TAGLIO $V \rightarrow \tau$

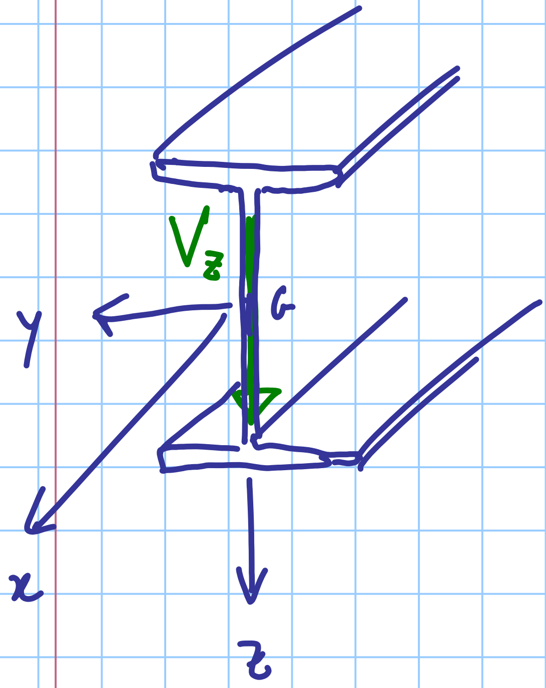
Titolo nota 16/12/2014

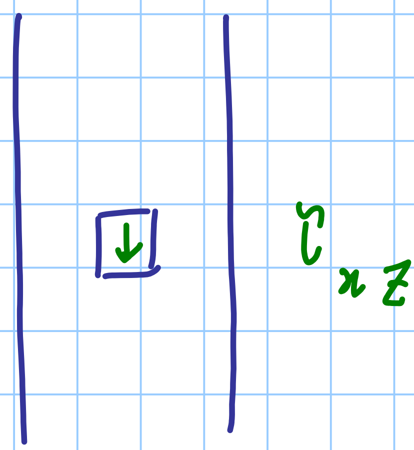
modello lineare

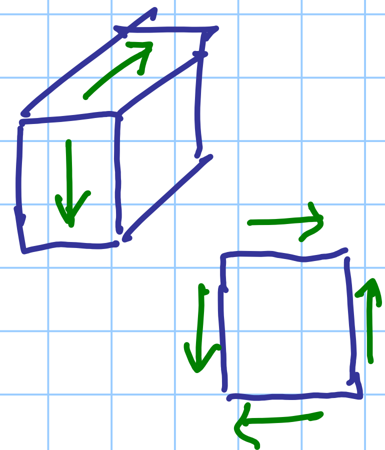
DE SAINT VENANT

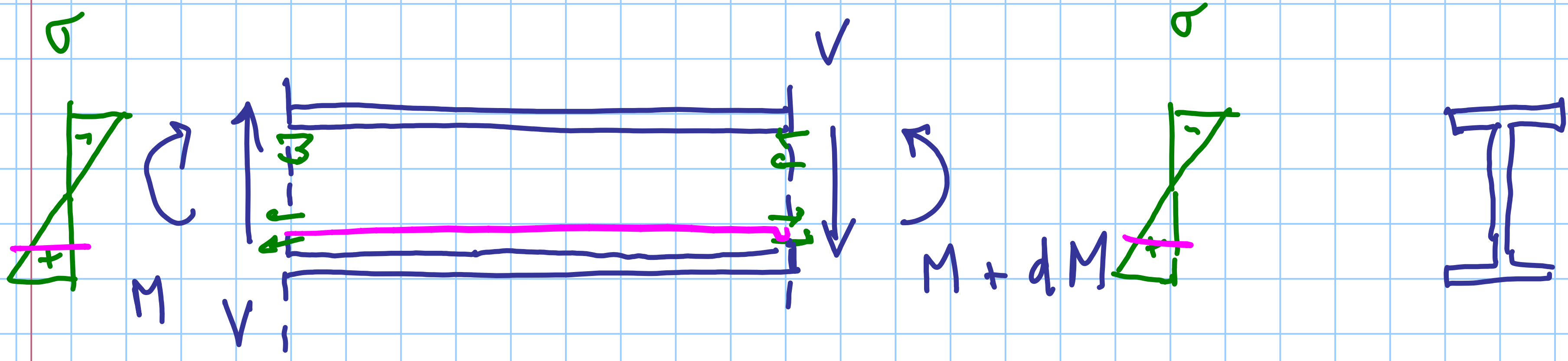
JOURAWSKI

$$V_z = \int \tau_{xz}\, dA$$





