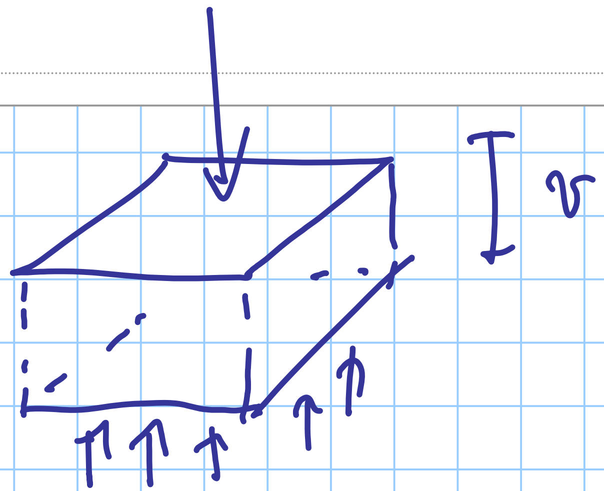
Titolo nota 03/06/2014

SUOLO ELASTICO ALLA WINKLER



K costante di Winkler

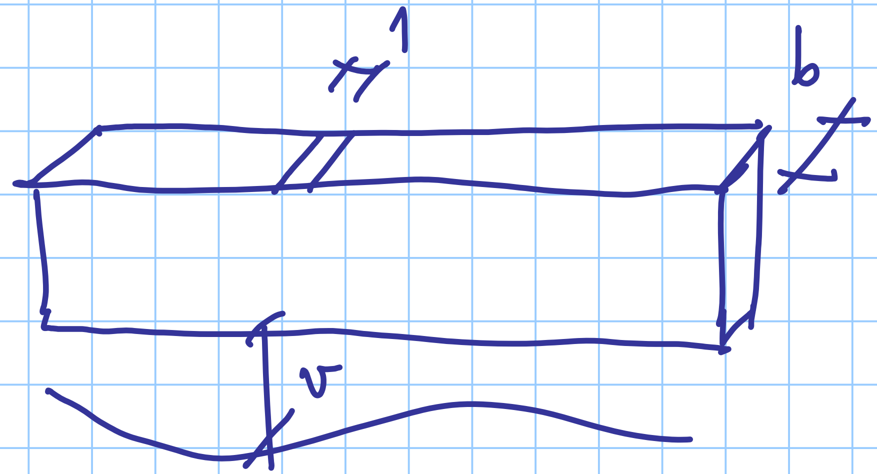
$[F L^{-3}]$

$$p = -K v$$

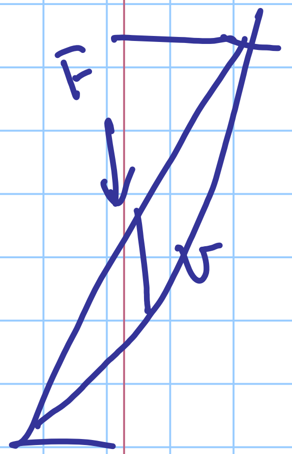
$[F L^{-2}]$ $\quad$ $[L]$

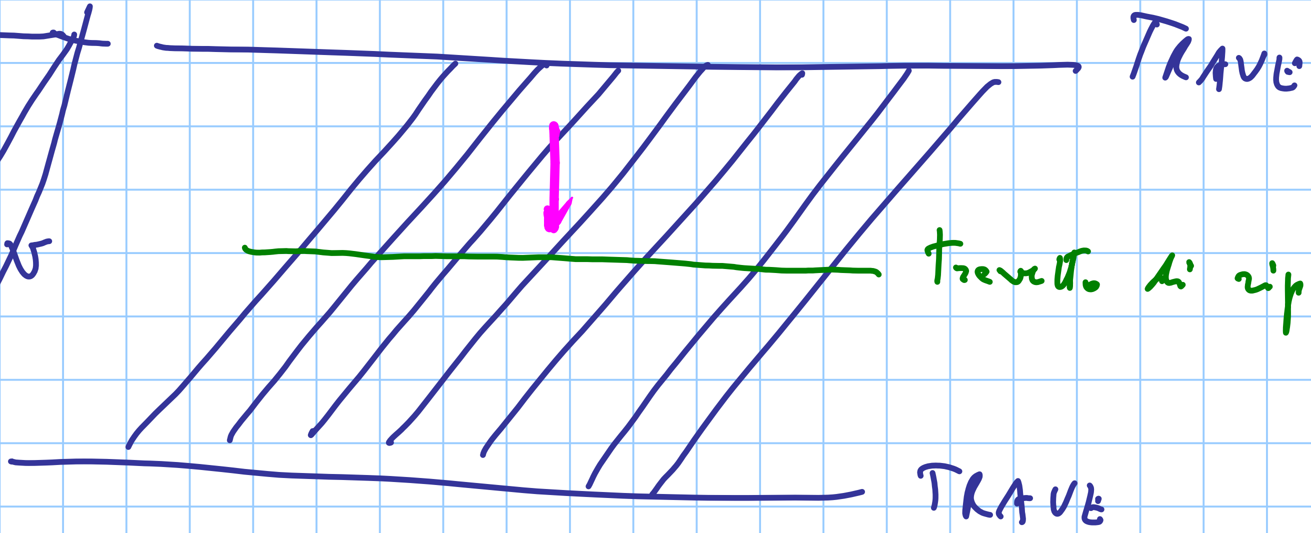
$[F L^{-1}]$

$$q_t = -K b v$$



