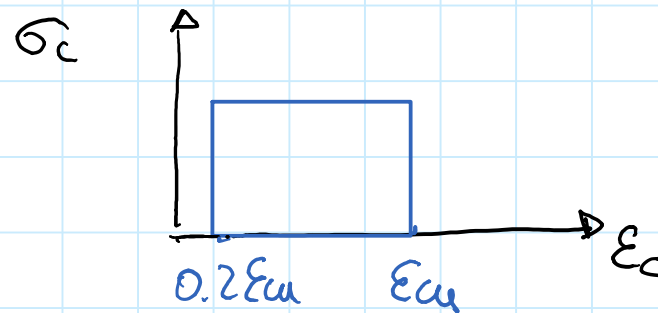
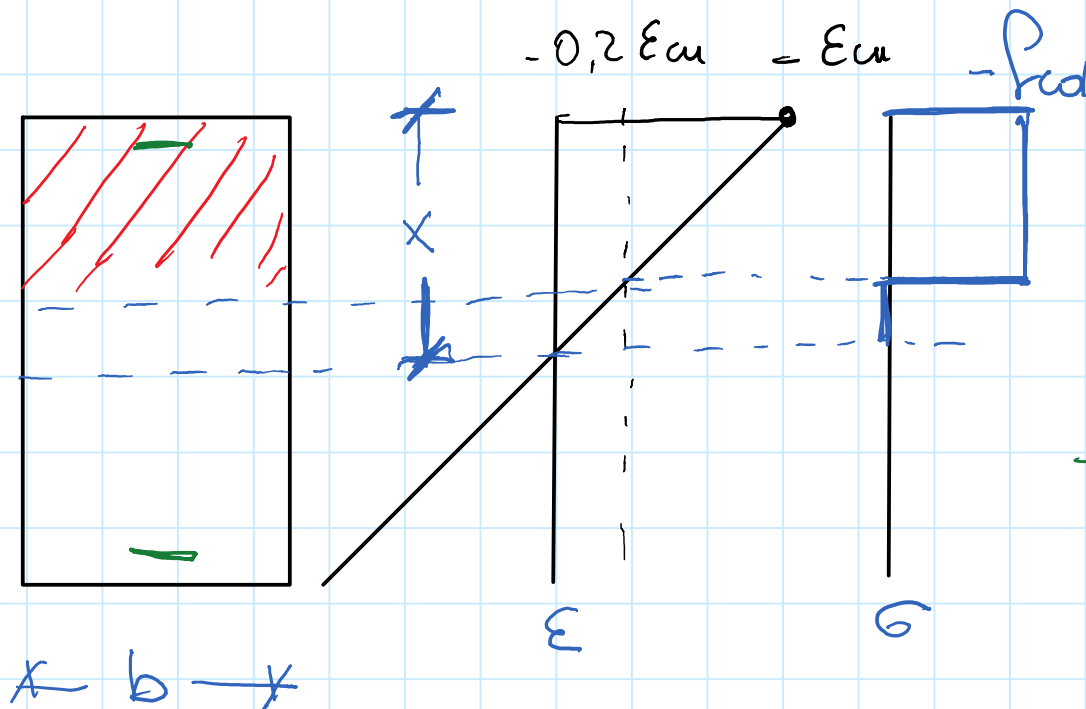


# STRESS

# BLOCK



RISULTATO  
SIMILE A QUELLO  
OTTENUTO CON  
LEGAME PARABOLA  
RETTANGOLO

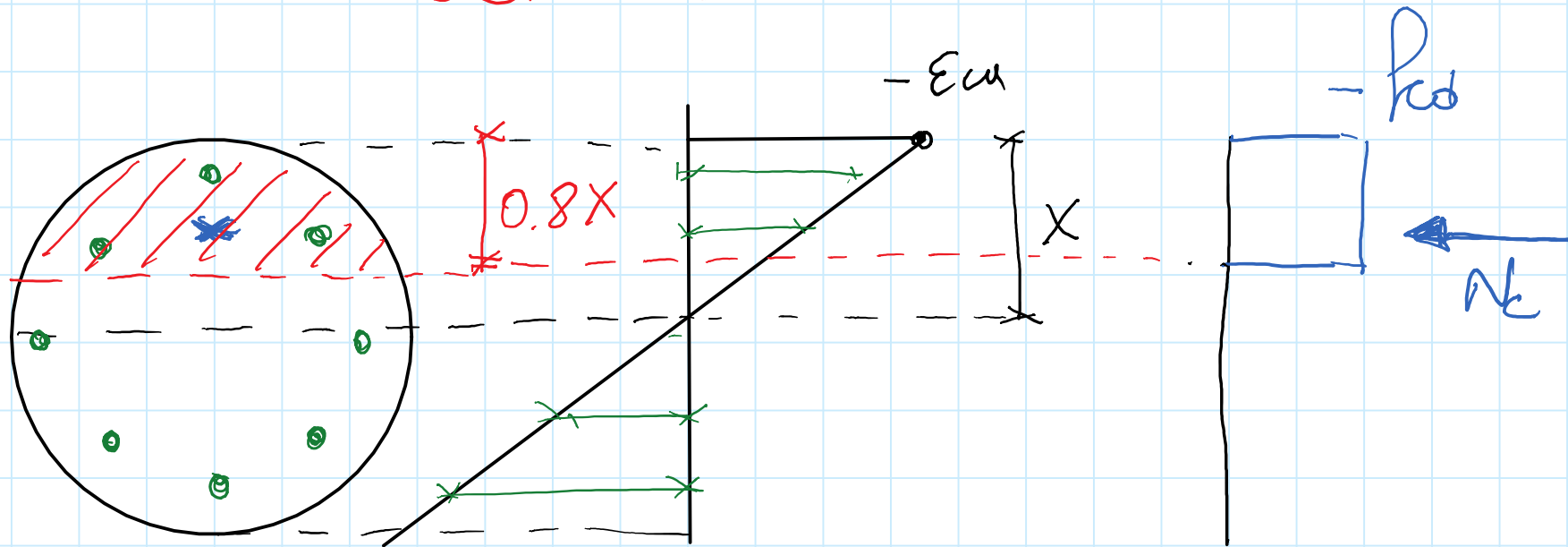


$$\begin{aligned} &0.8x \quad \leftarrow N_c \\ &\Delta x = 0.2x \end{aligned}$$


$$\begin{aligned} -\epsilon_{cu} : x &= -0.2\epsilon_{cu} : \Delta x \\ \Rightarrow \Delta x &= 0.2x \end{aligned}$$