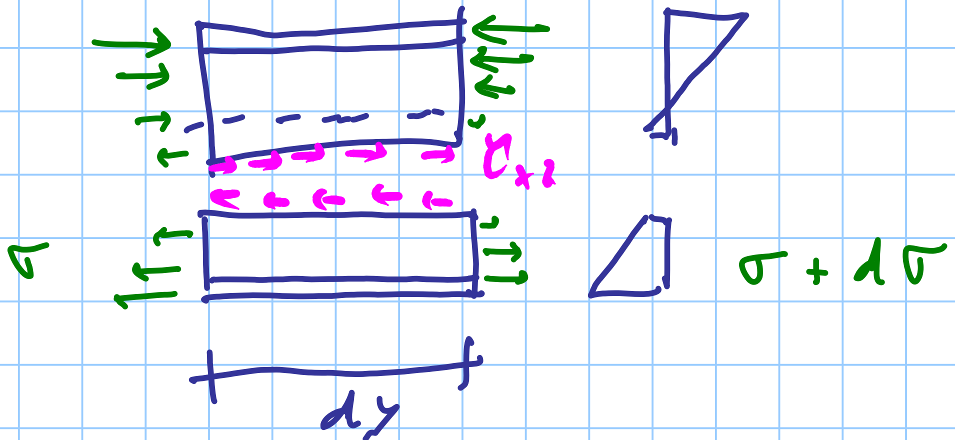




$$V = \frac{dM}{dx}$$

$$\sigma = \frac{M_y}{I_y} z$$

$$\sigma = \frac{M_y + dM}{I} z$$



$$d\sigma = \frac{dM_y}{I} z$$

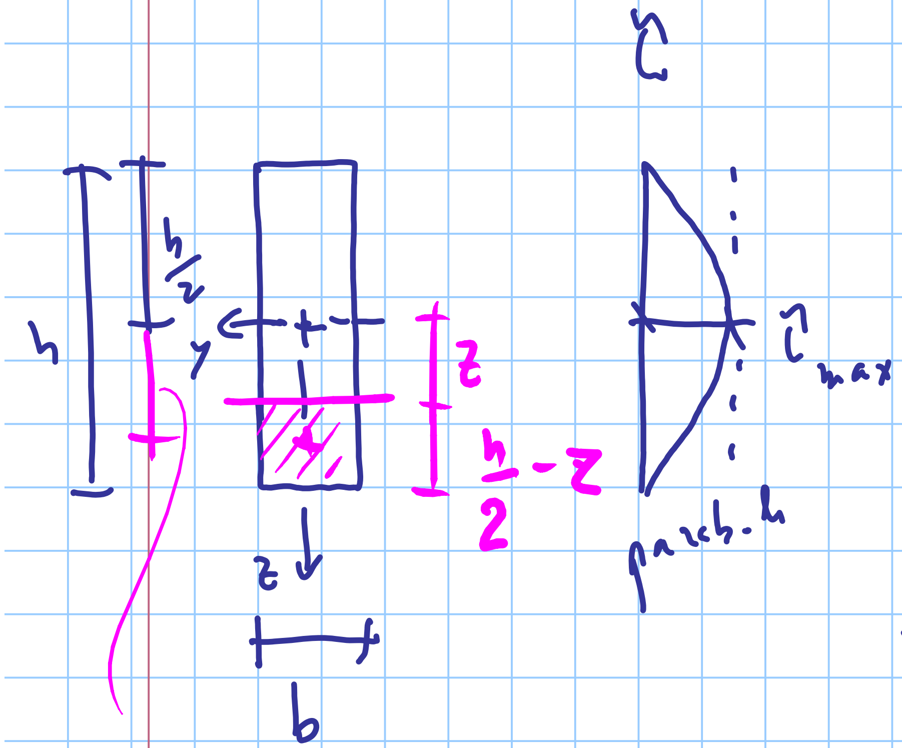
nascono τ per l'equilibrio alla traslazione orizzontale

