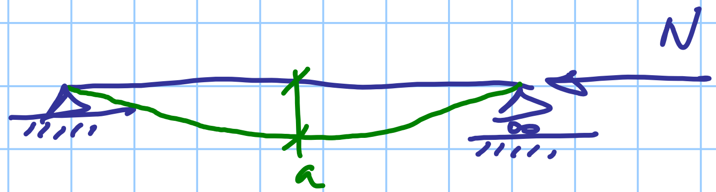
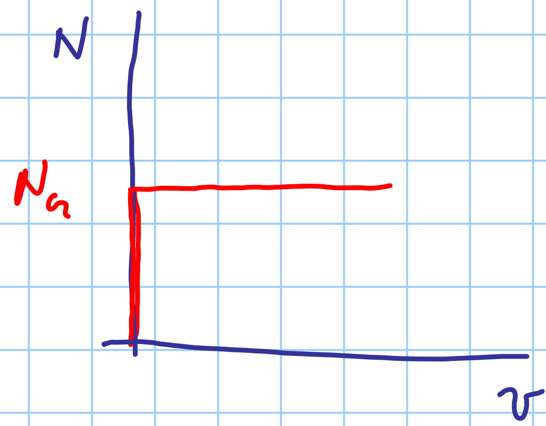
ASTA IDEALE, PERFETTA



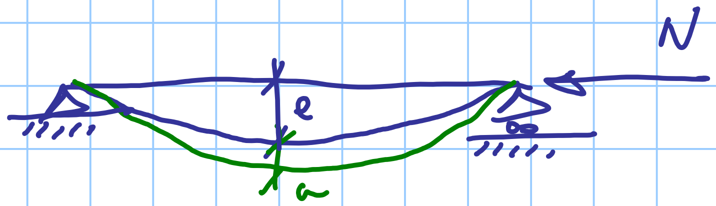
$$v = a \sin \frac{\pi x}{\ell}$$

$$N_{cr} = \frac{\pi^2 E I}{\ell^2}$$

per altri schemi: $N_{cr} = \dfrac{\pi^2 E I}{\ell_0^2}$



ASTA CON IMPERFEZIONI (materiale elastico-lineare)



$$v_0 = e \sin \frac{\pi x}{\ell}$$

deformazione iniziale (asta scarica)

per $N=0$ $M=0$

$$M = N v = N(e+a) \sin \frac{\pi x}{\ell}$$

$$v_1 = a \sin \frac{\pi x}{\ell} \qquad v_1'' = -a \frac{\pi^2}{\ell^2} \sin \frac{\pi x}{\ell}$$

$$M = -EI\, v_1'' = +EI\, a \frac{\pi^2}{\ell^2} \sin \frac{\pi x}{\ell} \qquad v = v_0 + v_1 = (e+a) \sin \frac{\pi x}{\ell}$$

$$N(e+a) \sin \cancel{\frac{\pi x}{\ell}} = +EI\, a \frac{\pi^2}{\ell^2} \sin \cancel{\frac{\pi x}{\ell}}$$