$$N_c = -0.8bx f_{cd}$$

# SEZIONE GENERICA



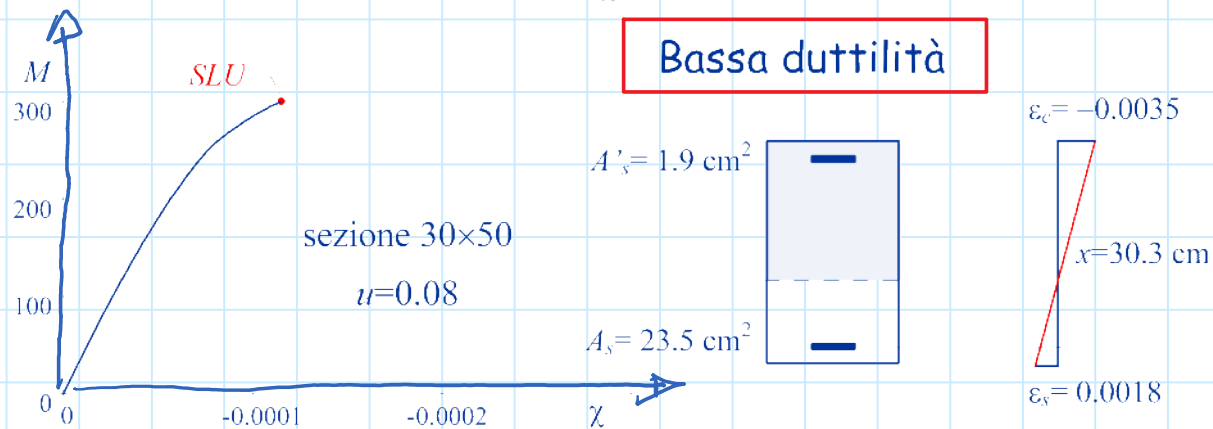
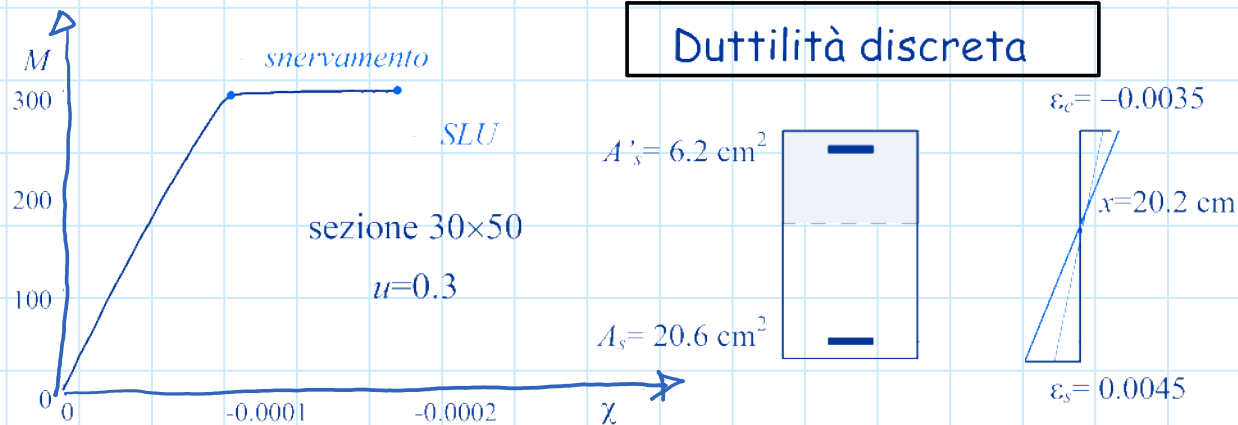
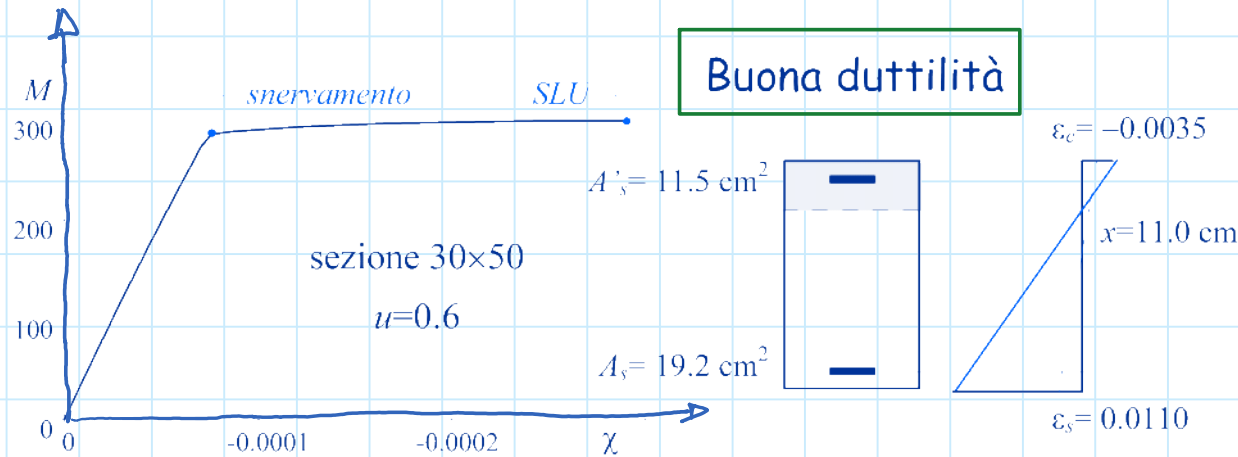
$$N_c = - f_{cd} \cdot A_{\text{setto}} \text{ circolare (TRATTO } 0.8x)$$

APPLICATO NEL BARICENTRO DI 

TROVO  $x$  :

$$N_c + \sum N_{s_i} = 0$$

# DUTTILITA'



$$M_{rd} = 300 \text{ kNm}$$

SEZIONE 30x50

CASO 1

$$\varepsilon_s > 1\%$$

$$X_u = 11 \text{ cm}$$

$$\mu = A'_s / A_s = 0.6$$

CASO 2:

