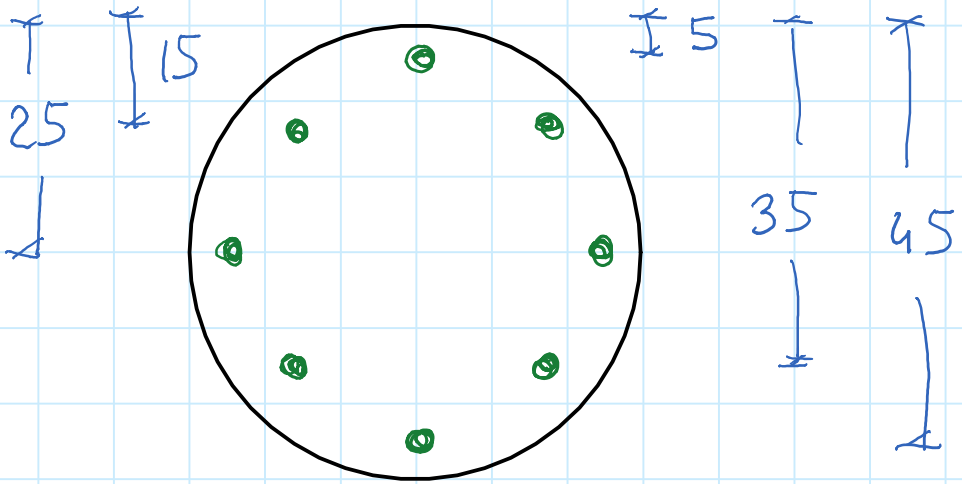
ESERCIZIO 1



DATI

$r = 25\ cm$

$c = 5\ cm$

$8\phi 16$

C30/37

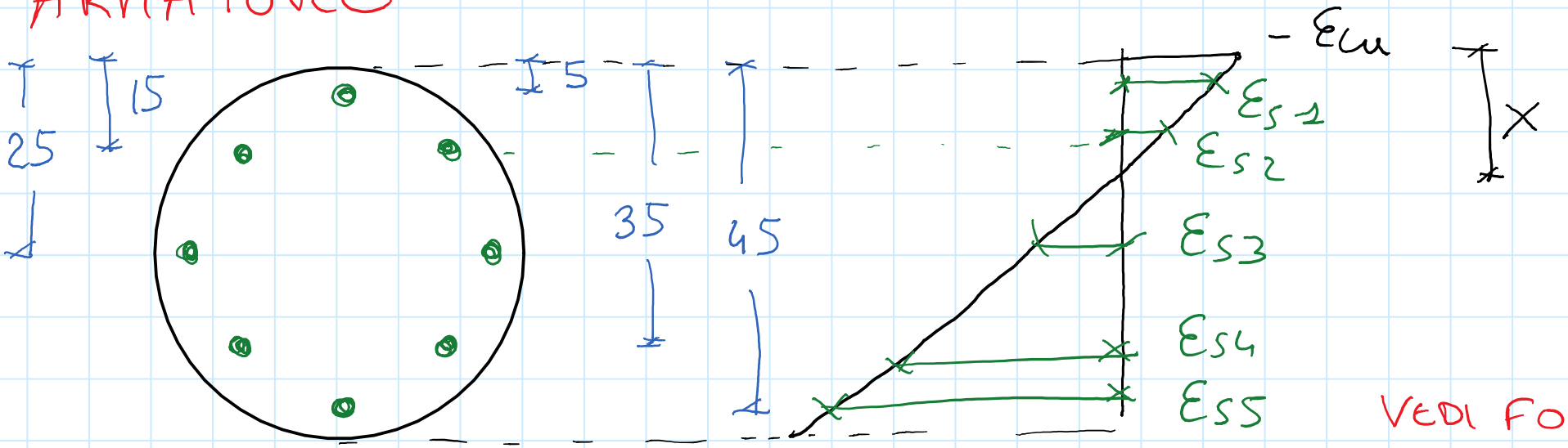
B450C

$\Rightarrow$

DETERMINARE

$M_{Rd} = ?$

ARMATURE



VEDI FOGLIO EXCEL

ARMATURA	A_s
1	$2\,cm^2$ $(1\phi16)$
2	$4\,cm^2$ $(2\phi16)$
3	$4\,cm^2$
4	$4\,cm^2$
5	$2\,cm^2$

