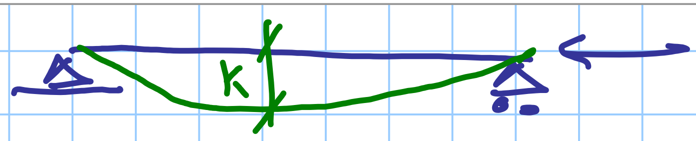
Titolo nota 20/11/2014



$$v = K \sin \frac{\pi x}{L}$$

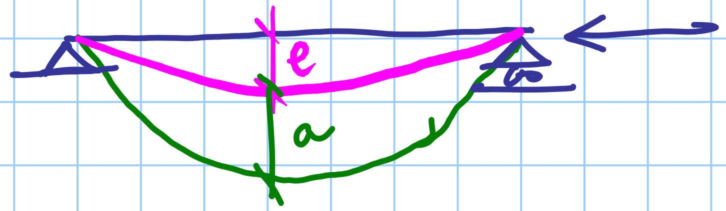
$$N = \frac{\pi^2 EI}{L^2} = N_{cr}$$

instabilità flessionale

Eulero

Hp. asta perfetta

materiale elastico lineare



$$v_0 = e \sin \frac{\pi x}{L}$$

$$v_1 = a \sin \frac{\pi x}{L}$$

$$e + a = K$$

$$v = v_0 + v_1$$

$$M = N(v_0 + v_1)$$

$$EI\, v_1'' + N(v_0 + v_1) = 0$$

$$M = -EI\, v_1''$$

$$v_1'' = -a \frac{\pi^2}{L^2} \sin \frac{\pi x}{L}$$

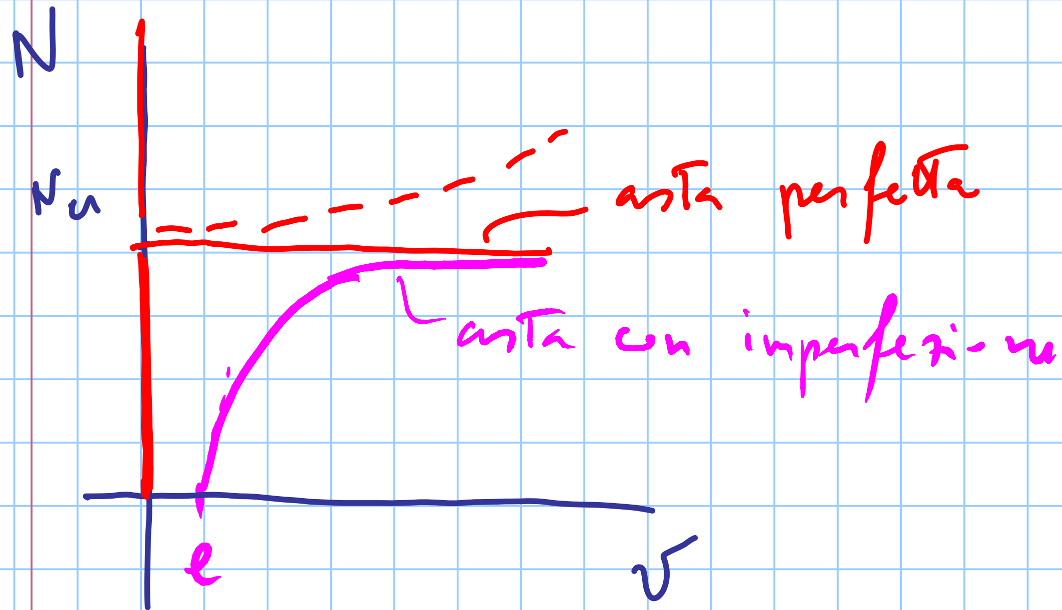
$$-EI\,a\frac{\pi^2}{L^2}\cancel{\sin\frac{\pi x}{L}}+N(e+a)\cancel{\sin\frac{\pi x}{L}}=0$$

$$-N_{cr}\,a+Ne+Na=0$$

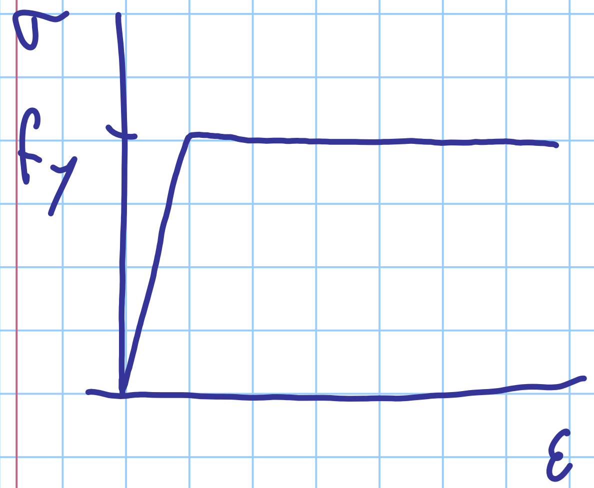
$$a(N_{cr}-N)=Ne \qquad a=\frac{N}{N_{cr}-N}e$$