$$\int d\sigma \, dA = \int \frac{dM_y}{I} z \, dA = \int \frac{V \, dy}{I} z \, dA =$$

$$dM_y = V \, dy \qquad = \frac{V \, dy}{I} \underbrace{\int z \, dA}_{S}$$

$$\tau_{xz\,(\text{medio})}\; b\, \cancel{dy} = \frac{V\, \cancel{dy}}{I}\, S$$

$$\tau = \frac{V S}{I\, b}$$



V

$$I = \frac{b h^3}{12}$$

$$S = b\left(\frac{h}{2} - z\right) \cdot \frac{1}{2}\left(\frac{h}{2} + z\right) =$$

$$= \frac{1}{2} b \left(\frac{h^2}{4} - z^2\right)$$

$$\frac{1}{2}\left(z + \frac{h}{2}\right)$$

$$\tau = \frac{V S}{I b}$$

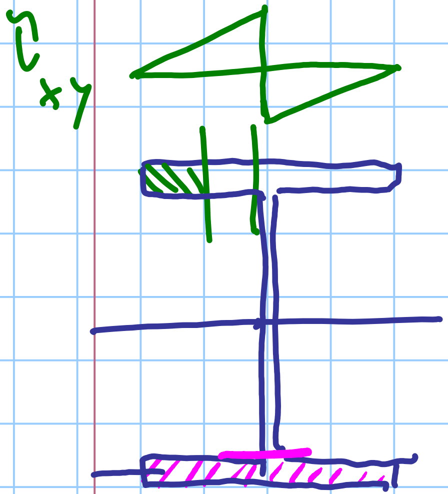
b

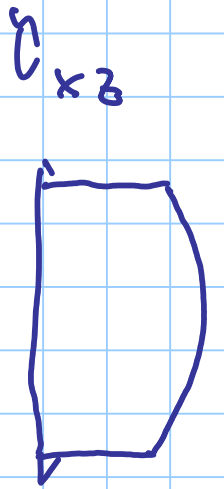
τ_{max} ricavo τ in funzione di z

calcolato per $z = 0$

$$V = \int \tau \, dA = \int \tau \, b \, dz = b \int \tau \, dz = b \, \frac{2}{3} \tau_{max} h$$

$$\tau_{max} = 1{,}5 \, \frac{V}{b\,h}$$





$$V_z = \int \tau_{xz} \, dA$$

$$t_w < t_f$$

$$S_{ala} = 2 \, S_{1/2\,ala}$$

$$\tau_{xy,max} = \frac{V \, S_{1/2\,ala}}{I \, t_f}$$

$$\tau_{xz,o} = \frac{V \, S_{al}}{I \, t_w}$$

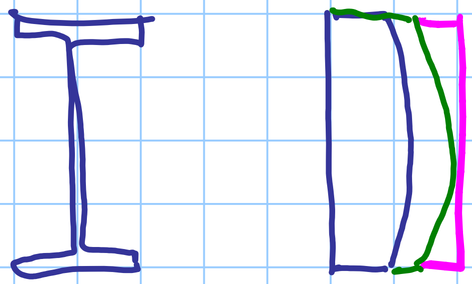
CRITERI DI RESISTENZA con σ_x τ_{xt}

Mises
$$\sigma_{id} = \sqrt{\sigma^2 + 3\tau^2} \leq \sigma_{limite}$$

in presenza di sola τ

$$\sigma_{id} = \tau\sqrt{3} \leq \sigma_{lim} \qquad \tau \leq \frac{\sigma_{lim}}{\sqrt{3}}$$

