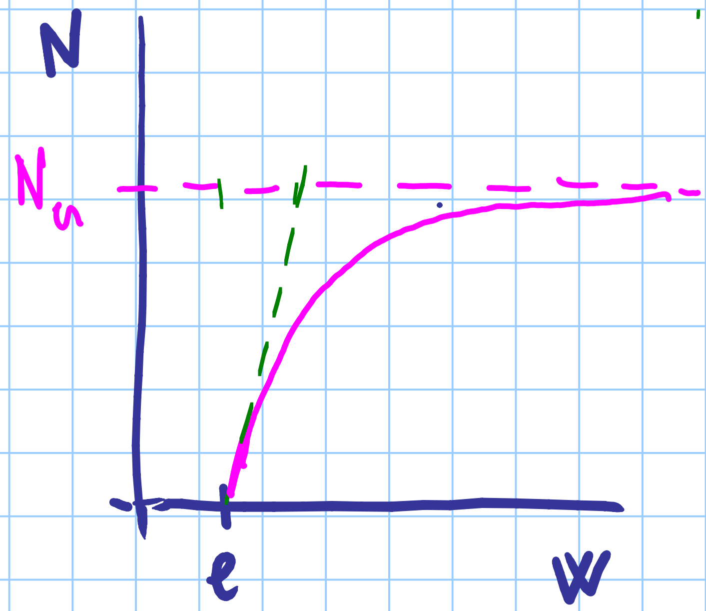
$$a = \frac{N}{N_u - N}\,e$$

$$a+e = \left(\frac{N}{N_u-N}+1\right)e = \frac{\cancel{N}+N_u-\cancel{N}}{N_u-N}\,e$$

$$a+e = \frac{1}{1-N/N_u}\,e$$



ASTA IDEALE



ASTA REALE con imperfezioni

MATERIALE lin. elastico, senza limiti di resistenza

$$\sigma_{max} = \frac{N}{A} + \frac{M}{\underbrace{\frac{I}{h/2}}_{W}}$$

$$M = N\,w$$

$$M_{max} = N\,(e+a)$$

$$\sigma_{max} = \frac{N}{A} + \frac{N\,(e+a)\,A}{A\,W}$$

$$e+a = \frac{1}{1-\frac{N}{N_u}}\,e$$

$$\sigma_{max} = \frac{N}{A}\left[1 + \frac{1}{1-\frac{N}{N_u}}\,\frac{e\,A}{W}\right]$$

$$\eta = \frac{e\,A}{W} = \frac{e}{i^2}\,h/2$$

$$\sigma_{max} = \frac{N}{A}\left[1 + \frac{\eta}{1 - \frac{N}{N_u}}\right] = \frac{N}{A} \frac{1 + \eta - \frac{N}{N_u}}{1 - \frac{N}{N_u}}$$

$$f_y = \frac{N_u}{A} \frac{1 + \eta - \frac{N_u}{N_{cr}}}{1 - \frac{N_u}{N_{cr}}}$$

$$1 = \frac{N_u}{N_y} \frac{1 + \eta - \frac{N_u}{N_{cr}}}{1 - \frac{N_u}{N_{cr}}}$$

$$1 - \frac{N_u}{N_{cr}} = \frac{N_u}{N_y}\left[1 + \eta - \frac{N_u}{N_{cr}}\right]$$

$$\bar{\lambda} = \sqrt{\frac{N_y}{N_{cr}}} \qquad \bar{\lambda}^2 = \frac{N_y}{N_{cr}} \Rightarrow N_{cr} = \frac{N_y}{\bar{\lambda}^2}$$

$$1 - \frac{N_u}{N_y}\bar{\lambda}^2 = \frac{N_u}{N_y}\left[1 + \eta - \frac{N_u}{N_y}\bar{\lambda}^2\right]$$

$$\frac{N_u}{N_y} = \chi \qquad 1 - \bar{\lambda}^2 \chi = \chi\left[1 + \eta - \bar{\lambda}^2 \chi\right]$$

$$\bar{\lambda}^2 \chi^2 - \underbrace{\left[1 + \eta + \bar{\lambda}^2\right]}_{2\phi}\chi + 1 = 0$$

$$\phi = \frac{1}{2}\left[1+\eta+\bar{\lambda}^2\right] \qquad \eta = \alpha\left(\bar{\lambda}-0.2\right)$$

$$\chi = \frac{\phi - \sqrt{\phi^2-\bar{\lambda}^2}}{\bar{\lambda}^2} \times \frac{\phi + \sqrt{\phi^2-\bar{\lambda}^2}}{\phi + \sqrt{\phi^2-\bar{\lambda}^2}}$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2-\bar{\lambda}^2}}$$

$$\phi = \frac{1}{2}\left[1+\alpha\left(\bar{\lambda}-0.2\right)+\bar{\lambda}^2\right]$$