$$K=a+e=\frac{N}{N_{cr}-N}e+e=\frac{\cancel{N}+N_{cr}-\cancel{N}}{N_{cr}-N}e=\frac{N_{cr}}{N_{cr}-N}e$$

$$K=a+e=\frac{1}{1-N/N_{cr}}e$$


N
N_u
asta perfetta
asta con imperfezione
e
v

MATERIALE ELASTICO - PERFETT. PLASTICO



asta ideale

$$\sigma = \frac{N}{A}$$

$$\sigma_u = \frac{N_u}{A}$$

$$\sigma_u = \frac{\pi^2 E I}{l_0^2 A}$$

$$\frac{I}{A} = i^2$$

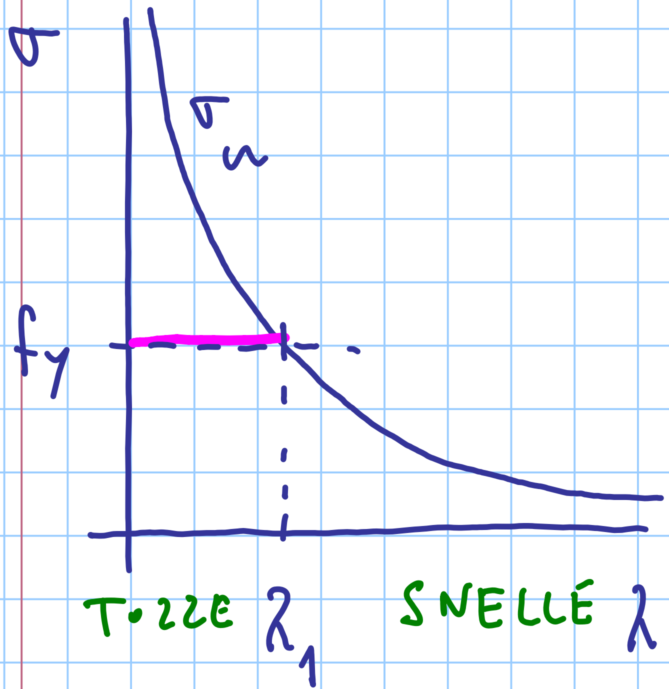
i raggio d'inerzia $[\rho]$

$$\sigma_{cr} = \frac{\pi^2 E \, i^2}{l_0^2} = \frac{\pi^2 E}{\left(\frac{l_0}{i}\right)^2}$$