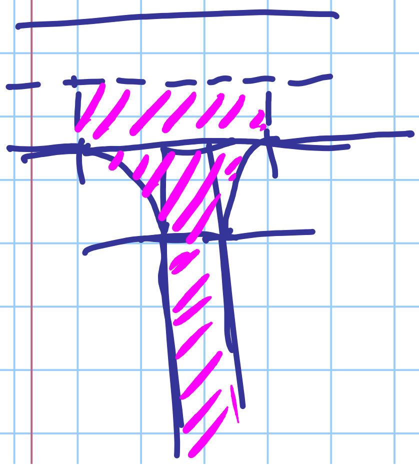
COSA SUCCEDE ALLO SLU



$$\frac{f_y/\sqrt{3}}{\gamma_{M0}}$$

$$V_{Rd} = A_v \frac{f_y/\sqrt{3}}{\gamma_{M0}}$$

area dell'anima



$$A_v = A - 2 b t_f + (t_w + 2r) t_f$$

ESEMPIO HE 160 A luce $L = 6.40\ m$

S 275

$g_1 + q_1 = 3.60\ KN/m$

$$M_{Ed} = \frac{ql^2}{8} = 18.43\ KNm$$

$$M_{Rd} = W_{pl} \frac{f_y}{\gamma_{M_0}} = 245.1 \times 10^3 \times \frac{275}{1.05} \times 10^{-6} = 64.19$$

(in realtà c'è anche M_z)

e limiti SLE

$b = 160$ mm

$t_f = 9$ mm $r = 15$ mm $A = 3880\ mm^2$

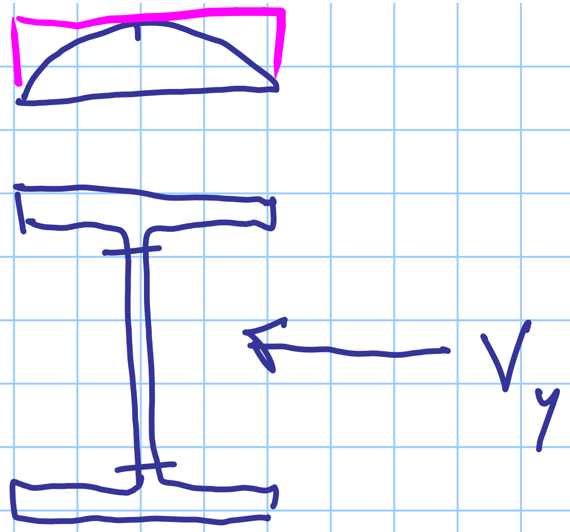
$t_w = 6$ mm

$$A_v = \underset{3880}{A} - \underset{2880}{2 b t_f} + \underset{324}{(t_w + 2r) t_f} = 1324 \text{ mm}^2$$

dal sagomario 1321 mm^2

$$V_{Ed} = \frac{q l}{2} = \frac{3.60 \times 6.40}{2} = 11.52 \text{ kN}$$

$$V_{Rd} = A_v \frac{f_y/\sqrt{3}}{\gamma_{M0}} = 1321 \times \frac{275/\sqrt{3}}{1.05} \times 10^{-3} = 599.2 \text{ kN}$$