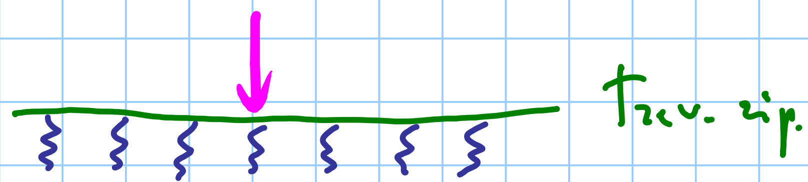
TRAVETTO DI RIPARTIZIONE



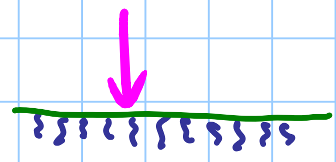


$F = -K v$

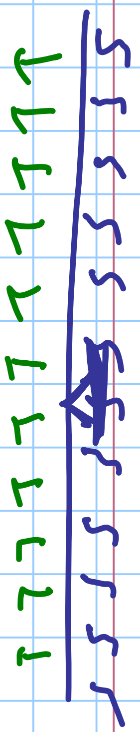
molle

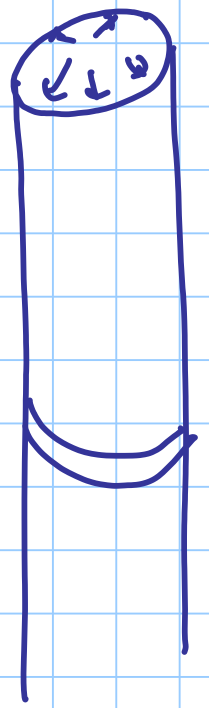


molle concentrate

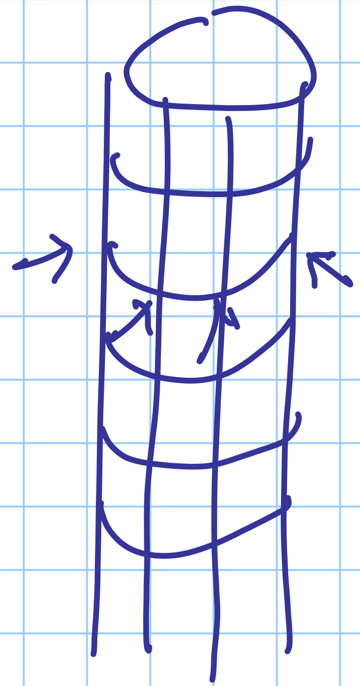


molle diffuse





def. impedita



$p = k\,\Delta z$

TRAVE SU SUOLO ELASTICO (alla Winkler)