$$\lambda = \frac{l_0}{i}$$

SNELLEZZA

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$



$\sigma \leq f_y$

$\lambda < \lambda_1 \qquad \sigma_u > f_y$

TOZZE

$\lambda > \lambda_1 \qquad \sigma_u < f_y$

vale Eulero fino a N_u

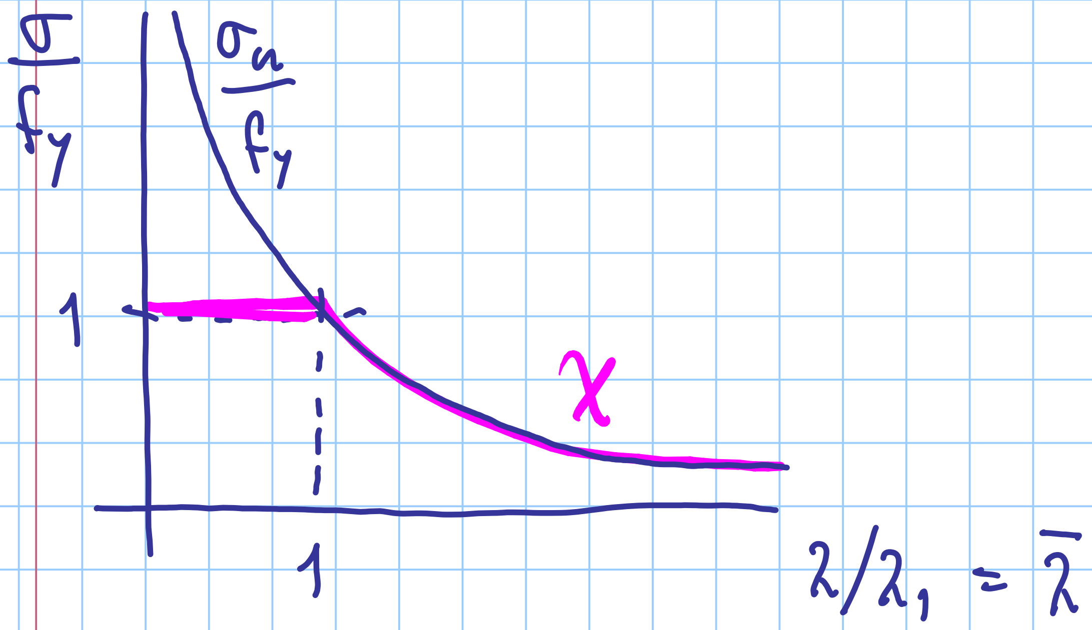
SNELLE

b = buckling
instabilità

$$N_b = \mathrm{MIN}\left(A\, f_y \;;\; N_u\right)$$

$$\frac{N_b}{A\, f_y} = \chi \quad \text{lettera greca chi} \qquad \chi = \text{MIN}\left(\frac{\sigma_u}{f_y}\, ;\, 1\right)$$

$$N_b = \chi\, A\, f_y$$



$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$

λ_1 corrisp. $\sigma_{cr} = f_y$

$$f_y = \frac{\pi^2 E}{\lambda_1^2}$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_1}$$

Es. $f_y = 235$ MPa

$E = 210000$ MPa

$$\lambda_1 = 93.9$$

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \qquad \lambda^2 = \frac{\pi^2 E}{\sigma_{cr}} \qquad \lambda = \pi \sqrt{\frac{E}{\sigma_{cr}}}$$

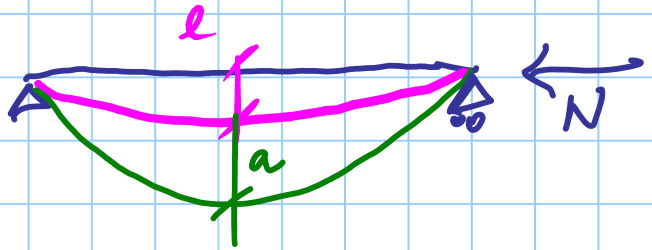
$$f_y = \frac{\pi^2 E}{\lambda_1^2} \qquad \lambda_1^2 = \frac{\pi^2 E}{f_y} \qquad \lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} = \sqrt{\frac{f_y}{\sigma_{cr}}} = \sqrt{\frac{A f_y}{N_{cr}}} \qquad \bar{\lambda}^2 N_{cr} = A f_y$$

$$N_b = \mathrm{MIN}\left(A f_y \, ; \, N_{cr}\right) = \chi \, A f_y$$

$$x = \mathrm{MIN}\left(\frac{1}{\lambda^2} ; 1\right)$$

ASTA IMPERFETTA ; $\sigma - \varepsilon$ ELASTO PLASTICO

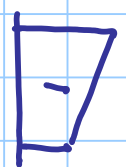


$$K = \frac{1}{1 - N/N_{cr}} \, e$$

$K = e + a$

$$M_{max} = N K$$

σ



$$\sigma_{max} = \frac{N}{A} + \frac{M}{W}$$

$$\sigma_{max} = \frac{N}{A} + N \frac{1}{1 - N/N_u} e \frac{1}{W} =$$

$$= \frac{N}{A} \left[1 + \frac{1}{1 - N/N_u} \frac{e\,A}{W} \right]$$

$$\frac{e\,A}{W} = \eta$$ lettera greca eta

$$\sigma_{max} = \frac{N}{A} \left[1 + \frac{1}{1 - N/N_u} \eta \right]$$

$$\sigma_{max} = \frac{N}{A} \frac{1 - N/N_u + \eta}{1 - N/N_u}$$

$$\sigma_{max} \leq f_y$$

quando $\sigma_{max} = f_y$ si ottiene la massima capacità portante N_b

$$f_y = \frac{N_b}{A} \frac{1 - N_b/N_u + \eta}{1 - N_b/N_u}$$

$$1 - \frac{N_b}{N_u} = \frac{N_b}{A f_y} \left[1 - \frac{N_b}{N_u} + \eta \right]$$

$$\frac{N_b}{A f_y} = \chi \qquad \frac{N_b}{N_u} = \frac{N_b}{A f_y} \frac{A f_y}{N_u} = \chi \bar{\lambda}^2$$

$$1 - \chi \bar{\lambda}^2 = \chi \left[1 - \chi \bar{\lambda}^2 + \eta \right]$$