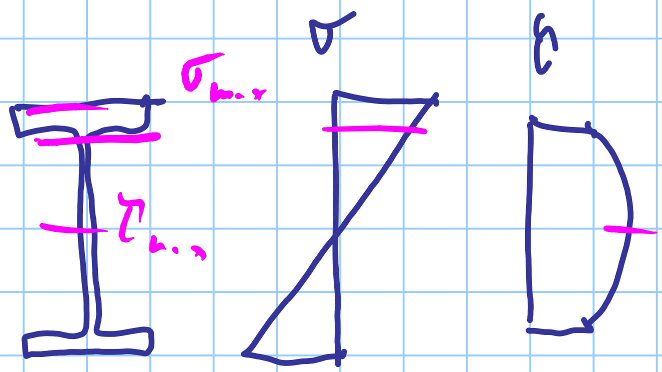


$$V_{Rd} = A_v \frac{f_y/\sqrt{3}}{\gamma_{M0}}$$

$$A_v \simeq 2\, b\, t_f$$

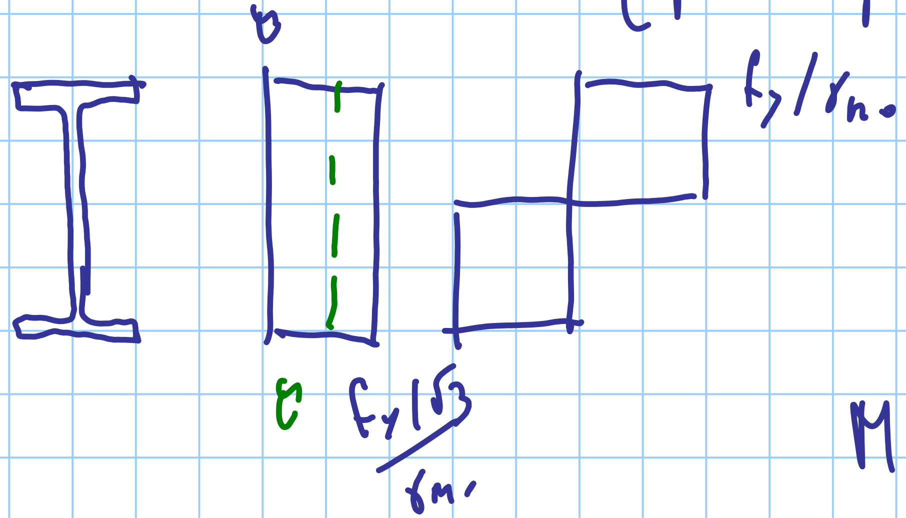
TAGLIO + FLESSIONE

modello lineare



$$\sqrt{\sigma^2 + 3\tau^2} < \sigma_{lim}$$

modello non lineare (piena plasticizzazione



se l'anima è plast. a taglio
non può portare M

$$M_{Rd} = (W_{pl,tot} - W_{pl,an}) \frac{f_y}{\gamma_{M0}}$$

$$\frac{\tau}{f_y/\sqrt{3}\,/\,\gamma_{M0}} = \frac{V_{Ed}}{V_{Rd}} \qquad \sqrt{3}\,\tau = \frac{V_{Ed}}{V_{Rd}}\,\frac{f_y}{\gamma_{M0}}$$

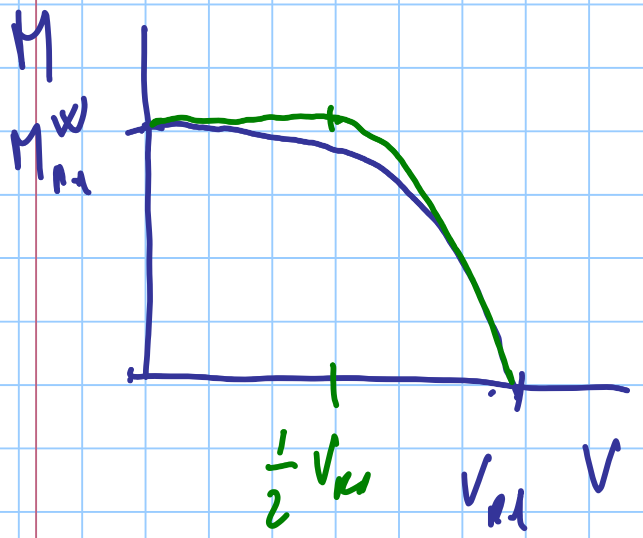
$$3\,\tau^2 = \left(\frac{V_{Ed}}{V_{Rd}}\right)^2 \left(\frac{f_y}{\gamma_{M0}}\right)^2$$