$$\varepsilon_{s1} = -\frac{\varepsilon_{cu}(x-5)}{x}$$

$$\varepsilon_{s2} = -\varepsilon_{cu}\frac{(x-15)}{x}$$

$$\varepsilon_{s3} = -\varepsilon_{cu}\frac{(x-25)}{x}$$

$$\varepsilon_{s4} = -\varepsilon_{cu}\frac{x-35}{x}$$

$$\varepsilon_{s5} = -\varepsilon_{cu}\frac{x-45}{x}$$

CONFRONTO $|\varepsilon_{si}|$ CON ε_{yd} PER TROVARE σ_{si}

$N_{si} = A_{si}\,\sigma_{si}$

CONTRIBUTO CLS

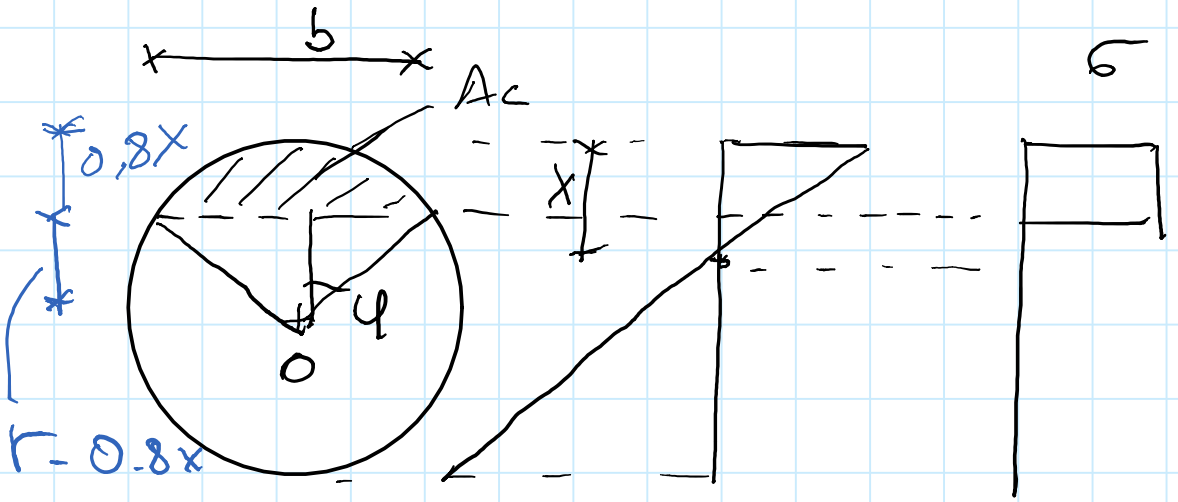


DIAGRAMMA STRESS BLOCK

Fisso x : $r - 0,8x = r\cos\varphi \Rightarrow \varphi = \arccos\frac{r-0.8x}{r}$

$$A_c = \varphi r^2 - \frac{1}{2} b \cdot r\cos\varphi$$

area settore — area triangolo

DOVE $b = 2r\,\mathrm{sen}\varphi$

$$\Rightarrow A_c = \varphi r^2 - r^2 \mathrm{sen}\varphi\cos\varphi$$

$2\,\mathrm{sen}\varphi\cos\varphi = \mathrm{sen}\,2\varphi \rightarrow$

$$\boxed{A_c = \frac{r^2}{2}(2\varphi - \mathrm{sen}\,2\varphi)}$$

$$N_c = -A_c \cdot f_{cd}$$

METODO DIMEZZAMENTO

FISSO $X = 25\,cm \rightarrow \varphi = 1.37$ rad

armature	As (cm2)	d sup (cm)	eps	eps/eyd	s	sigma s (Mpa)	Ns (kN)
1	2	5	-0.280%	-1.431	-1.000	-391.30	-78.3
2	4	15	-0.140%	-0.716	-0.716	-280.00	-112.0
3	4	25	0.000%	0.000	0.000	0.00	0.0
4	4	35	0.140%	0.716	0.716	280.00	112.0
5	2	45	0.280%	1.431	1.000	391.30	78.3