$$N(e+a) = EI\, a \frac{\pi^2}{\ell^2} = a N_{cr}$$

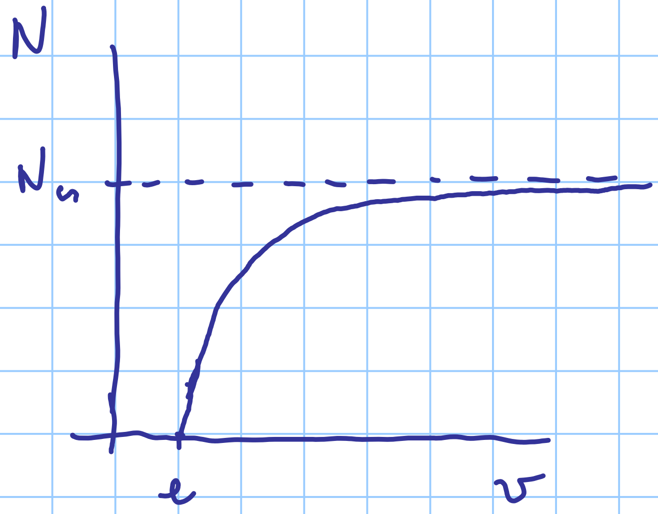
$$N e = -N a + N_{cr} a$$

$$a = \frac{N e}{N_{cr} - N}$$

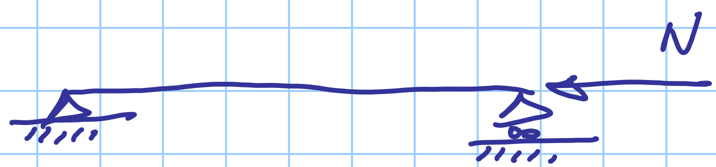
$$a = \frac{N e}{N_n - N}$$

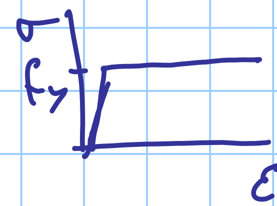
$$e + a = e + \frac{N}{N_n - N} e = \frac{N_n}{N_n - N} e$$

$$e + a = \frac{1}{1 - N/N_n} e$$



ASTA PERFETTA — materiale elastico - perfettamente plastico



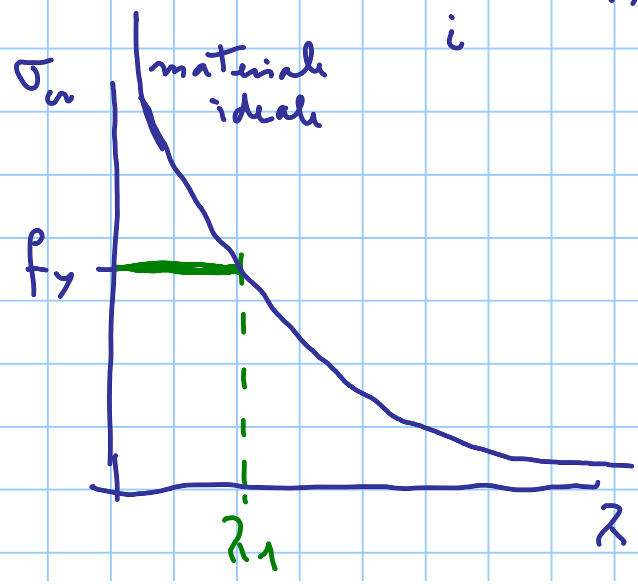


$$N_{cr} = \frac{\pi^2 E I}{l_0^2} \qquad \sigma = \frac{N}{A} \qquad \sigma_{cr} = \frac{\pi^2 E I}{l_0^2 A} = \frac{\pi^2 E i^2}{l_0^2}$$

$\sqrt{\frac{I}{A}} = i$ raggio d'inerzia

$\frac{l_0}{i} = \lambda$ snellezza

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$



se $\lambda < \lambda_1$ aste si osservano TOZZE

se $\lambda > \lambda_1$ aste vanno in carico critico elastico SNELLE

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \qquad \sigma_{cr} = f_y \qquad f_y = \frac{\pi^2 E}{\lambda_1^2} \Rightarrow \lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