$$\sqrt{\sigma^2 + 3\,\tau^2} \leq \frac{f_y}{\gamma_{M0}}$$

$$\sigma^2 \leq \left(\frac{f_y}{\gamma_{M0}}\right)^2 - 3\,\tau^2 = \left(\frac{f_y}{\gamma_{M0}}\right)^2 \left[1 - \left(\frac{V_{Ed}}{V_{Rd}}\right)^2\right]$$

quindi $\sigma_{r.r} = \frac{f_y}{\gamma_{M0}} \sqrt{1 - \frac{V_{Ed}^2}{V_{Rd}^2}}$

contributo dell' anima.



$$\rho = \left(\frac{2 V_{Ed}}{V_{Rd}} - 1 \right)^2$$

per $V_{Ed} \geq \frac{1}{2} V_{Rd}$ $\qquad 0 \leq \rho \leq 1$

$V_{Ed} = \frac{1}{2} V_{Rd} \qquad\qquad V_{Rd}$

$$W_{\cancel{el}\,pl,w} = t_w \frac{h^2}{\cancel{6}\,4} = \frac{t_w^2 \, h^2}{\cancel{6}\,4 \, t_w} = \frac{A_v^2}{\cancel{6}\,4 \, t_w}$$

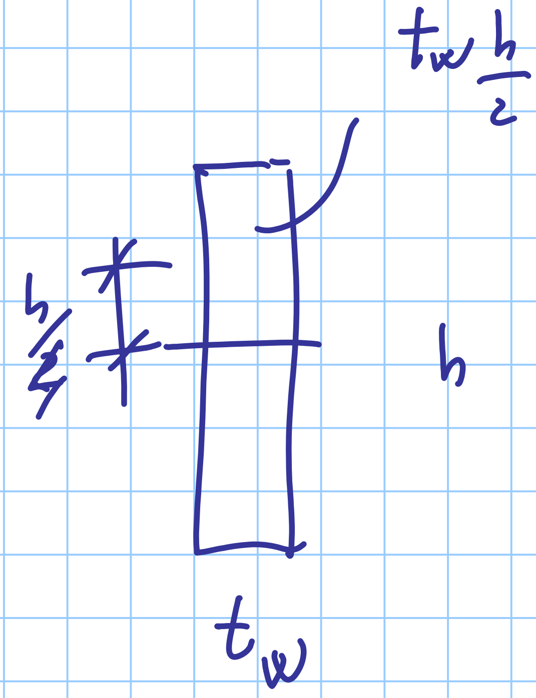
$t_w \times h$

$$M_{Rd} = \left(W_{pl} - \rho \frac{A_v^2}{4 t_w} \right) \frac{f_y}{\gamma_{M0}}$$