Ac (cm2)	733.425
Nc (kN)	-1246.82

$\Rightarrow N_{TOT} = \Sigma N_S + N_C = -1246.8\,kN$

$\rightarrow$ DEVE RIDURSI PARTE COMPRESSA $\rightarrow$ $0 < X < 25$

PROCEDURA ITERATIVA

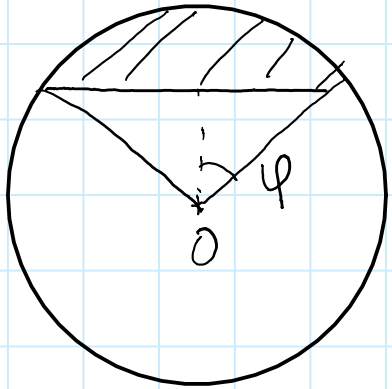
$X = 11.23$ cm

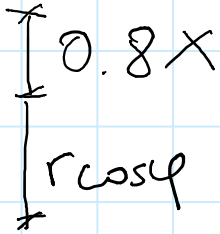
Ac (cm2)	239.757
Nc (kN)	-407.587

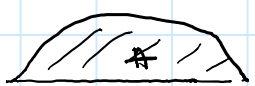
armature	As (cm2)	d sup (cm)	eps	eps/eyd	s	sigma s (Mpa)	Ns (kN)
1	2	5	-0.194%	-0.993	-0.993	-388.37	-77.7
2	4	15	0.117%	0.600	0.600	234.90	94.0
3	4	25	0.429%	2.193	1.000	391.30	156.5
4	4	35	0.741%	3.786	1.000	391.30	156.5
5	2	45	1.052%	5.379	1.000	391.30	78.3

$\Rightarrow N_{TOT} = 0$

BARICENTRO DEL SEGMENTO CIRCOLARE







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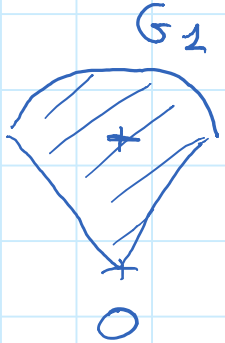

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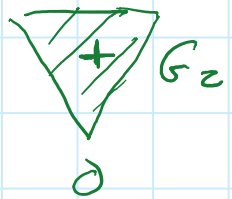


$$A_c \cdot d_{GO} = A_1 \cdot d_{G1,O} - A_2 d_{G2,O}$$

$$d_{G1,O} = -\frac{2r\,\mathrm{sen}\,\varphi}{3\varphi} \quad ; \quad A_1 = \varphi r^2$$