limiti : – molle indipendenti

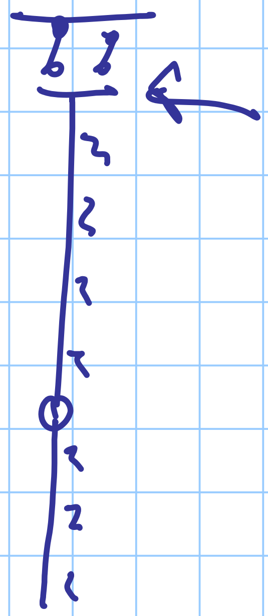
– lineare

TRAVE

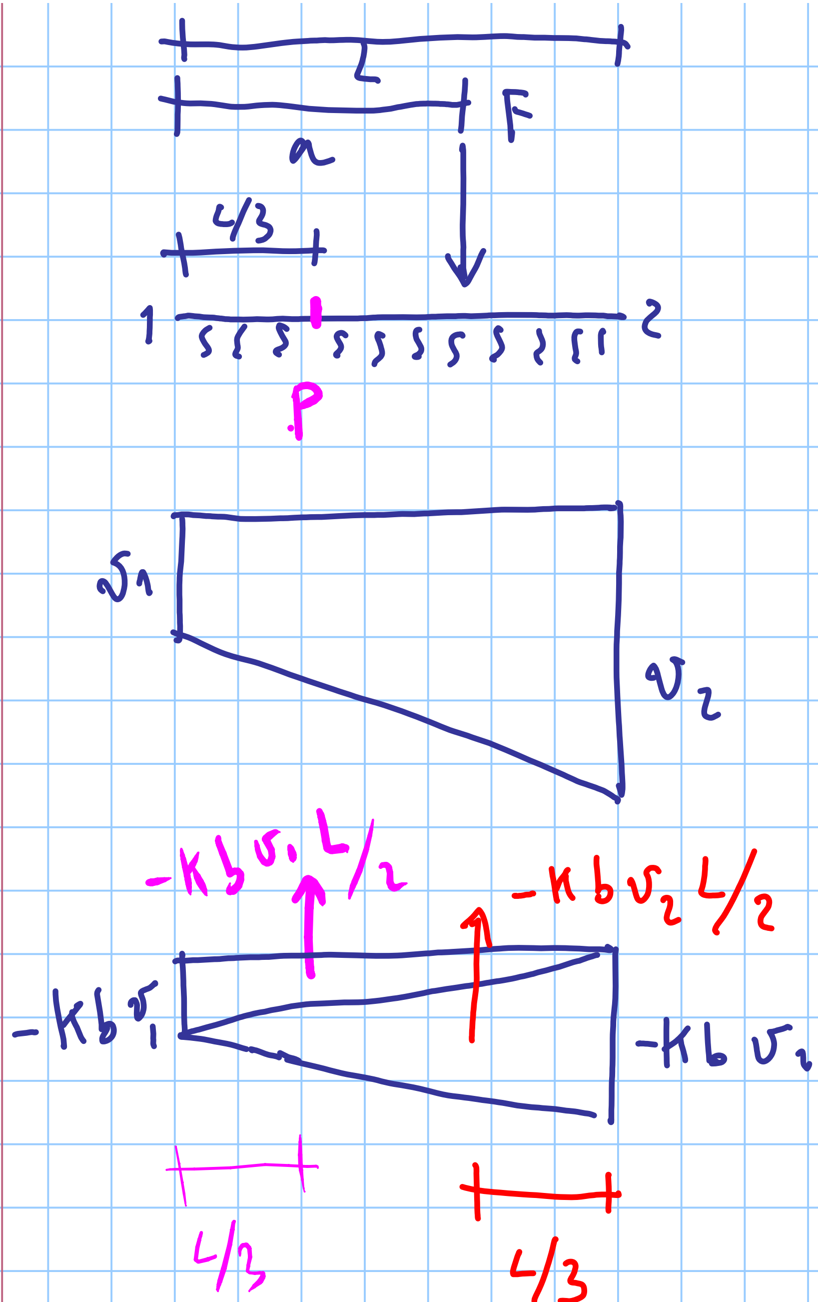
RIGIDA comportamento limite

ELASTICA reale

TRAVE RIGIDA SU SUOLO ELASTICO

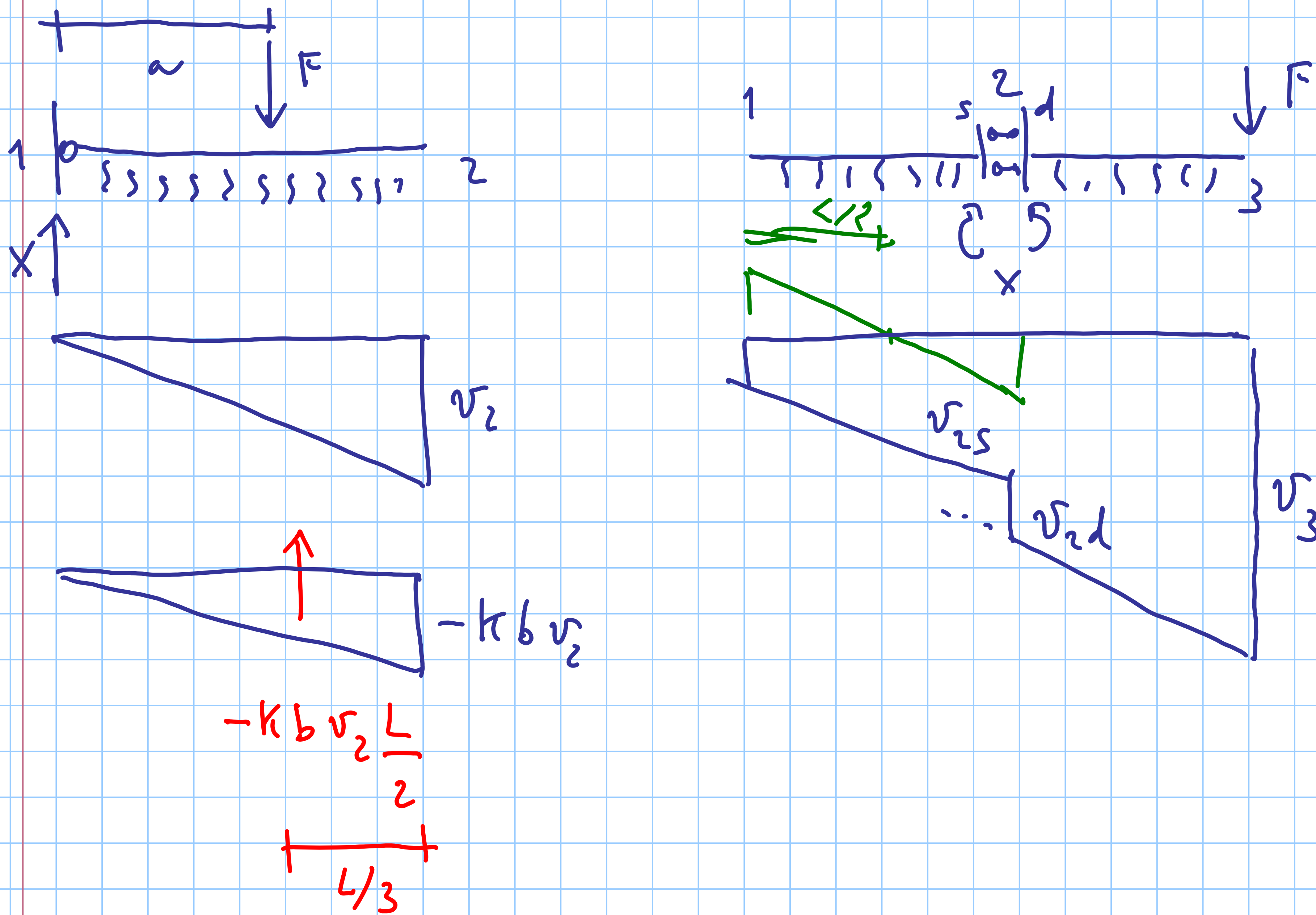


2 incognite per tratto



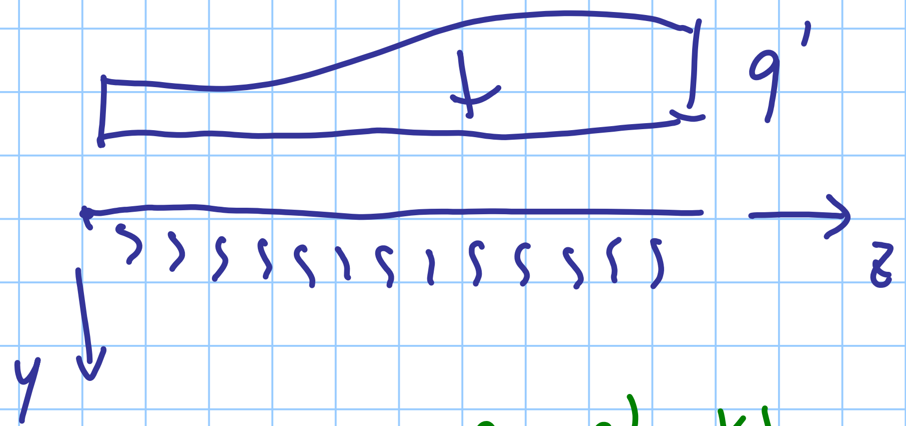
Eq. rotaz. intorno a P

$$F\left(a-\frac{L}{3}\right)=-Kb\,v_2\,\frac{L}{2}\cdot\frac{L}{3}$$



a
F
1
0
2
x
v_2
$-kbv_2$
$-kbv_2\frac{L}{2}$
L/3
1
2
s
d
F
3
L/2
x
v_{2s}
v_{2d}
v_3

TRAVE ELASTICA SU SUOLO ELASTICO



$v(z)$

$$q = q' - kbv$$

$$\frac{dv}{dz} = -\varphi$$