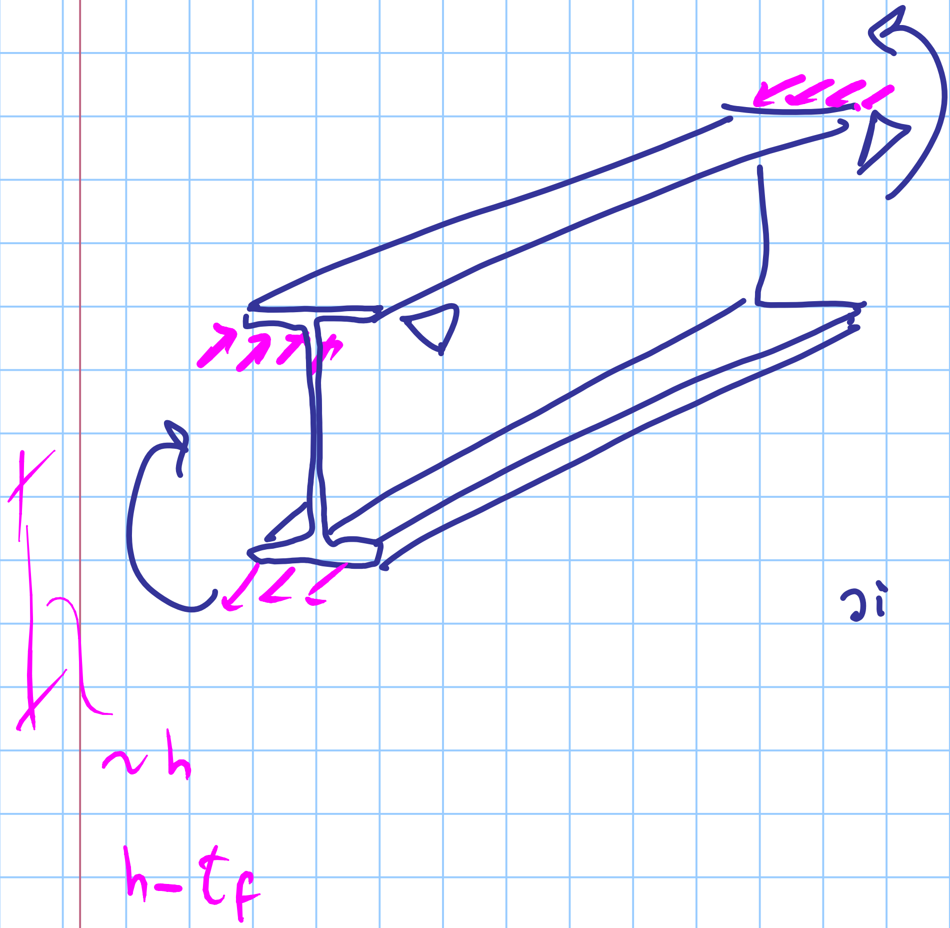
$$W_{pl} = 2 \; S_{y/2} = t_w \frac{h^2}{4}$$

$$\downarrow$$

$$t_w \frac{h^2}{8}$$



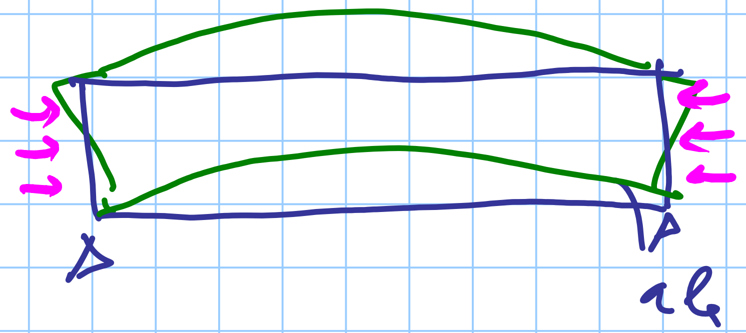
INSTABILITÀ FLESSO-TORSIONALE



ALA COMPRESSA

(da sola, ma vincolata dall'anima)