$$\bar{\lambda}^2 x^2 - \left(\bar{\lambda}^2 + 1 + \eta\right) x + 1 = 0$$

$$\phi = \frac{1}{2}\left(1 + \eta + \bar{\lambda}^2\right)$$

$$\bar{\lambda}^2 x^2 - 2\phi x + 1 = 0$$

$$x = \frac{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}{\bar{\lambda}^2}$$

$$\chi = \frac{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}{\bar{\lambda}^2} \qquad \frac{\phi - \sqrt{\phi^2 - \bar{\lambda}^2}}{\phi - \sqrt{\phi^2 - \bar{\lambda}^2}} =$$

$$= \frac{\phi^2 - (\phi^2 - \bar{\lambda}^2)}{\bar{\lambda}^2 \left[\phi - \sqrt{\phi^2 - \bar{\lambda}^2}\right]} = \frac{1}{\phi - \sqrt{\phi^2 - \bar{\lambda}^2}}$$

$$\phi = \frac{1}{2}\left(1 + \eta + \bar{\lambda}^2\right) \qquad \eta = \alpha\left(\bar{\lambda} - 0.2\right)$$

coeff. di imperfez.

calcolare $\bar{\lambda}$

definire α

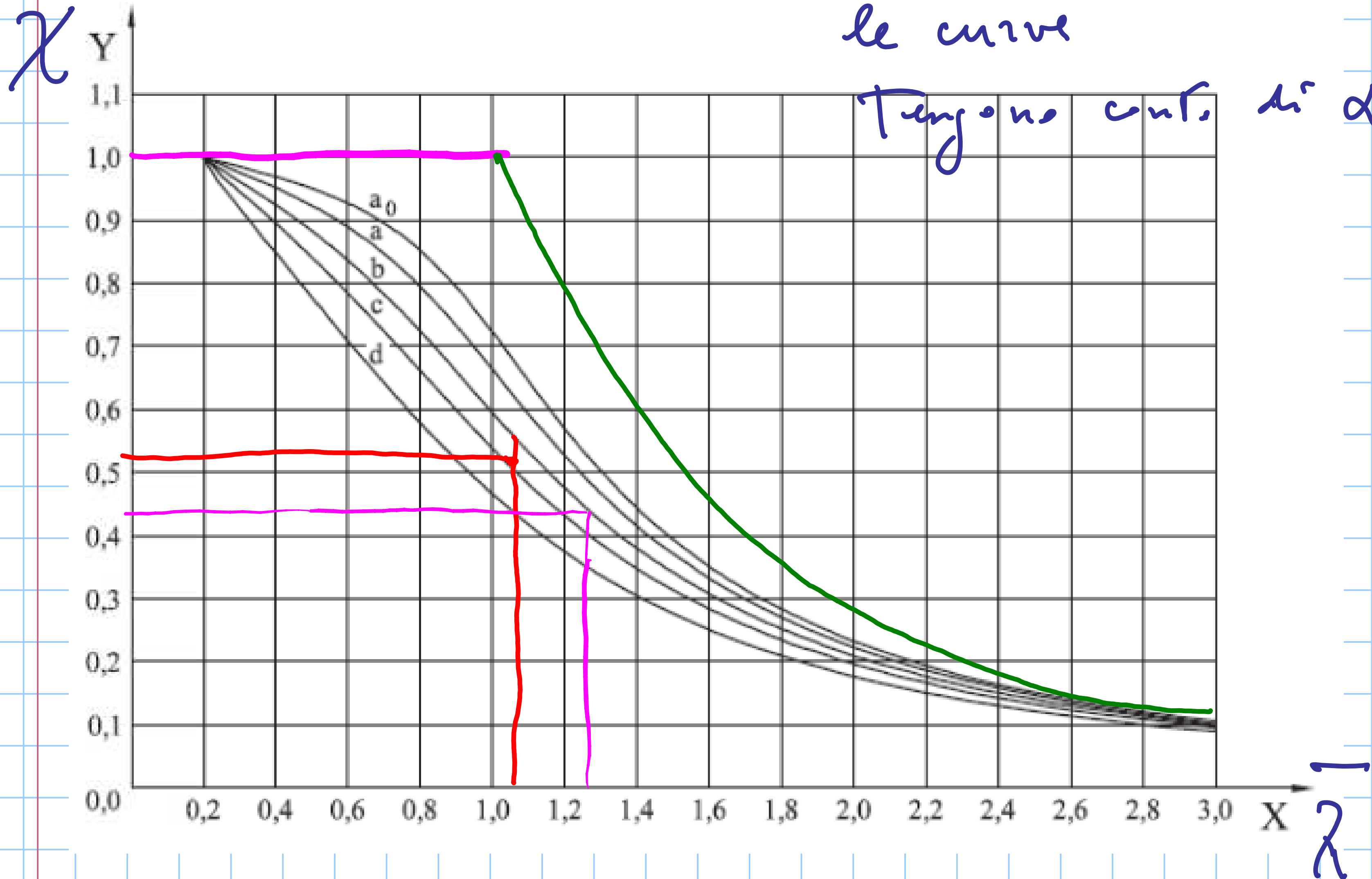
calcolare $\phi = \frac{1}{2}\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$

calcolare $\chi = \frac{1}{\phi - \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1$