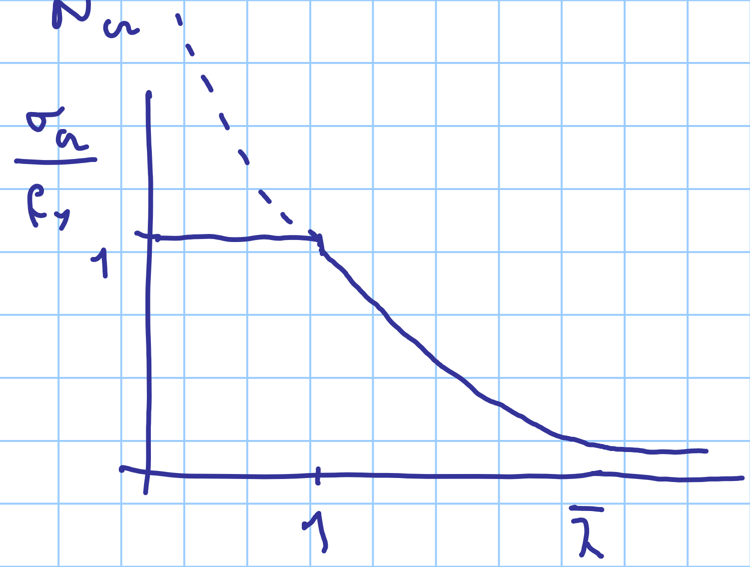
$$\bar{\lambda} = \frac{\lambda}{\lambda_1}$$ snellezza normalizzata

$$\lambda = \pi \sqrt{\frac{E}{\sigma_{cr}}} \qquad \bar{\lambda} = \frac{\lambda}{\lambda_1} = \sqrt{\frac{f_y}{\sigma_{cr}}} = \sqrt{\frac{A f_y}{N_{cr}}}$$

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} \Rightarrow \underbrace{N_{cr}}_{N_b} = \underbrace{\frac{1}{\bar{\lambda}^2}}_{\chi} \underbrace{A f_y}_{N_{Rd}}$$

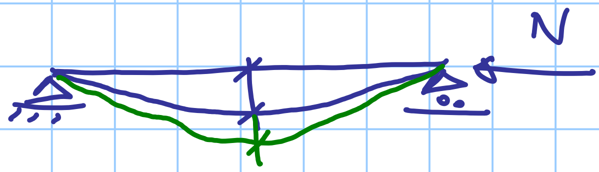
$$N_b = \chi \, N_{Rd}$$

$$\chi = \mathrm{MIN}\left(1; \frac{1}{\bar{\lambda}^2}\right)$$



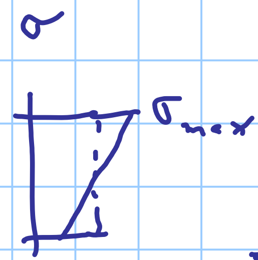
b buckling (instabilità)

ASTA IMPERFETTA – materiale con snervamento



$e + a$

$$e + a = \frac{1}{1 - N/N_{cr}} \, e$$



$$M_{max} = N\,(e + a)$$

N

$$\frac{I}{y_{max}} = W$$

$$\sigma = \frac{N}{A} + \frac{M}{I} y \qquad \sigma_{max} = \frac{N}{A} + \frac{M}{W}$$

$$\sigma_{max} = \frac{N}{A} + \frac{N}{1 - N/N_{cr}} \, \frac{e}{W}$$

$$f_y = \frac{N_b}{A} + \frac{N_b}{1 - N_b/N_{cr}} \, \frac{e}{W}$$

$$f_y = \frac{N_b}{A} + \frac{N_b}{1 - N_b/N_{cr}}\frac{e}{W} = \frac{N_b}{A}\left[1 + \frac{1}{1 - N_b/N_{cr}}\underbrace{\frac{eA}{W}}_{\eta}\right]$$

$$f_y = \frac{N_b}{A}\,\frac{1 - N_b/N_{cr} + \eta}{1 - N_b/N_{cr}}$$

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}}$$

$$1 - N_b/N_{cr} = \underbrace{\frac{N_b}{A f_y}}_{x}\left(1 + \eta - N_b/N_{cr}\right)$$

$$N_{cr} = \frac{A f_y}{\bar{\lambda}^2}$$

$$1 - \underbrace{\frac{N_b}{A f_y}}_{x}\bar{\lambda}^2 = x\left(1 + \eta - \underbrace{\frac{N_b}{A f_y}}_{x}\bar{\lambda}^2\right)$$