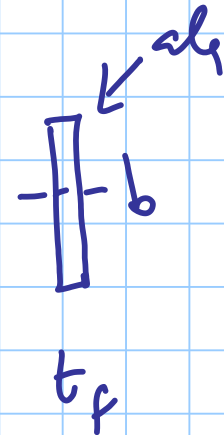
si può instabilizzare nel piano orizzontale dall'ali



l'instabilità nel piano verticale è impedita dall'anima

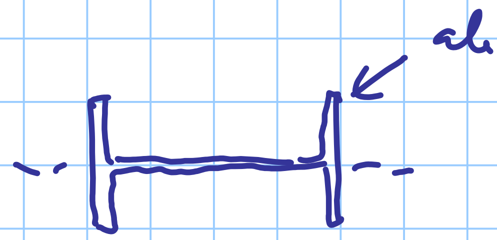
$$N_{cr} = \frac{\pi^2 E I}{l_0^2}$$

$$I = \frac{t_f\, b^3}{12}$$



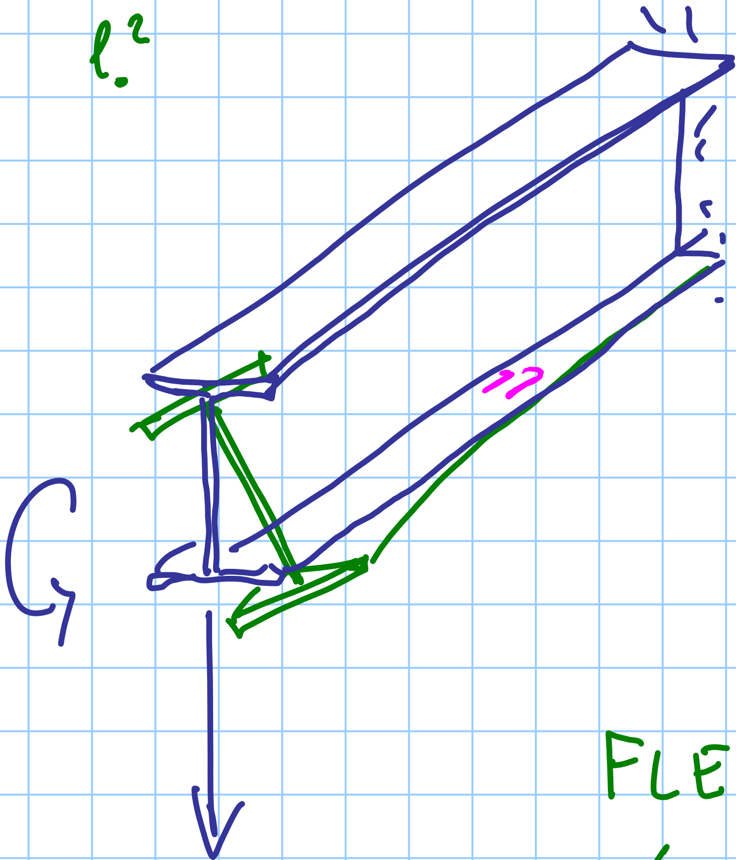
$$N = \frac{M}{h-t_f} \Rightarrow M_{cr} = N_{cr}(h-t_f)$$

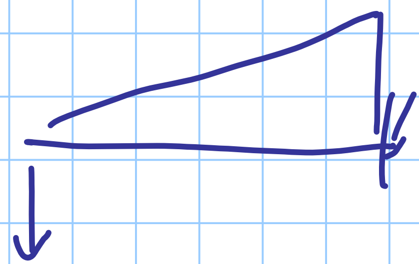
$$\simeq \frac{1}{2} I_z$$



$$M_u = \frac{\pi^2 E I_z}{l_0^2} / 2\,(h - t_f)$$

M var. lineare





l'ala inferiore sbanda

FLESSO - TORSIONALE

momenti da M

rotazioni torsionali

$$M_u = \sqrt{\frac{\pi^2}{L^2} E I_z \; G I_t} \sqrt{1 + \frac{\pi^2}{L^2} \frac{E I_w}{G I_t}}$$

$$= \frac{\pi}{L} \sqrt{E I_z \, G I_t} \sqrt{1 + \frac{\pi^2}{L^2} \frac{E I_w}{G I_t}}$$

$$G = \frac{E}{2(1+\nu)}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl}\, f_y}{M_{cr}}} \longrightarrow \chi_{LT}$$

LT lateral torsional