$$\frac{d\varphi}{dz} = \frac{M}{EI}$$

$$\frac{d^4v}{dz^4} = \frac{q}{EI} = \frac{q'}{EI} - \frac{kb}{EI}v$$

$$\frac{dM}{dz} = V$$

$$\frac{d^4v}{dz^4} + \frac{kb}{EI}v = \frac{q'}{EI}$$

$$\frac{dV}{dz} = -q$$

$$\frac{Kb}{EI} = 4\lambda^4 \qquad \lambda = \sqrt[4]{\frac{Kb}{4EI}}$$

$$\frac{d^4 v}{dz^4} + 4\lambda^4 v = \frac{q'}{EI}$$

$$\frac{d^4 v}{dz^4} + 4\lambda^4 v = 0$$ omogenea asociata

v	$e^{\lambda z} \sin \lambda z$	$e^{\lambda z} \cos \lambda z$
v'	$\lambda e^{\lambda z} \sin \lambda z + \lambda e^{\lambda z} \cos \lambda z$	$\lambda e^{\lambda z} \cos \lambda z - \lambda e^{\lambda z} \sin \lambda z$
v''	$\lambda^2 \cancel{e^{\lambda z}} \sin \lambda z + \lambda^2 e^{\lambda z} \cos \lambda z$	
	$+ \lambda^2 e^{\lambda z} \cos \lambda z - \lambda^2 \cancel{e^{\lambda z}} \sin \lambda z$	
	$= 2 \lambda^2 e^{\lambda z} \cos \lambda z$	$-2 \lambda^2 e^{\lambda z} \sin \lambda z$
v^{IV}	$-4 \lambda^4 e^{\lambda z} \sin \lambda z$	$-4 \lambda^4 e^{\lambda z} \cos \lambda z$

$$v = C_1 e^{\lambda z} \sin \lambda z + C_2 e^{\lambda z} \cos \lambda z + C_3 e^{-\lambda z} \sin \lambda z + C_4 e^{-\lambda z} \cos \lambda z$$