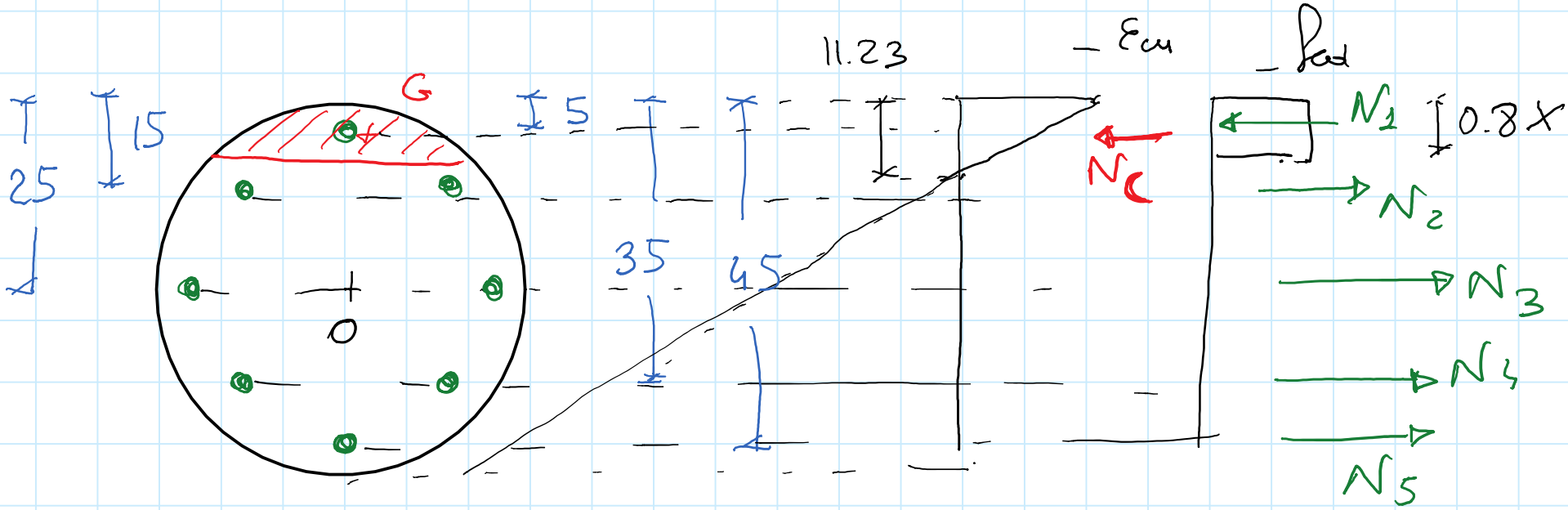
$$d_{G2} = -\frac{2r\cos\varphi}{3} \qquad A_2 = \frac{r^2}{2}\,\mathrm{sen}\,2\varphi \rightarrow$$

$$d_{GO} = -\frac{\varphi r^2 \cdot \frac{2}{3}\frac{r\,\mathrm{sen}\,\varphi}{\varphi} - \frac{r^2}{2}\,2\,\mathrm{sen}\,\varphi\cos\varphi \cdot \frac{2}{3}r\cos\varphi}{A_c} =$$

$$= -\frac{\frac{2}{3}r^3\,\mathrm{sen}\,\varphi\,(1-\cos^2\varphi)}{A_c} \rightarrow$$

$$\boxed{d_{GO} = -\frac{2}{3}\frac{r^3\,\mathrm{sen}^3\varphi}{A_c}}$$

CALCOLO M_{Rd}



CALCOLO M_{Rd} RISPETTO A O

$$M_{Rd} = N_C \cdot d_{GO} + \sum N_{si} \left(d_{si} - r \right)$$

DISTANZA ARMATURE DA O.

d_{si}: DISTANZA ARMATURE DA BORDO SUPERIORE

CALCOLO M_{Rd}

ARM	Ns (kN)	d 0	M0
1	-77.7	-20	15.53
2	94.0	-10	-9.40
3	156.5	0	0.00
4	156.5	10	15.65
5	78.3	20	15.65

CLS :

Nc (kN)	-407.587	
dG0 (cm)	-19.7	
Mc	80.18	kNm

$\Rightarrow M_{Rd} = 117.62$ kNm

ESEMPIO 2

$M_{Ed} = 100\ kNm$

$h_{SOLAIO} = 25\ cm$

$c = 4\ cm$

C30/37

B450C

$d = 21\ cm$

PROGETTARE TRAVE A SPESSORE

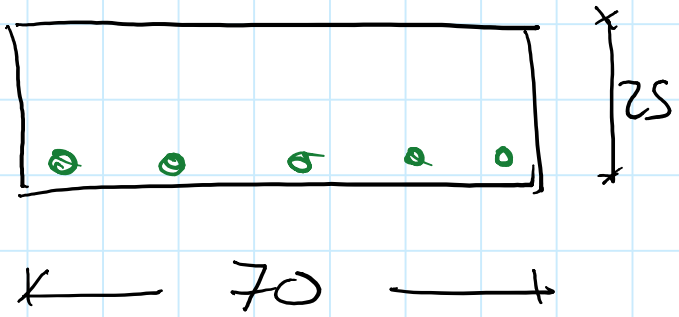
FISSO $r' = 0.017$ (VA BENE SE $\gamma = \frac{c}{d} \simeq 0.20$)

$$M = \frac{bd^2}{z^{12}} \Rightarrow b = 0.017^2 \cdot \frac{100}{0,21^2} = 0,65\ m$$