$$1-\bar{\lambda}^2 x = x\left(1+\eta-\bar{\lambda}^2 x\right)$$

$$1-\bar{\lambda}^2 x = (1+\eta)x-\bar{\lambda}^2 x^2$$

$$\bar{\lambda}^2 x^2-\left(1+\eta+\bar{\lambda}^2\right)x+1=0 \qquad \frac{1}{2}\left(1+\eta+\bar{\lambda}^2\right)=\varphi$$

$$\bar{\lambda}^2 x^2-2\varphi x+1=0 \qquad x=\frac{\varphi-\sqrt{\varphi^2-\bar{\lambda}^2}}{\bar{\lambda}^2}$$

$$x=\frac{\varphi-\sqrt{\varphi^2-\bar{\lambda}^2}}{\bar{\lambda}^2}\,\frac{\varphi+\sqrt{\varphi^2-\bar{\lambda}^2}}{\varphi+\sqrt{\varphi^2-\bar{\lambda}^2}}=\frac{\cancel{\varphi^2}-\left(\cancel{\varphi^2}-\cancel{\bar{\lambda}^2}\right)}{\cancel{\bar{\lambda}^2}\left(\varphi+\sqrt{\varphi^2-\bar{\lambda}^2}\right)}$$

$$\chi = \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}^2}}$$

$$\varphi = \frac{1}{2}\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right]$$

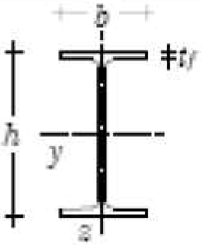
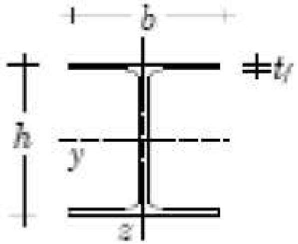
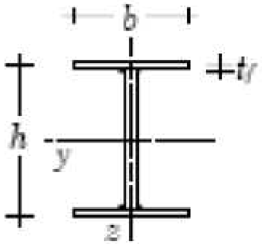
$$\cancel{\eta = \frac{e\,A}{W}}$$

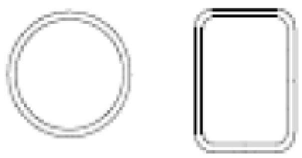
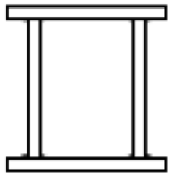
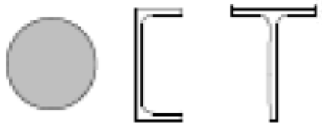
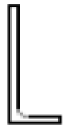
$$\eta = \alpha(\bar{\lambda} - 0.2)$$