$+$ integrale particolare

se $q = \text{cost}$

$$\cancel{\frac{d^4 v}{dz^4}} + 4 \lambda^4 v = \frac{q}{EI} \qquad v = \frac{q}{4 \lambda^4 EI} = \frac{q}{kb}$$

La soluzione generale della (13) è

$$v(z) = C_1\, e^{\lambda z}\, \sin\lambda z + C_2\, e^{\lambda z} \cos\lambda z + C_3\, e^{-\lambda z} \sin\lambda z + C_4\, e^{-\lambda z} \cos\lambda z$$

La derivata della (14) fornisce la rotazione sezione per sezione

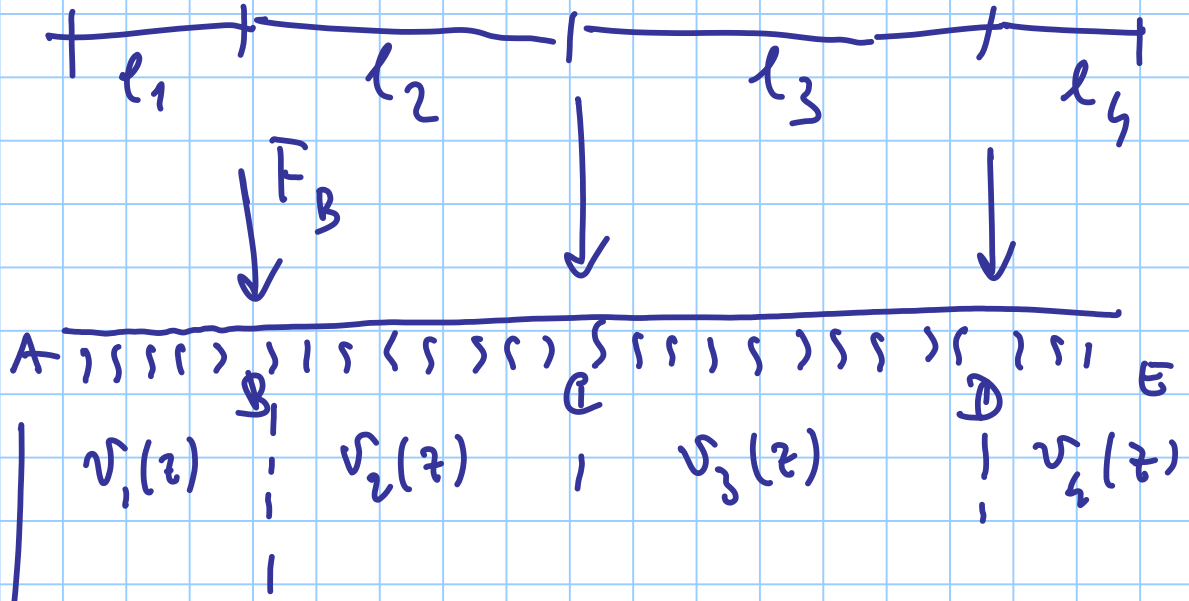
$$v'(z) = C_1\, \lambda\, e^{\lambda z}\, (\sin\lambda z + \cos\lambda z) + C_2\, \lambda\, e^{\lambda z}\, (-\sin\lambda z + \cos\lambda z) +$$
$$+ C_3\, \lambda\, e^{-\lambda z}\, (-\sin\lambda z + \cos\lambda z) + C_4\, \lambda\, e^{-\lambda z}\, (-\sin\lambda z - \cos\lambda z) = -\varphi$$

La derivata seconda è in relazione al momento flettente, ed è

$$v''(z) = 2\, C_1\, \lambda^2\, e^{\lambda z} \cos\lambda z - 2\, C_2\, \lambda^2\, e^{\lambda z} \sin\lambda z +$$
$$-2\, C_3\, \lambda^2\, e^{-\lambda z} \cos\lambda z + 2\, C_4\, \lambda^2\, e^{-\lambda z} \sin\lambda z = -\frac{M}{EI}$$