$$\varepsilon_{yd} < \varepsilon_s < 1\%$$

$$X_u = 20.2 \text{ cm}$$

$$\mu = 0.30$$

CASO 3

$$\varepsilon_s < \varepsilon_{yd}$$

$$X_u = 30.3 \text{ cm}$$

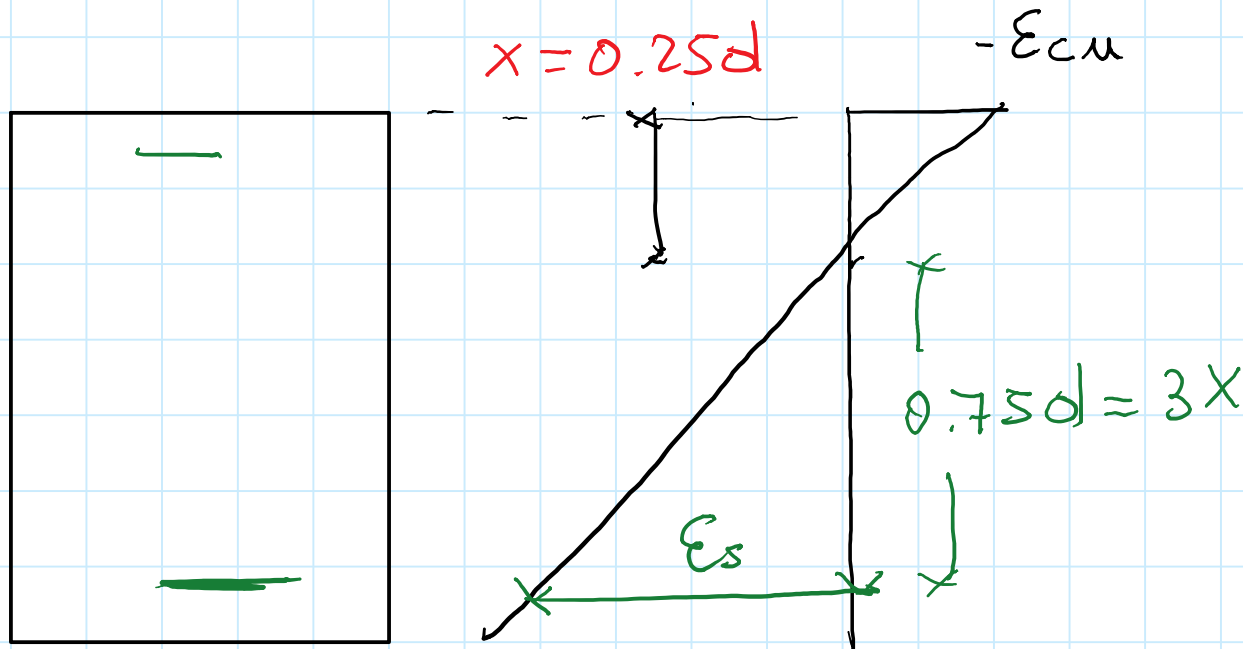
$$\mu = 0.08$$

## RIEPILOGANDO :

ALTA DUTTILITA'  $\chi_u : \epsilon_s > 1\% \leftarrow$

MEDIA DUTTILITA'  $\chi_u : \epsilon_{yd} < \epsilon_s < 1\%$

BASSA DUTTILITA'  $\chi_u : \epsilon_s < \epsilon_{yd}$



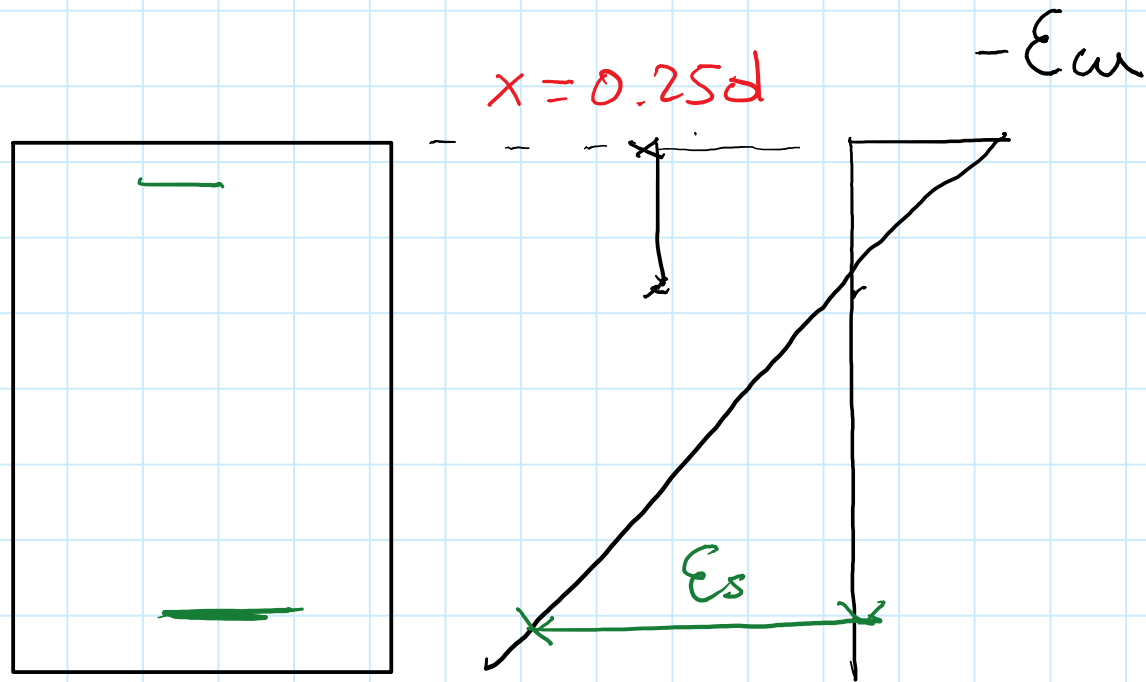
PER AVERE ALTA  
DUTTILITA'  
IMPONGO

$$x = 0.25d$$

$$\begin{aligned} \epsilon_s &= \frac{0.75d}{0.25d} \epsilon_{cu} = 3\epsilon_{cu} \\ &= 1.05\% \end{aligned}$$



# PROGETTO A FLESSIONE



DATI:

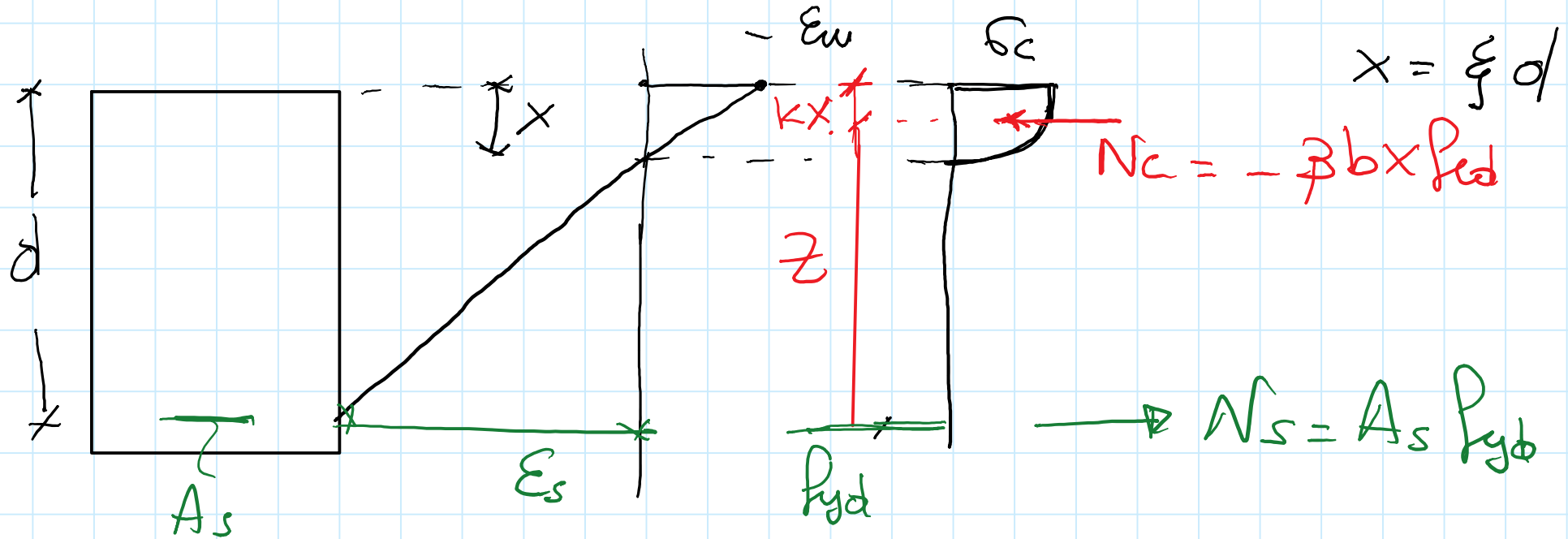
$M_{Ed}$ , BUONA DUTT

$$\Rightarrow x = 0.25d = \xi d$$

INCOGNITE:

$b, h, A_s, A_s'$

# PROGETTO SEZIONE A SEMPLICE ARMATURA



EQ. ROTAZIONE RISPETTO ALL'ARMATURA

$$\begin{aligned}
 M_{ed} &= -N_c (d - kx) \\
 &= \beta b x f_{cd} (d - kx) = \beta b \xi d f_{cd} (d - k\xi d) \\
 &= bd^2 \cdot \underbrace{\beta \xi f_{cd} (1 - k\xi)}_{1/\gamma^2}
 \end{aligned}$$

# FORMULA PROGETTO SEZIONE

$$M = \frac{b d^2}{\gamma^2}$$

DOVE  $\frac{1}{\gamma^2} = \beta \xi f_{cd} (1 - \kappa \xi) \Rightarrow$

$$\gamma = \sqrt{\frac{1}{\beta \xi f_{cd} (1 - \kappa \xi)}}$$

DIPENDE DA  $f_{cd}$   
-  $\xi$

$M$  [kNm]  $\rightarrow f_{cd}$  [kN/m<sup>2</sup>]  
 $b, d$  [m]

ES: C30/37  $\rightarrow f_{cd} = 17 \text{ MPa} = 17 \cdot 10^3 \text{ kN/m}^2$

$$\gamma = \sqrt{\frac{1}{0,81 \cdot 0,25 \cdot 17'000 (1 - 0,416 \cdot 0,25)}} \Rightarrow \gamma = 0,018$$

SE C25/30  $\rightarrow$

$$\gamma = 0,0197$$

NOTO  $M = \frac{bd^2}{\gamma^2}$   $\rightarrow$

TRAVE EMERGENTE

FISSO  $b \rightarrow$

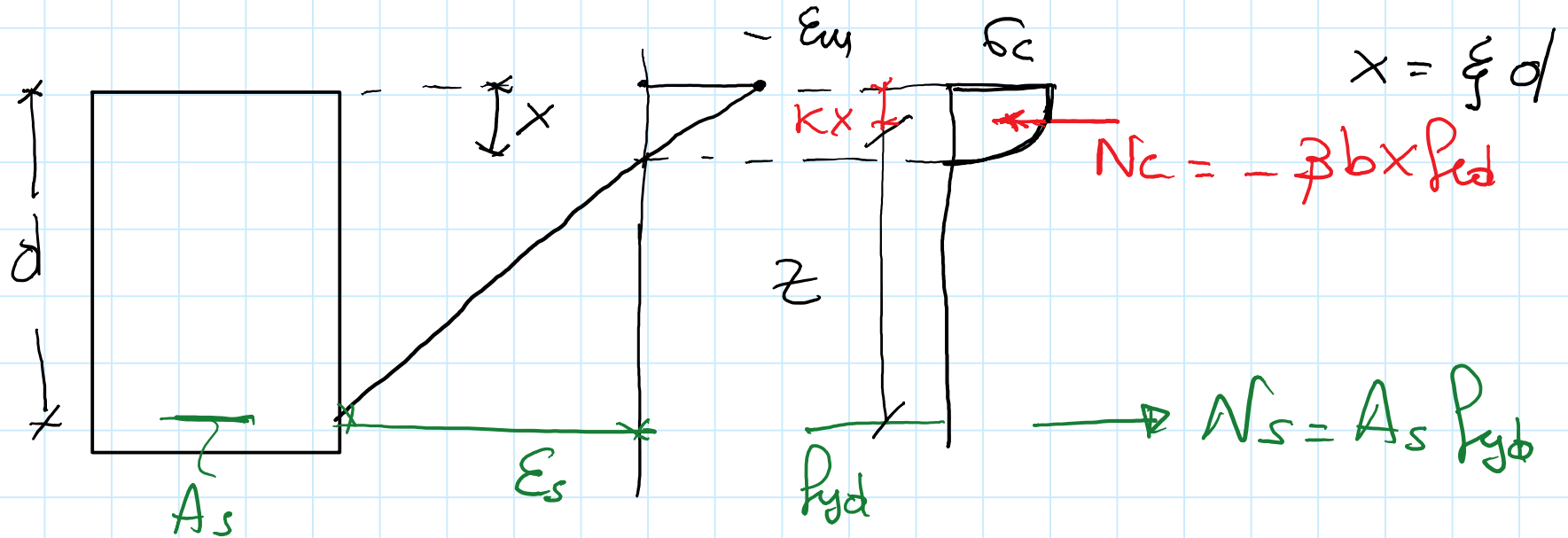
$$d = \gamma \sqrt{\frac{M}{b}}$$
$$h = d + c$$