FISSO $b = 70\ cm$

$$A_s = \frac{M}{0,9 d f_{yd}} = \frac{100\ kNm \times 10}{0,9 \times 0,21m \times 391.3 \frac{N}{mm^2}} = 13,52\ cm^2$$

$A_s = 5\phi 20$



ARMATURA COMPRESSA

CALCOLO M_{Rd} IN ASSENZA DI ARMATURA COMPRESSA

SE $A'_s = 0$

- $b = 70$ cm $\quad M_{Rd} = \frac{bd^3}{\alpha^2} = \frac{0{,}7 \times 0.21^2}{0.018^2} = 95.28$ kNm
- $b = 65$ cm $\rightarrow M_{Rd} = 88{,}47$ kNm

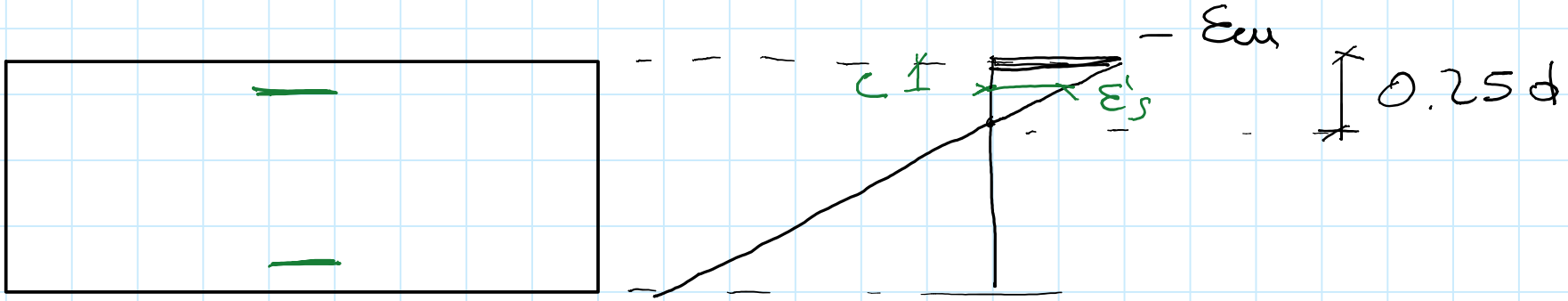
se $b = 70 \Rightarrow A'_s$ DEVE PORTARE

$$\Delta M = 100 - 95{,}28 = 4{,}72 \text{ kNm}$$

se $b = 65$

$$\Delta M = 100 - 88{,}47 = 11{,}53 \text{ kNm}$$

TASSO DI LAVORO ARMATURA COMPRESSA



$$\varepsilon'_s = -\varepsilon_{cu}\frac{x-c}{x} = -\varepsilon_{cu}\frac{(0.25-\gamma)}{0.25}$$

$$\gamma = \frac{4}{21} = 0{,}19 \rightarrow .$$

$$s' = -\frac{\varepsilon'_s}{\varepsilon_{yd}} = \frac{3.5}{1.96}\cdot\frac{(0.25-0.19)}{0.25} = 0.42$$

$$A'_s = \frac{\Delta M}{(d-c)\, s' f_{yd}}$$

N.B SE $\gamma = 0.25 \Rightarrow$ $s' = 0 \Rightarrow A'_s \rightarrow \infty$

ARMATURA COMPRESSA NON CONTRIBUISCE $\Rightarrow r' = r$

PROGETTO A'_s

Se $b = 70$ cm $\Rightarrow$

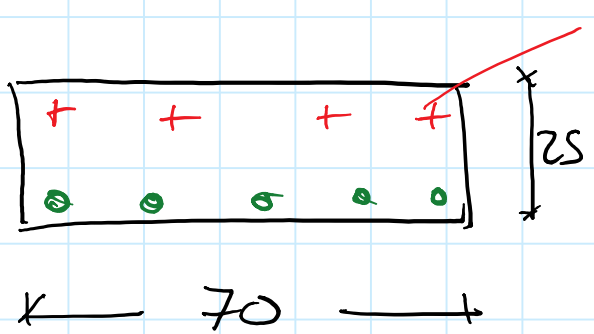
$$A'_s = \frac{4.72 \text{ kNm} \times 10}{(0.21 - 0.04)\text{m} \times 0.42 \cdot 391.3 \frac{N}{mm^2}}$$