$N_{b,Rd} = \chi A \frac{f_y}{\gamma_{M1}}$ $\gamma_{M1} = 1.05$

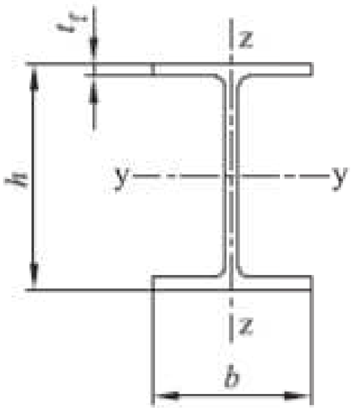
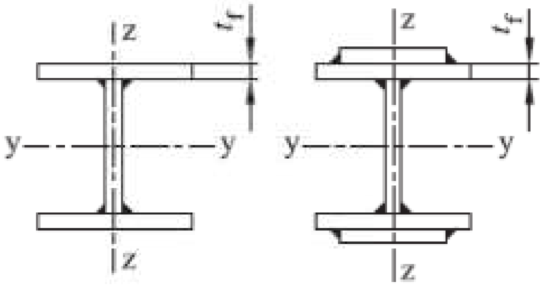
X Snellezza adimensionale $\bar{\lambda}$

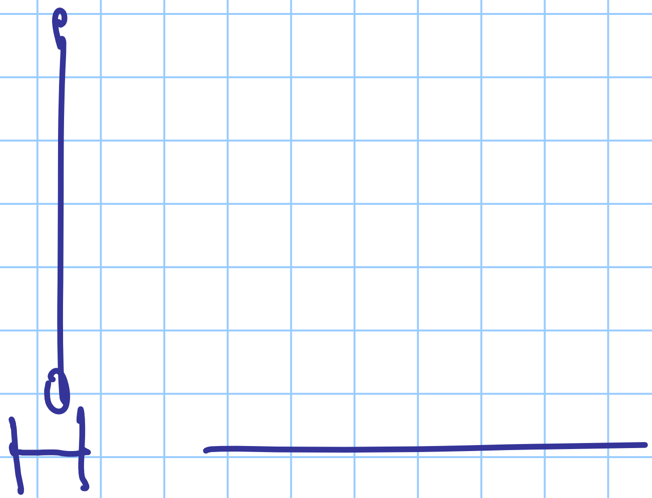
Y Coefficiente di riduzione χ

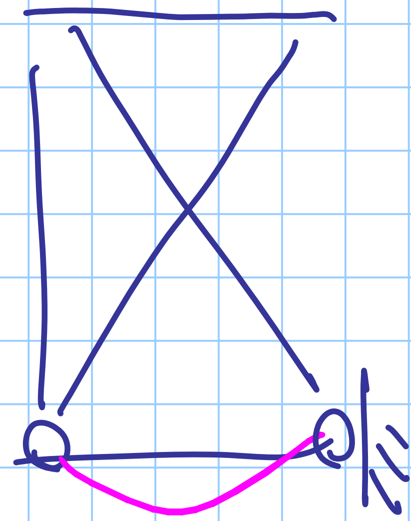


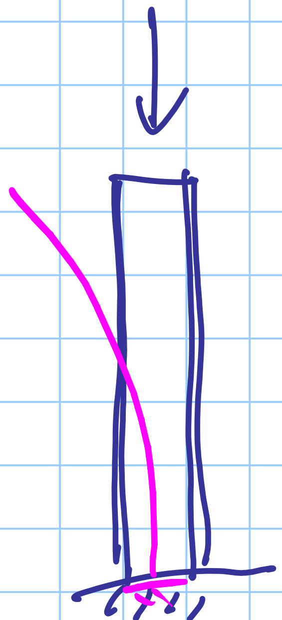
le curve tengono conto di α

Curva di instabilità	a_0	a	b	c	d
Coefficiente di imperfezione α	0,13	0,21	0,34	0,49	0,76