TRAVE A SPESSORE

FISSO  $d = h_{\text{SOLAIO}} - c \rightarrow$

$$b = \gamma^2 \frac{M}{d^2}$$

# PROGETTO ARMATURA



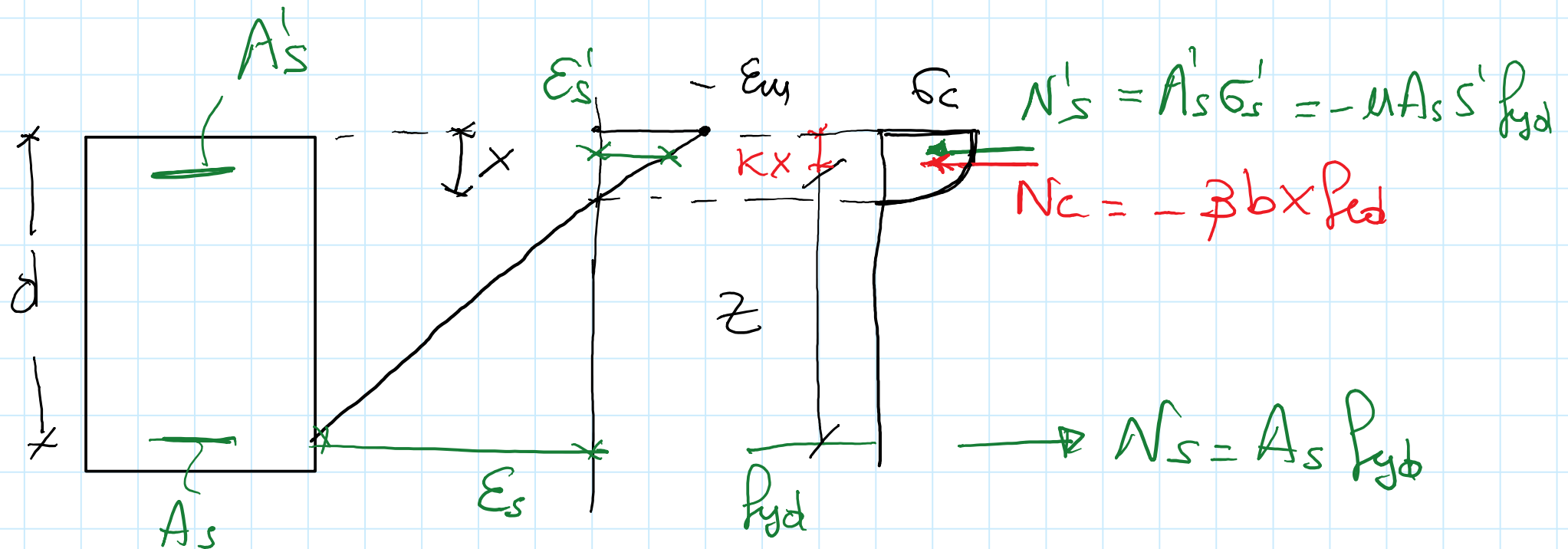
EQ. ROTAZIONE RISPETTO A  $N_c$

$$M_{\bar{\epsilon}_d} = A_s f_{yd} z = A_s f_{yd} (d - \kappa x) = A_s f_{yd} (d - \kappa \xi d) \\ = A_s f_{yd} d (1 - \kappa \xi) \rightarrow 1 - 0,416 \cdot 0,25$$

$$\Rightarrow M_{\bar{\epsilon}_d} = A_s f_{yd} \cdot 0,896 d \rightarrow A_s = \frac{M_{ed}}{0,9 d f_{yd}}$$

# SEZIONE A DOPPIA ARMATURA

FISSO  $X = \xi d = 0.25d$  (DUTTILITÀ)



$$A'_s = \mu A_s \quad ; \quad \sigma'_s = -s' f_{yd} \quad ; \quad s' = -\frac{\epsilon'_s}{\epsilon_{yd}} \leq 1$$

$$\epsilon'_s = -\epsilon_{uy} \frac{(x-c)}{x} = -\epsilon_{uy} \frac{x/d - c/d}{x/d}$$

## SEZIONE A DOPPIA ARMATURA

PONGO  $\gamma = c/d \rightarrow \epsilon'_s = -\epsilon_w \frac{\xi - \gamma}{\xi}$

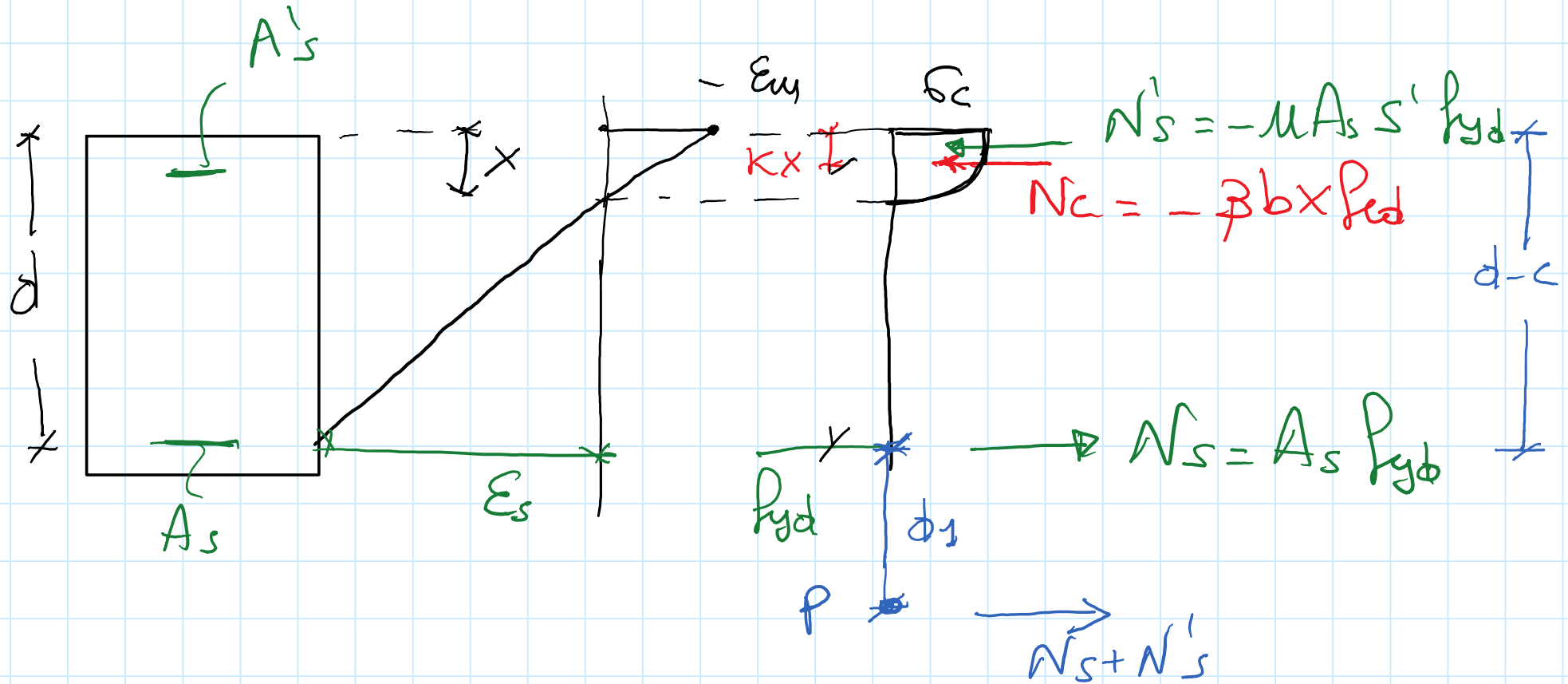
PER TRAVI EMERGENTI (ES. 30x60)  $\rightarrow$

$$\gamma = \frac{c}{d} \approx \frac{5}{55} = 0.091 \rightarrow \epsilon'_s = -3.5\% \cdot \frac{0.25 - 0.091}{0.25} = -2\% \Rightarrow \text{SNERVATA}$$

PER TRAVI A SPESSORE (ES. 80x25)  $\rightarrow$

$$\gamma = \frac{5}{20} = 0.25 \rightarrow \epsilon'_s = -3.5\% \cdot \frac{0.25 - 0.25}{0.25} = 0 \Rightarrow \text{NON SNERVATA}$$