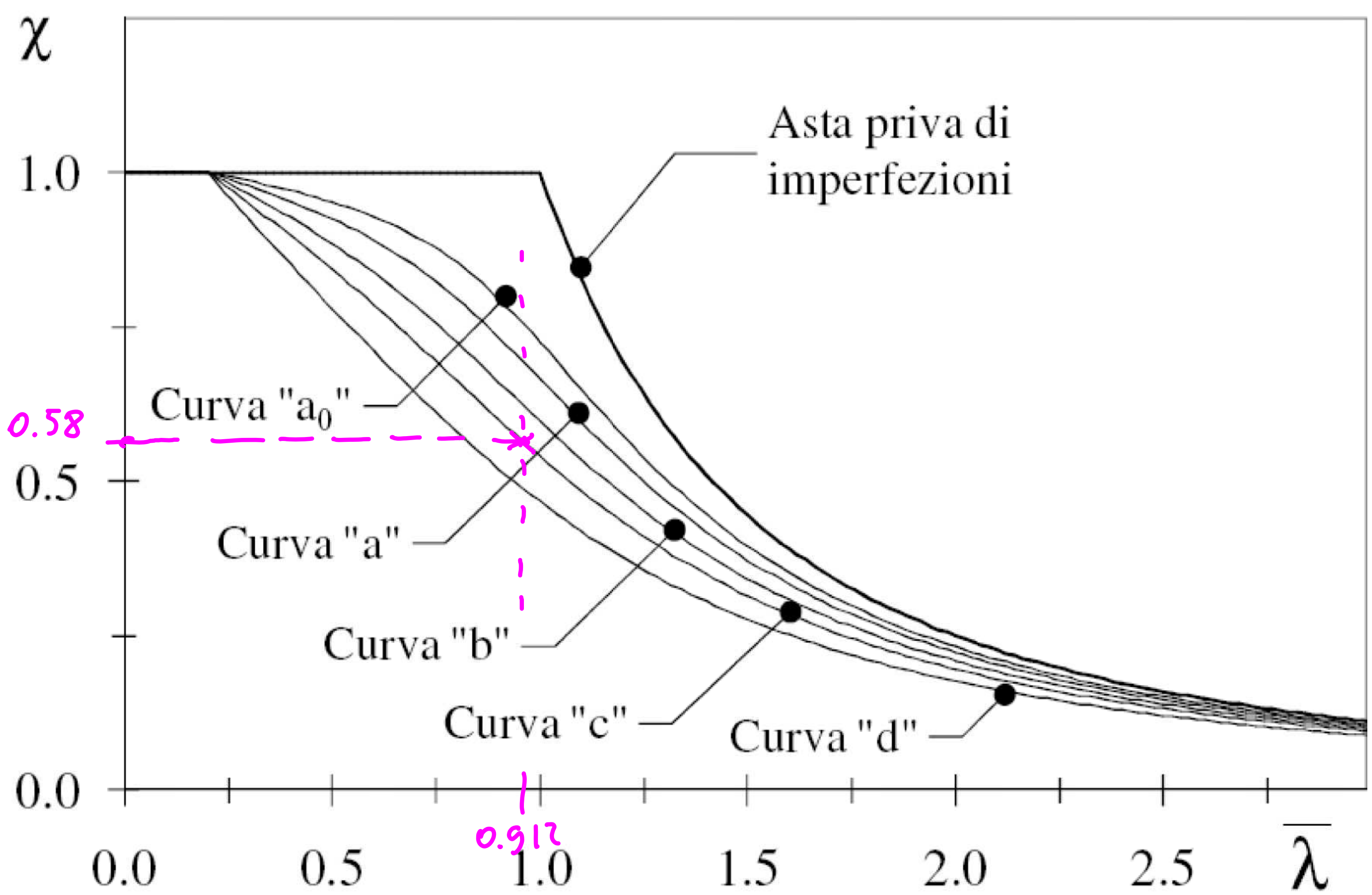
$\uparrow$ coefficiente di imperfezione

dipende dalla forma e altro

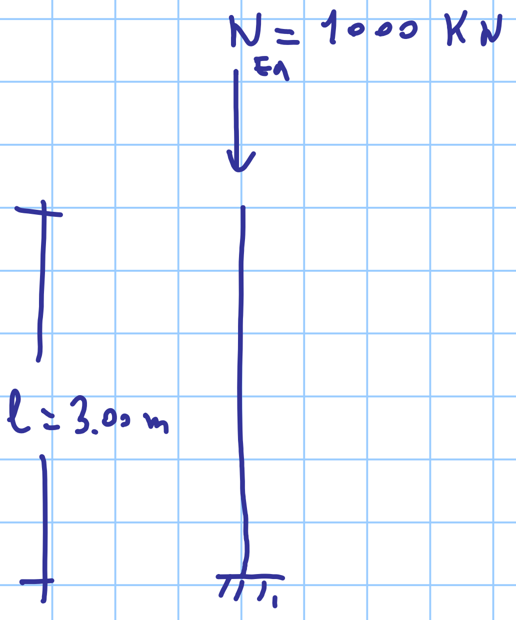
a_0	$\alpha = 0.13$
a	0.21
b	0.34
c	0.49
d	0.76