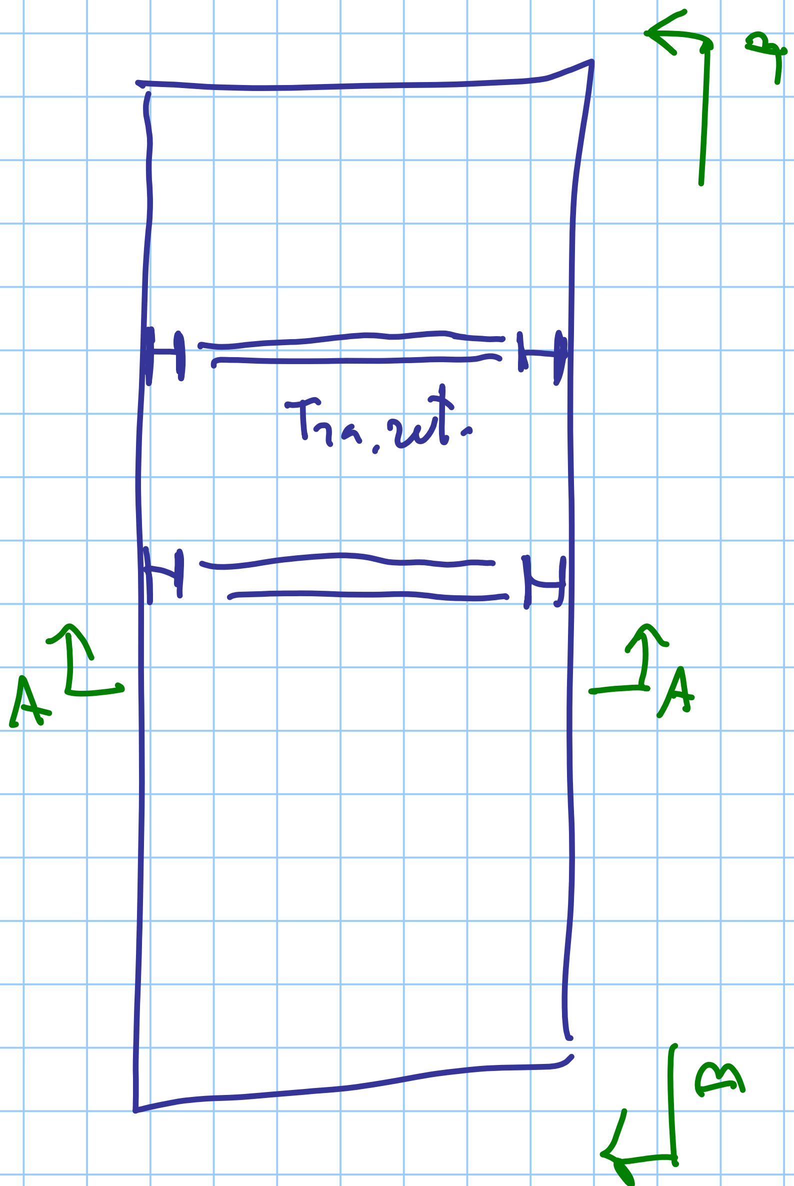
Sezione trasversale		Limiti		Instabilità intorno all'asse	Curva di instabilità S 235 S 275 S 355 S 420	Curva di instabilità S 460
Sezioni laminate		$h/b > 1{,}2$	$t_f \leq 40$ mm	y - y z - z	a b	a_0 a_0
			40 mm $< t_f \leq$ 100 mm	y - y z - z	b c	a a
		$h/b \leq 1{,}2$	$t_f \leq$ 100 mm	y - y z - z	b c	a a
			$t_f >$ 100 mm	y - y z - z	d d	c c
Sezioni al saldate		$t_f \leq 40$ mm		y - y z - z	b c	b c
		$t_f > 40$ mm		y - y z - z	c d	c d