# RESULTANTE $N_s + N'_s$



$$N_s d_1 + N'_s (d_2 + d - e) = 0 \Rightarrow$$

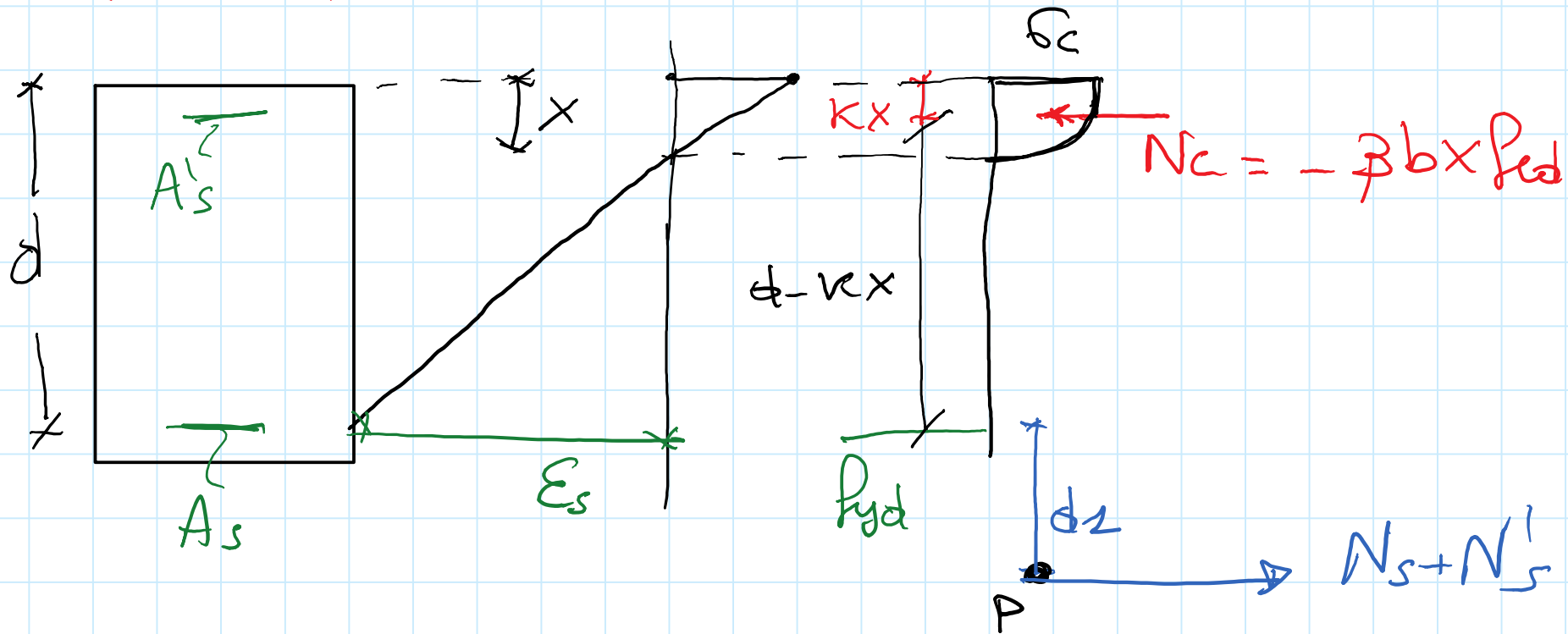
$$\cancel{A_s f_{yd}} d_1 - \mu \cancel{A_s s' f_{yd}} (d_1 + d - e) = 0 \Rightarrow$$

$$d_1 (1 - \mu s') = \mu s' (d - e) \rightarrow$$

$$d_1 = \frac{\mu s' (1 - \gamma)}{1 - \mu s'} d$$



# PROGETTO DELLA SEZIONE



EQ. RISPETTO A P

$$M_{Ed} = -N_c (d_2 + d - \kappa x)$$

$$M_{Ed} = \underbrace{\beta b x f_{cd}}_{\xi d} \cdot \left[ \frac{\sigma_s' (1 - \gamma) d}{1 - \mu s'} + d - \kappa \xi d \right]$$

$$M = bd^2 \beta \xi f_{cd} \left[ \frac{\mu s' (1-\gamma)}{1-\mu s'} + 1 - k \xi \right] \rightarrow$$

$$M = bd^2 \underbrace{\beta \xi f_{cd} (1 - k \xi)}_{1/\gamma^2} \cdot \underbrace{\left[ \frac{\mu s' (1-\gamma)}{(1-\mu s')(1-k\xi)} + 1 \right]}_{1/k^2}$$

$\underbrace{\hspace{15em}}_{1/\gamma'^2}$

$$\frac{1}{\gamma'^2} = \frac{1}{\gamma^2} \cdot \frac{1}{k^2}$$

$$\rightarrow \gamma' = \gamma \cdot k$$

$$\frac{1}{k^2} = \frac{\mu s' (1-\gamma)}{(1-\mu s')(1-k\xi)} + 1$$

PER TRAVI EMERGENTI

$$\rightarrow \gamma \simeq 0.1 \rightarrow 1-\gamma = 0.9$$

$$1-k\xi \simeq 0.9$$

IN MODO APPROSSIMATO

$$\frac{1}{k^2} \approx \frac{\mu s'}{1 - \mu s'} + 1 = \frac{\cancel{\mu s'} + 1 - \cancel{\mu s'}}{1 - \mu s'} \rightarrow$$

$$k = \sqrt{1 - \mu s'}$$

$$k < 1 \Rightarrow r' < r$$

PROGETTO SEZIONE:

$$M = \frac{b d^2}{\gamma^2} \Rightarrow$$

$$d = \gamma' \sqrt{\frac{M}{b}}$$

È NECESSARIA UN'ALTEZZA MINORE  
RISPETTO AL CASO DI SEMPLICE  
ARMATURA

Diagram illustrating the internal forces and stress distribution in a rectangular cross-section of a beam under a point load  $P$ .

The cross-section has a total height  $d$  and an effective depth  $d_1$ . The neutral axis is at a depth  $x$  from the top.

The internal forces are:

- Concrete compression force:  $N_c = -\beta b x f_{cd}$
- Steel tension force:  $N_s + N'_s$

The diagram also shows the internal forces  $N_c$  and  $N_s + N'_s$  acting on the cross-section.

EQ. AUA ROTAZ. RISPETTO A  $N_C \Rightarrow$

$$\begin{aligned} M_{ed} &= (N_s + N'_s) (d_1 + d - kx) \\ &= (A_s p_{yd} - u A_s s' p_{yd}) \left[ \frac{u s' (1 - \alpha)}{1 - u s'} d + d - k \xi d \right] \\ &= A_s p_{yd} (1 - u s') d \left[ \frac{u s' (1 - \alpha)}{1 - u s'} + 1 - k \xi \right] \end{aligned}$$

$$\begin{aligned}
M_{ed} &= A_s p_{yd} (1 - u s') d \left[ \frac{u s' (1 - \alpha)}{1 - u s'} + 1 - k \xi \right] \\
&= A_s p_{yd} \cdot d (1 - k \xi) (1 - u s') \cdot \left[ \frac{u s' (1 - \alpha)}{(1 - u s') (1 - k \xi)} + 1 \right] \\
&= A_s p_{yd} d (1 - k \xi) \left[ \frac{u s' (1 - \alpha)}{1 - k \xi} + 1 - u s' \right] \\
&\approx A_s p_{yd} d (1 - k \xi) [u s' + 1 - u s']
\end{aligned}$$

$$M_{ed} \approx A_s p_{yd} \cdot \underbrace{0.9d}_z$$

VALORI DI  $z/d$  (C30/37, B500C,  $\xi = 0.25$ )