$$= 1.69 \text{ cm}^2$$

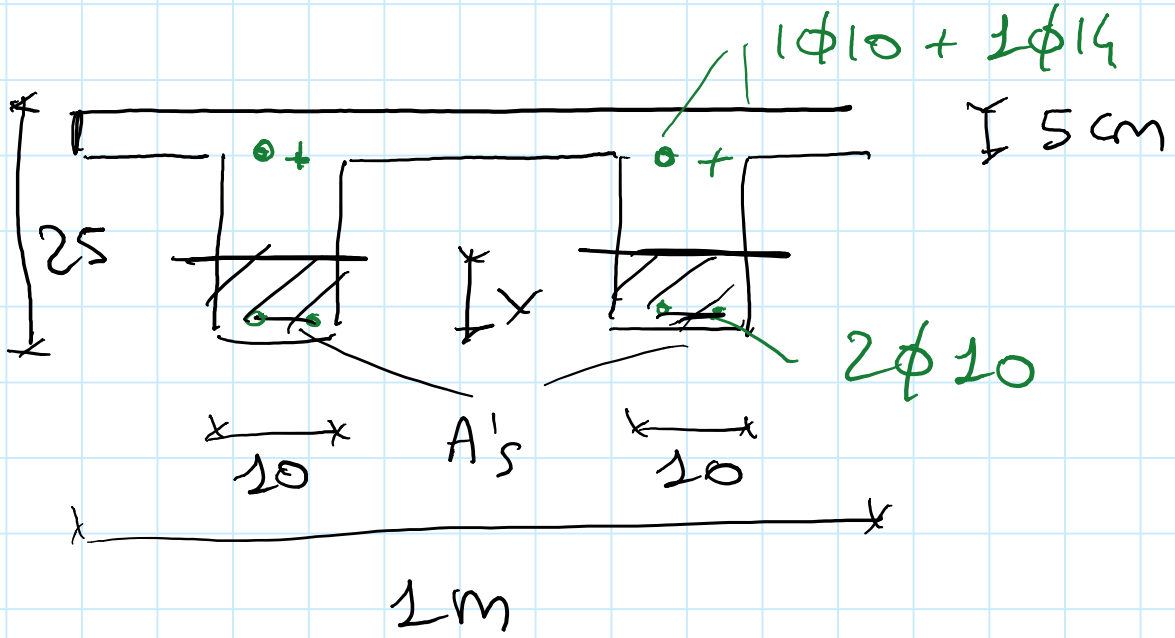
Se $b = 65$ cm $\Rightarrow \Delta M = 11.53$ kNm $\Rightarrow$

$$A'_s = 4.13 \text{ cm}^2$$

IN OGNI CASO IN TRAVE A SPESSORE DISPONGO ALMENO 4 BARRE $\Rightarrow$ 4 ϕ 14



ESERCIZIO 3



M

M = -20 KNM

(COMB. RARA, CARICHI LUNGA DURATA)

C = 3 cm

σ_{cmax} = ?

CARICHI LUNGA DURATA → n = 15

$A'_s = 2 \times 2\phi 10 = 4\phi 10 = 4 \times \pi \frac{1^2}{4} = 3.14\ cm^2$

$A_s = 2 \times (1\phi 10 + 1\phi 14) = 1.56 + 2 \cdot 1.54 = 4.65\ cm^2$

POSIZIONE ASSE NEUTRO

(DISTANZA x DAL BORDO INFERIORE)

$$S_N = 0 \Rightarrow$$

$$20\frac{x^2}{2} + 15 \times 3.14 \left(x - 3\right) + 15 \times 4.65 \underbrace{\left(22 - x\right)}_{d} = 0$$