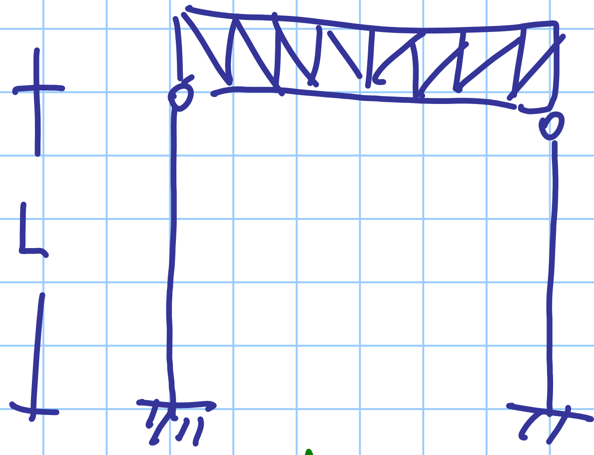






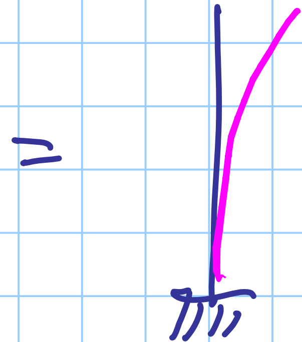
ASTA CON VINCOLO DIVERSO NEI DUE PIANI

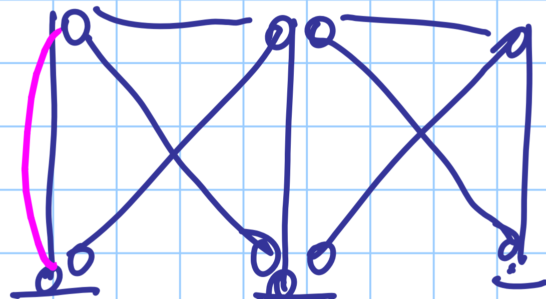




H

A - A





I

B - B

RESTIVO

colonna HE 240 B $L = 6.00\ m$

DEFINISCO $\lambda = \frac{l_0}{i}$ NEL PIANO B-B

$l_0 = L = 6{,}00\ m$

$i_y = 103{,}1\ mm$

$i_z = 60{,}8\ mm$

$i = 60{,}8\ MM$

$$\lambda = \frac{6.000\ MM}{60{,}8\ MM} = \simeq 100 = 98{,}7$$

$\lambda_1 = 93,9$ PER ACCIAIO S235

$\bar{\lambda} = \dfrac{\lambda}{\lambda_1} = 1,05$

CURVA C $\Rightarrow \alpha = 0,49$

$\chi = 0,51$

$$N_{b,rd} = \chi A \cdot \frac{f_y}{\gamma_{M1}} = 0,51 \cdot \underset{\times 10^2}{106,0} \cdot \frac{235}{1,05} \times 10^{-3} = 1209 \text{ kN}$$

INTERLANDI

$$\lambda = \frac{l_0}{i} \qquad l_0 = 2l = 12\,m$$

$$i = 103{,}1\ mm$$

$$\lambda = \frac{12000}{103{,}1} = 116{,}4$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} = \frac{116{,}4}{93{,}9} = 1{,}24$$

CURVA b $\rightarrow$ $\chi = 0{,}44$

$$N_{b,Rd} = \chi A \frac{f_y}{\gamma_{M1}} \qquad A = 106 \times 10^2\ mm^2$$

$$N_{b,Rd} = 0{,}44 \times 106 \times 10^2 \times \frac{235}{1{,}05} = 1048\ kN$$

PROGETTO

N_{Ed}

$$N_{b,Rd} = \chi A \frac{f_y}{\gamma_{M1}}$$

suggerimenti

ipotizzare χ

calcolare A

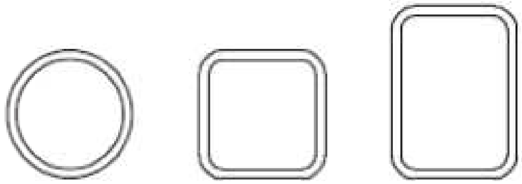
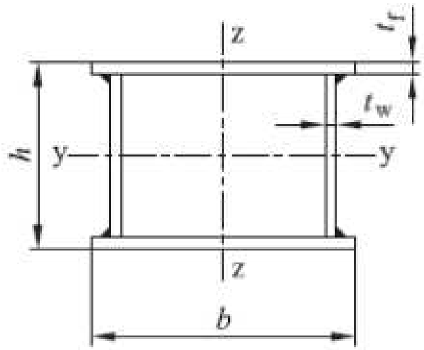
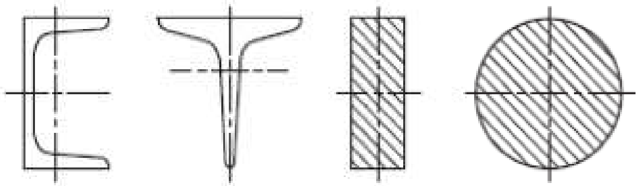
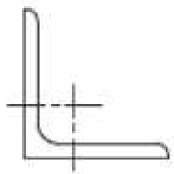
scegliere il profilo

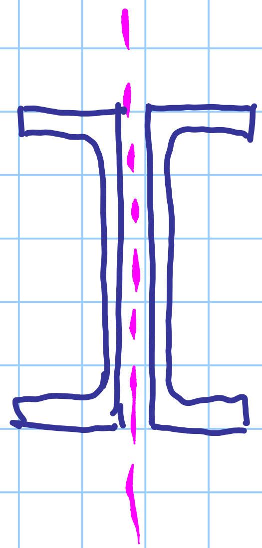
calcolare χ

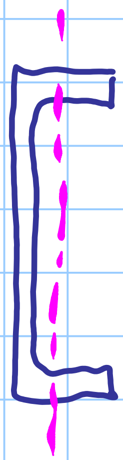
$$A \geq \frac{N_{Ed}\, \gamma_{M1}}{\chi\, f_y}$$