	$\gamma = 0.10$	$\gamma = 0.15$	$\gamma = 0.20$
u	$s' = 1.00$	$s' = 0.72$	$s' = 0.36$
0	0.896		
0.25	0.897	0.888	0.887
0.50	0.898	0.880	0.879

TASSO DI LAVORO  
ARMATURA  
COMPRESSA

$z \approx 0.9d$   
SEMPRE

TRAVI EMERGENTI

TRAVI A SPESSORE  
NON TROPPO  
BASSE

# PROGETTO ARMATURA COMPRESSA

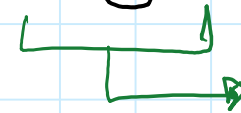
NOTO  $M_{ed}$   $\Rightarrow$  SEZIONE :  $d = \sqrt[3]{\frac{M_{ed}}{b}}$

ARMATURA TESA :  $A_s = \frac{M_{ed}}{0.9 d f_{yd}}$

$$A'_s = ?$$

PROGETTO  $A'_s$  PER PORTARE

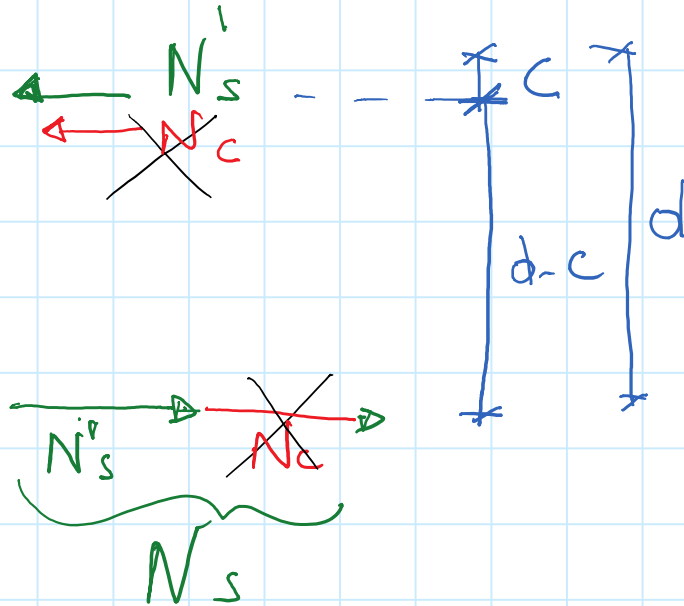
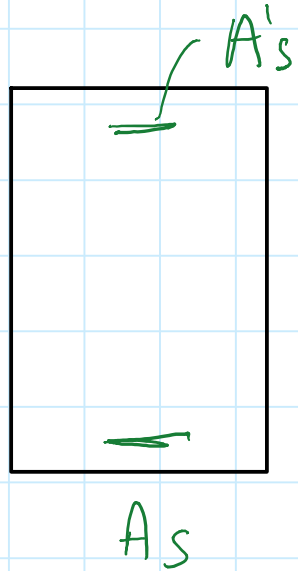
$$\Delta M = M_{ed} - \frac{b d^2}{\gamma^2}$$



MOMENTO  
RESISTENTE DEL CLS  
IN ASSENZA DI  $A'_s$

$$\text{Se } M_{ed} < \frac{b d^2}{\gamma^2} \rightarrow A'_s = 0$$

# PROGETTO ARMATURA COMPRESSA



$$\left. \begin{aligned} N'_s &= - \frac{\Delta M}{(d-c)} \\ N'_s &= - s' A'_s f_{yd} \end{aligned} \right\} \rightarrow A'_s = \frac{\Delta M}{s' f_{yd} (d-c)}$$

$$s' = - \frac{\epsilon'_s}{\epsilon_{yd}} \leq 1 \Rightarrow$$

$$\epsilon'_s = - \epsilon_{cu} \frac{(x-c)}{x} = - \epsilon_{cu} \frac{\xi - \gamma}{\xi} \rightarrow s' = \frac{\epsilon_{cu}}{\epsilon_{yd}} \frac{\xi - \gamma}{\xi}$$



VALORI  $r'$  (C30/37; B450C;  $\xi = 0.25$ )

	$\gamma = 0.10$	$\gamma = 0.15$	$\gamma = 0.20$
u	$s' = 1.00$	$s' = 0.72$	$s' = 0.36$
0	0.0180		
0.25	0.0156	0.0164	0.0173
0.50	0.0127	0.0146	0.0165

$r'$  SI RIDUCE POCO  
RISPETTO A  $r$

PERCHÉ IN  
TRAVI A SPESORE  
 $s'$  È BASSO

↓  
AL CRESCERE DI  $u$  SI  
RIDUCE MOLTO  $r'$  PERCHÉ  
 $A's$  È SNERVATA IN TRAVI  
EMERGENTI

QUALE VALORE  
DI  $r'$  USARE?

VALORI SUGGERITI  $r'$

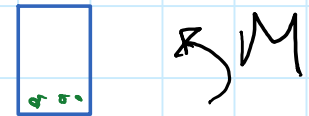
SE USO  $r'$  BASSI  $\rightarrow d = r' \sqrt{\frac{M}{b}} \Rightarrow d$  BASSA

$$\Rightarrow A_s = \frac{M}{0,9 d f_{yd}} \quad (\text{FORTE ARMATURA TESA})$$

MINIMO VALORE DI  $r'$  TALE CHE

$A_s \simeq 1\% \cdot b d$  TRAVI EMERGENTI

$A_s \simeq 2,5\% \cdot b d$  TRAVI A SPESSORE



IN GENERALE  $A_s = \rho_s b d \rightarrow$

$$M = A_s 0,9 d f_{yd} = \rho_s b d \cdot 0,9 d f_{yd} = b d^2 \cdot \underbrace{\rho_s 0,9 f_{yd}}_{2/\zeta_s^2} = \frac{b d^2}{\zeta_s^2}$$

$$\zeta_s = \frac{1}{\sqrt{\rho_s 0,9 f_{yd}}}$$

$$\rightarrow \zeta_s = \frac{1}{\sqrt{\frac{1}{100} \cdot 0,9 \times 381,3 \cdot 10^3}} \rightarrow$$

$$\zeta_s = 0,017$$

## ESEMPIO

PROGETTARE TRAVE EMERGENTE CON

$$M_{ed} = 300 \text{ kNm}$$

MATERIALI:

C30/37

B450C

$$\text{DA } M = \frac{b d^2}{\gamma^2} \rightarrow d = \gamma' \sqrt{\frac{M}{b}}$$

$$b = 0,30 \text{ m} \rightarrow d = 0,017 \sqrt{\frac{300}{0,30}} = 0,54 \text{ m}$$

$$h \geq d + e = 0,54 + 0,05 = 0,59 \text{ m}$$

$$\text{FISSO } h = 60 \text{ cm} \Rightarrow d = 55 \text{ cm}$$

$$A_s = \frac{M}{0,9 d f_{yd}} = \frac{300 \text{ kNm}}{0,9 \times 0,55 \text{ m} \times 391,3 \text{ N/mm}^2} \times 10 = 15,48 \text{ cm}^2$$

$$A_{\phi 20} = 3,14 \rightarrow 15,48 / 3,14 = 4,93 \Rightarrow 5 \phi 20$$

È NECESSARIA  $A'_s$ ?