La derivata terza è legata al taglio

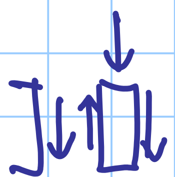
$$v'''(z) = 2\, C_1\, \lambda^3\, e^{\lambda z}\, (-\sin\lambda z + \cos\lambda z) + 2\, C_2\, \lambda^3\, e^{\lambda z}\, (-\sin\lambda z - \cos\lambda z) +$$
$$+ 2\, C_3\, \lambda^3\, e^{-\lambda z}\, (\sin\lambda z + \cos\lambda z) + 2\, C_4\, \lambda^3\, e^{-\lambda z}\, (-\sin\lambda z + \cos\lambda z) = -\frac{V}{EI}$$



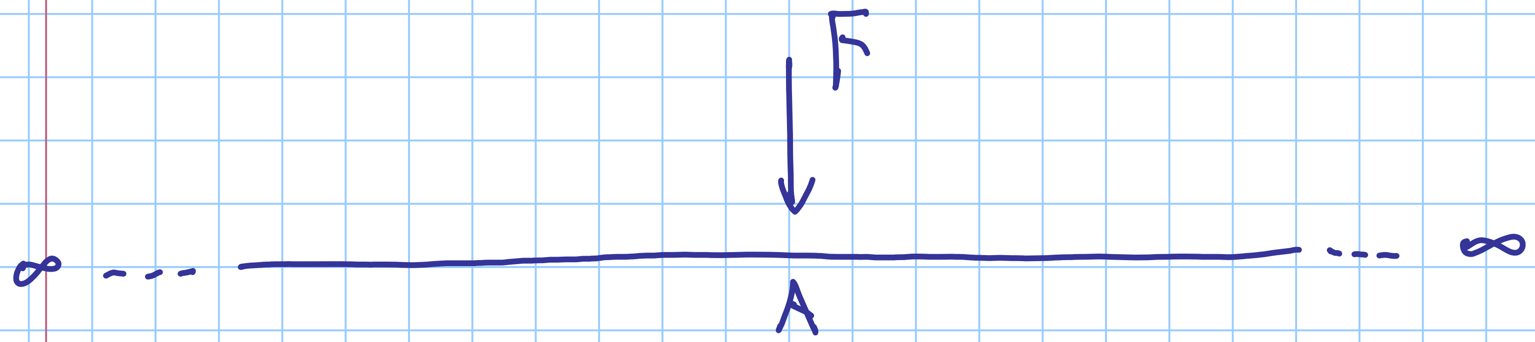
$v_1'' = 0$ $\quad v_1(l_1) = v_2(0)$

$v_1''' = 0$ $\quad v_1'(l_1) = v_2'(0)$

$v_1''(l_1) = v_2''(0)$



$V_1(l_1) = F_B + V_2(0)$



$$C_3 = C_4 \Leftarrow \varphi = 0$$

$$v = 0$$

$$\varphi = 0$$

$$\Downarrow$$

$$C_1 = C_2 = 0$$

$$4 C_3 \lambda^3 = -\frac{V}{EI}$$

$$V = -\frac{F}{2}$$

$$C_3 = \frac{F}{8 \lambda^3 EI}$$

$$v(z) = \frac{F}{8\,\lambda^3\,E\,I} e^{-\lambda z}\left(\sin\lambda z + \cos\lambda z\right) = \frac{F\,\lambda}{2\,k\,b} A_{\lambda z}$$

$$\varphi(z) = \frac{F}{4\,\lambda^2\,E\,I} e^{-\lambda z}\sin\lambda z = \frac{F\,\lambda^2}{k\,b} B_{\lambda z}$$

$$M(z) = \frac{F}{4\,\lambda} e^{-\lambda z}\left(-\sin\lambda z + \cos\lambda z\right) = -\frac{F}{4\,\lambda} C_{\lambda z}$$

$$V(z) = -\frac{F}{2} e^{-\lambda z}\cos\lambda z = -\frac{F}{2} D_{\lambda z}$$