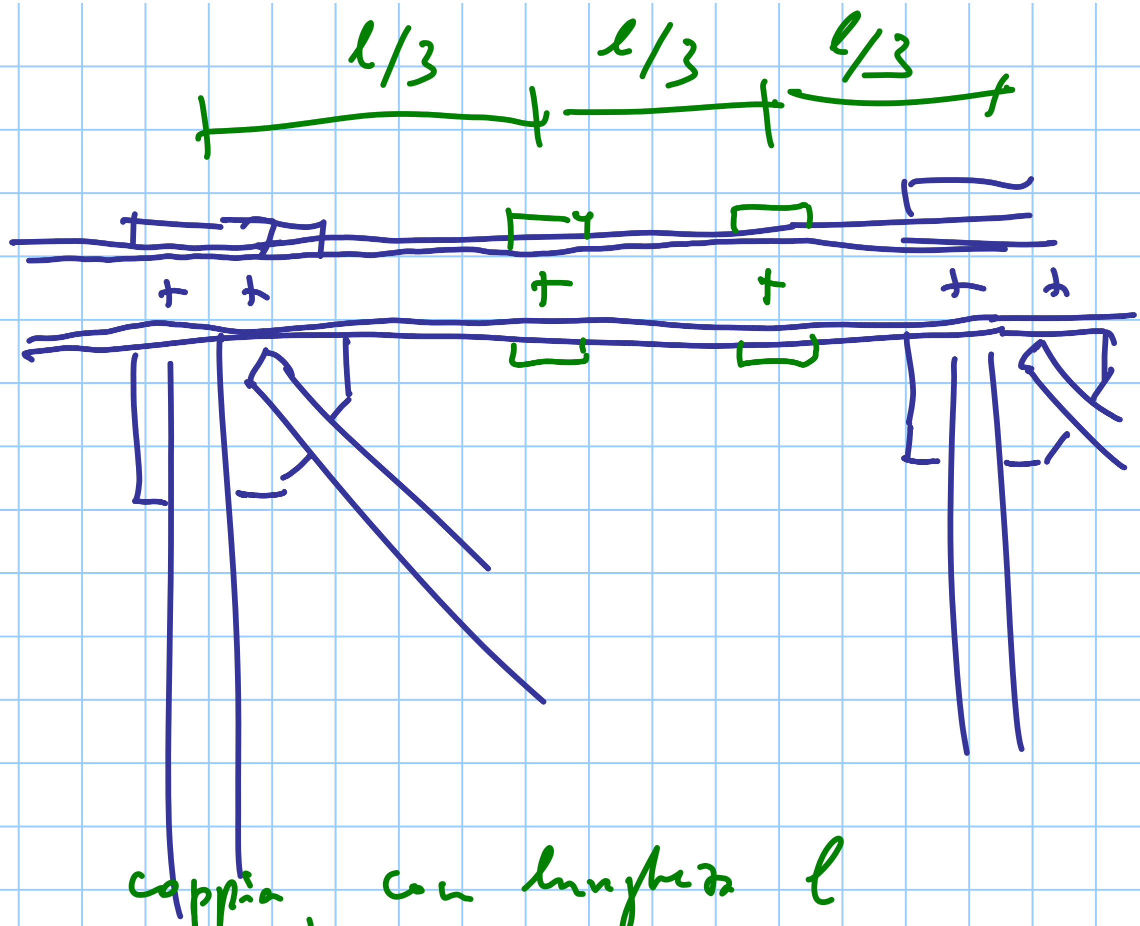
?

EVITARE SNELLEZZE ECCESSIVE

Sezioni tubolari		Laminate a caldo	qualunque	*a*	a_0
		Formate a freddo	qualunque	*c*	*c*
Sezioni a cassone saldate		In generale (ad eccezione di quanto riportato sotto)	qualunque	*b*	*b*
		Saldature spesse: $a > 0,5\ t_f$ $b/t_f < 30$ $h/t_w < 30$	qualunque	*c*	*c*
Sezioni a U, T e sezioni piene			qualunque	*c*	*c*
Sezioni a L			qualunque	*b*	*b*







λ_2 coppia, con lunghezza l

λ_1 singolo, con lunghezza $l/3$

$$\lambda_{eq} = \sqrt{\lambda_1^2 + \lambda_2^2}$$