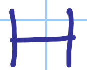
		Limiti	Inflessione intorno all'asse	Curve di instabilità	
				S235, S275, S355, S420	S460
Sezioni laminate $h/b > 1.2$		$t_f \leq 40$ mm	y-y	a	ao
			z-z	b	ao
		da 40 mm a 100 mm	y-y	b	a
			z-z	c	a
Sezioni laminate $h/b \leq 1.2$		$t_f \leq 100$ mm	y-y	b	a
			z-z	c	a
		> 100 mm	y-y	d	c
			z-z	d	c
Sezioni a doppio T saldate		$t_f \leq 40$ mm	y-y	b	b
			z-z	c	c
		> 40 mm	y-y	c	c
			z-z	d	d

Sezioni cave		formate a caldo	qualunque	a	a_0
		formate a freddo	qualunque	c	c
Sezioni scatolari saldate		in generale	qualunque	b	b
		saldature spesse	qualunque	c	c
Sezioni piene, a U e T		nessuno	qualunque	c	c
Sezioni a L		nessuno	qualunque	b	b



χ
1.0
0.5
0.0
Asta priva di
imperfezioni
Curva "a0"
Curva "a"
Curva "b"
Curva "c"
Curva "d"
0.58
0.912
0.0
0.5
1.0
1.5
2.0
2.5
λ