$$10x^2 + 47.1x - 141.3 + 69.75x - 1534,5 = 0$$

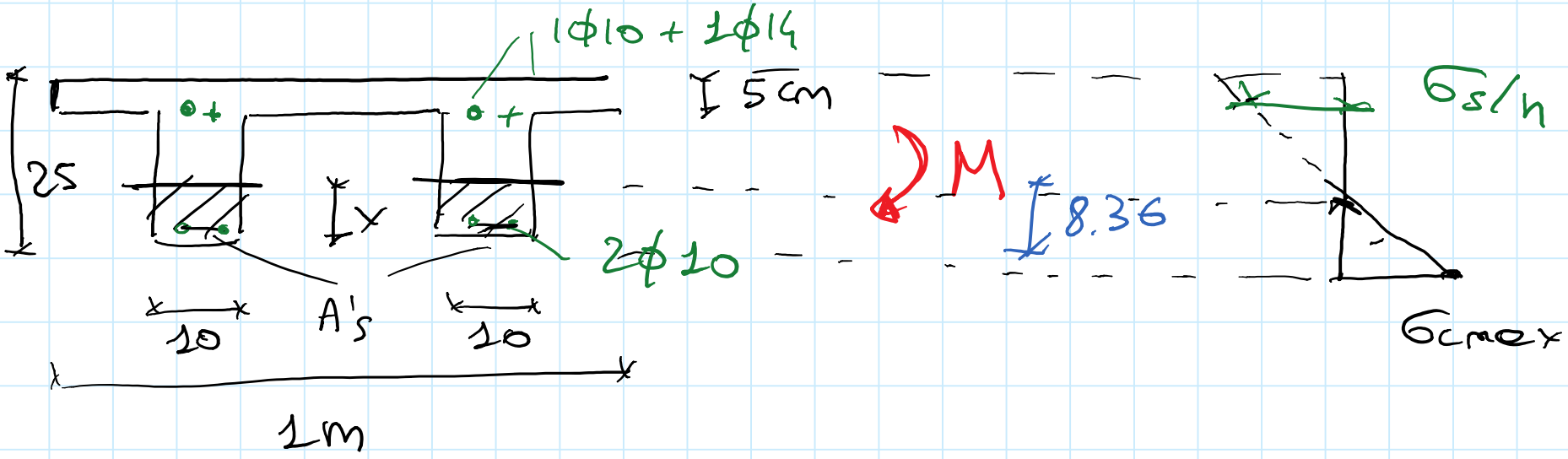
$$x^2 + 11.68x - 167.5 = 0$$

$$x = -\frac{11.68}{2} \pm \sqrt{\left(\frac{11.68}{2}\right)^2 + 167.5} =$$

$$= 8.36\,cm \Rightarrow$$

LA SEZ. REAGENTE E' RETTANGOLARE

CALCOLO TENSIONE MASSIMA



$$\sigma_{cmax} = \frac{M}{I} y \qquad \text{con} \qquad M = -20 \text{ KNm}$$

$$y = x = 8.36 \text{ cm}$$

$$I = 20 \cdot \frac{x^3}{3} + 15 \times 3.14 (x-3)^2 + 15 \times 4.65 (22-x)^2 =$$

$$= 20 \times \frac{8.36^3}{3} + 15 \times 3.14 (8.36-3)^2 + 15 \times 4.65 (22-8.36)^2 =$$

$$= 18220 \text{ cm}^4$$

$$\sigma_{c\,max}^{-} = \frac{-20\ kNm}{18220\ cm^4\ 3}\ 8.36\ cm\ \ 10^3 \cdot \frac{10^3}{10^3} = -9.2\ MPa$$

$$0.6\,f_{ck} = 0.6 \times 30\ MPa = -18\ MPa \Rightarrow$$ VERIFICA SODDISFATTA

$$\sigma_{s,max}^{+} = -\frac{M}{I}(d-x)\cdot n = \frac{20\ kNm}{18220\ cm^4}\cdot(22-8.36)\cdot 15$$

$$= 224\ MPa < 0.8\,f_{yk}$$