$$M_{rd, A'_s=0} = \frac{b d^2}{\gamma^2} = \frac{0,30 \times 0,55^2}{0,018^2} = 280,1 \text{ kNm}$$
$$M_{Gd} = 300 \text{ kNm}$$

}  $\Rightarrow$

PROGETTO  $A'_s$  PER  $\Delta M = 300 - 280,1 = 20 \text{ kNm}$

$$A'_s = \frac{\Delta M}{s' f_{yd} (d - c)}$$

$$s' = - \frac{\varepsilon'_s}{\varepsilon_{yd}} = \frac{\varepsilon_{cu}}{\varepsilon_{yd}} \left( \frac{\xi - \gamma}{\xi_s} \right) = \frac{3,5}{1,96} \times \frac{(0,25 - 5/55)}{0,25} = \frac{1,14}{1,0}$$

$$A'_s = \frac{20 \text{ kNm}}{1,391,3 \frac{\text{N}}{\text{mm}^2} (0,55 - 0,05) \text{ m}} \times 10 = 1 \text{ cm}^2$$

DISPONGO  
ALMENO  
 $A'_s = 0,25 A_s$

## CONSIDERAZIONI

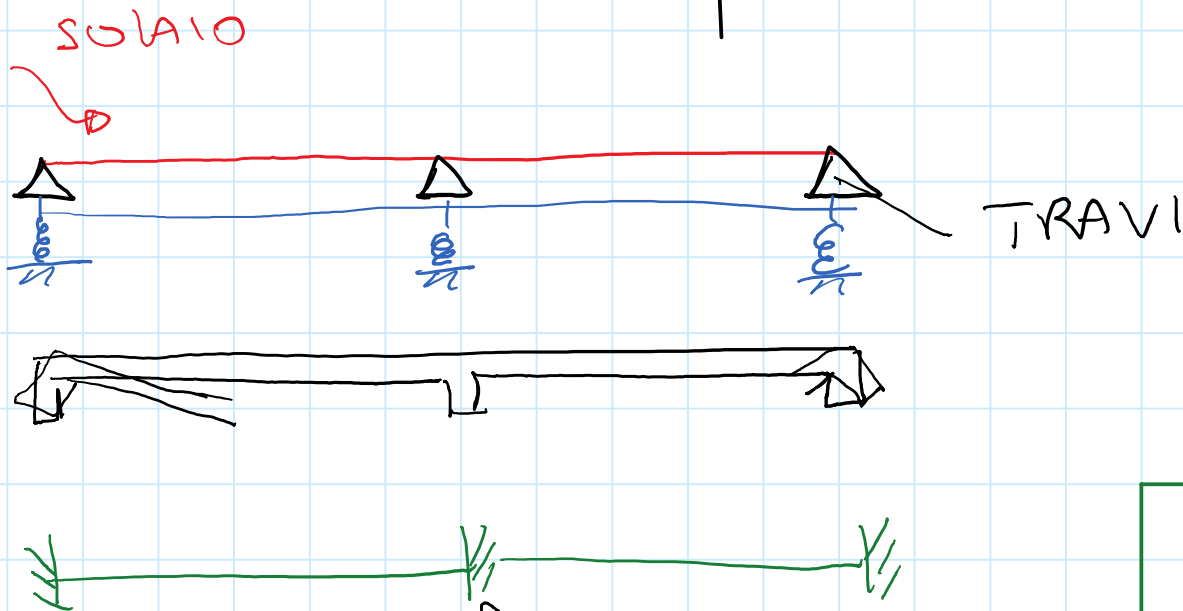
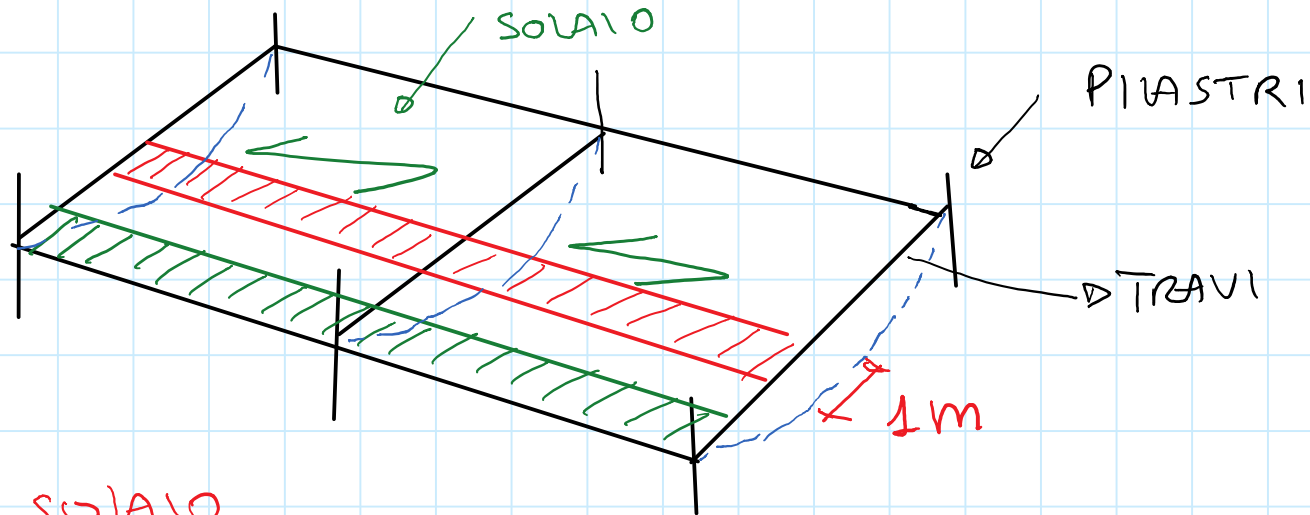
ARMATURA COMPRESSA NECESSARIA MODESTA

- PER  $\mu \leq$
- PER C30/37  $\gamma = 0,018$
  - PER LIMITARE  $A_s$   $\gamma' = 0,017$
  - $\gamma$  e  $\gamma'$  SONO SIMILI

PER C25/30  $\Rightarrow \gamma = 0,0197$

$\Rightarrow$  SERVIREBBE MAGGIORE  
 $A'_s$

# MODELLO SOLAIO



SOLO I CEDIMENTI  
DIFFERENZIALI PRODUCONO  
SOLLECITAZIONI SUL SOLAIO

↓  
CONSIDERO APPOGGI  
(RIGIDEZZA TORSIONALE  
TRAVI MODESTA)

CRITERIO DI FASCIA

IN PROSSIMITA' DEI PIASTRI  
ANCHE LA LORO  $K$  FLESSIONALE  
VINCOLA LA ROTAZIONE