S275

HE 300B

$A = 149.1 \times 10^2 \ mm^2$

$i_y = 129.9$ mm

$i_z = 75.8$ mm

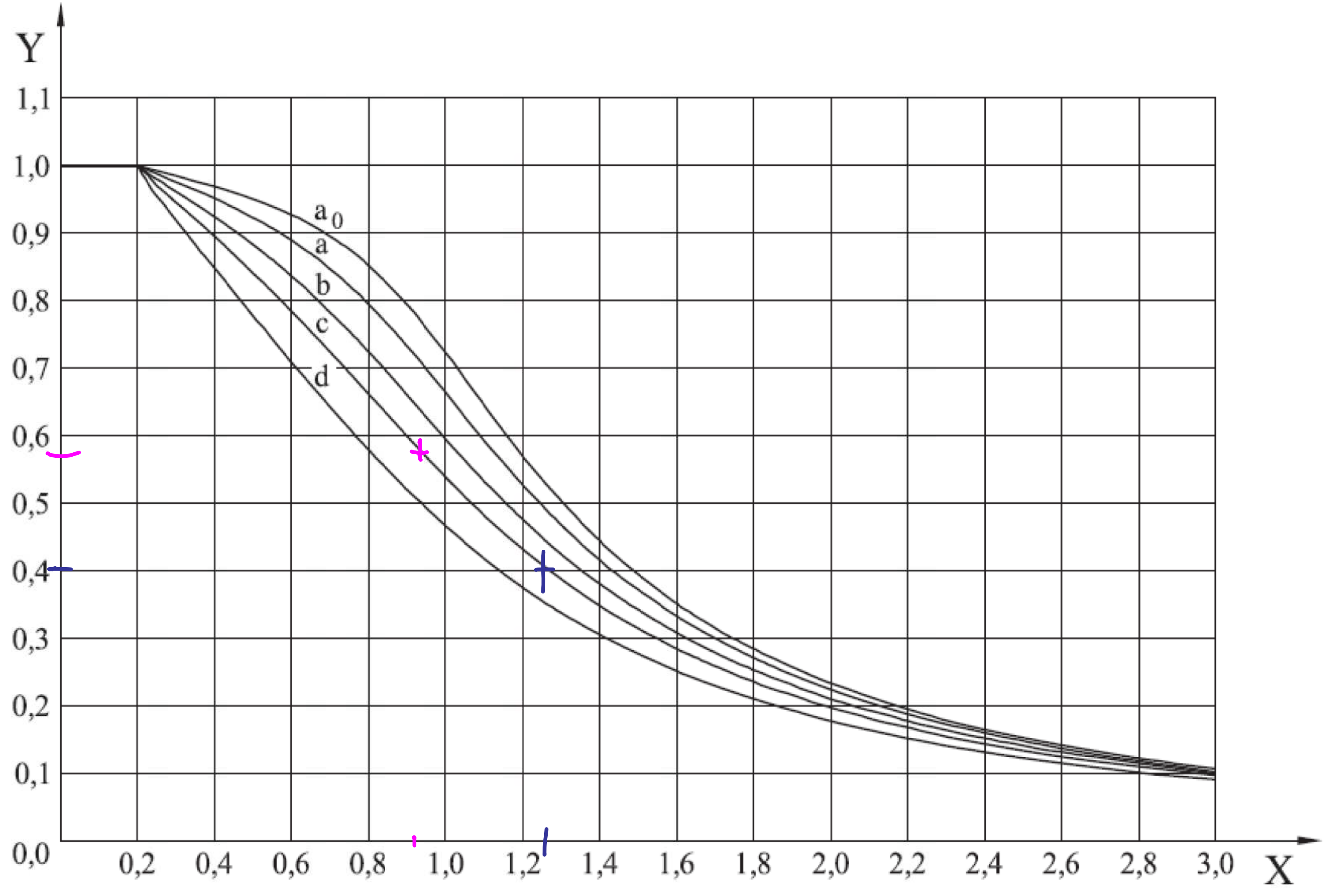
$l_0 = 6.00$ mm

$$\lambda_y = \frac{6000}{129.9} = 46.19$$

$$\lambda_z = \frac{6000}{75.8} = 79.16$$

$$\bar{\lambda}_z = \frac{79.16}{86.8} = 0.912$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = 3.14 \sqrt{\frac{210000}{275}} = 86.8$$



Y
1,1
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0,0
0,2
0,4
0,6
0,8
1,0
1,2
1,4
1,6
1,8
2,0
2,2
2,4
2,6
2,8
3,0
X
a 0
a
b
c
d

$$\chi = \frac{1}{\varphi + \sqrt{\varphi^2 - \bar{\lambda}^2}}$$

$$\varphi = \frac{1}{2}\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right] \qquad \alpha = 0.49$$

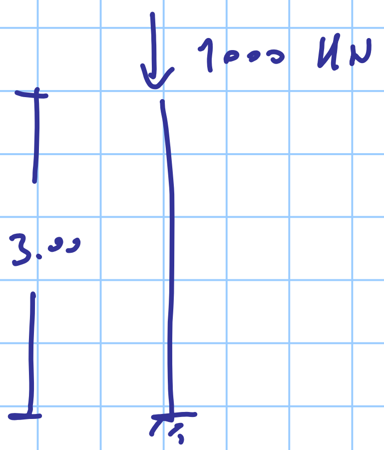
$$\varphi = \frac{1}{2}\left[1 + 0.49\left(0.912 - 0.2\right) + 0.912^2\right] = 1.13$$

$$\chi = \frac{1}{1.13 + \sqrt{1.13^2 - 0.912^2}} = 0.556$$

$$N_{b,Rd} = \chi A \frac{f_y}{\gamma_{M0}} = 0.556 \times 149.1 \times 10^2 \times \frac{275}{1.05} \times 10^{-3} = 2171.2 \text{ kN}$$

OK

progetto



$$N_{b,Rd} = \chi A \frac{f_y}{\gamma_{M0}}$$

$$A \geq \frac{N_{Ed}\ \gamma_{M0}}{\chi\ f_y} = \frac{1000 \times 10^3 \times 1.05}{0.5 \times 275} \times 10^{-2}$$

χ: 0.5

$$= 76.4 \times 10^2\ mm^2$$

HE 220 B $\rightarrow$ HE 240 B

No

$A = 91.0 \times 10^2\ mm^2$

$$N_{b,Rd} = \overset{0.40}{\cancel{\checkmark}}\ 91.0 \times 10^2 \times \frac{275}{1.05} \times 10^{-3} = 953.3\ kN$$

$\chi = 0.40$

$i_y =$

$i_z = 55.9\ mm$

$$\lambda_z = \frac{6000}{55.9} = 107.3$$

$$\bar{\lambda}_z = \frac{107.3}{86.8} = 1.237$$