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On the evaluation of seismic response of structures by nonlinear static methods

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SUMMARY

In the most recent seismic codes, the assessment of the seismic response of structures may be carried out by comparing the displacement capacity, provided by nonlinear static analysis, with the displacement demand. In many cases the code approach is based on the N2 method proposed by Fajfar, which evaluates the displacement demand by defining, as an intermediate step, a single degree-of-freedom (SDOF) system equivalent to the examined structure. Other codes suggest simpler approaches, which do not require equivalent SDOF systems, but they give slightly different estimation of the seismic displacement demand. The paper points out the differences between the methods and suggests an operative approach that provides the same accuracy as the N2 method without requiring the evaluation of an equivalent SDOF system. A wide parametric investigation allows an accurate comparison of the different methods and demonstrates the effectiveness of the proposed operative approach. Copyright © 2009 John Wiley & Sons, Ltd.

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1. INTRODUCTION

Since the second half of the XX century, the possibility of using more and more powerful computers is enhancing the comprehension of the structural response to seismic events. The full time history of elastic dynamic response to a single accelerogram may be evaluated by means of step-by-step integration of the equations of motion (*elastic dynamic analysis*). The maximum values achieved by all parameters (displacements, internal forces) may be adequately foreseen by means of *modal*

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response spectrum analysis. Note that this last approach is based on the evaluation of the maximum response of single degree-of-freedom (SDOF) systems, corresponding to the structure restrained to move according to the modal shapes. A simplified approach is the *lateral force* (*static*) *method of analysis*, which approximately accounts for the first modal shape. However, actual structures cannot remain elastic during strong seismic events. For this reason, seismic codes stipulate that the strength of the structural members be larger than the internal forces obtained by lateral forces or modal response spectrum analysis, performed by a pseudo-acceleration response spectrum reduced by a coefficient, named as behavior factor in Eurocode 8, which accounts for the global ductility of the structure. This approach, introduced by codes since the beginning of the XX century, is usually named as *force-based method*.

Indeed, the use of behavior factor allows only a rough estimation of the nonlinear structural response, which depends on the displacement and strain levels that the structure experiences during the seismic event. Therefore, a proper assessment of the structures requires a *displacement-based approach*, which compares the displacement capacity to the displacement demand. The diffusion of more powerful computers made possible the evaluation of the time history of inelastic response to a given accelerogram (*nonlinear dynamic analysis*). Also in this case, the maximum values achieved by response parameters may be determined. In particular, it is possible to evaluate the maximum values of base shear and top displacement for increasing values of peak ground acceleration (PGA), performing an incremental dynamic analysis (IDA), and to plot them versus PGA obtaining the so-called IDA curves; alternatively, it is possible to plot maximum base shear versus top displacement for increasing values of PGA, obtaining a curve which hereinafter will be named as *dynamic pushover curve*. Unfortunately, the difficulties in correctly modeling all the relevant characteristics of nonlinear cyclic behavior of structural members and in properly simulating the seismic input make this kind of analysis virtually inaccessible except to a few experts of seismic engineering and not recommended for every day design use. The need for a tool that explicitly considers the inelastic deformation experienced by the structural members during earthquakes without carrying out complex and computational costly nonlinear dynamic analysis led researchers to develop *nonlinear static methods*, which aim at predicting in a simplified way the base shear versus top displacement relationship and to relate each point of this curve to the value of PGA.

In this paper two of the most important nonlinear static methods existing in the literature (the N2 method proposed by Fajfar *et al.* [1–3] and the capacity spectrum method proposed by Freeman [4]) and their variants implemented in the seismic codes are examined, discussing their advantages and limitations, e.g. the different degree of reliability of the results that they provide or their operational complications. In particular, it is pointed out that the explicit reference to a SDOF system, which characterizes theoretical approach, is not necessary (as it is not necessary when performing lateral forces or modal response spectrum analysis). A different operational method, which is based on the same theoretical background and is simpler in practical applications, is therefore suggested. Finally, the effectiveness of the N2 method, of its variants stipulated in seismic codes and of the suggested method, is demonstrated by comparison with the results of nonlinear dynamic analyses performed on a wide set of frames.

2. NONLINEAR STATIC METHODS FOR SEISMIC ANALYSIS OF STRUCTURES

The nonlinear static methods, nowadays allowed by many seismic codes (as Eurocode 8 [5] in E.U., FEMA 356 [6] and FEMA 368 [7] in U.S., D.M. 14/1/2008 [8] and OPCM 3431 [9] in

Italy), are mainly related to two approaches available in the literature: the N2 method proposed by Fajfar *et al.* [1–3] and the capacity spectrum method proposed by Freeman [4]. In spite of some important conceptual differences, all the nonlinear methods are organized in two fundamental steps.

a. First step: *determination of the capacity curve of the structure*. The *capacity curve* of the structure, represented in terms of base-shear force versus top displacement relationship, is evaluated by monotonically increasing horizontal forces applied to the structure (pushover analysis). This analysis aims at describing how the dynamic structural response evolves when the PGA of the seismic input increases. The distribution of the horizontal forces F_i along the height is obtained by multiplying the floor masses m_i by a displacement profile $\mathbf{\Phi}$

$$F_i = m_i \Phi_i \tag{1}$$

Every reasonable profile $\mathbf{\Phi}$ can be used, but often the contributions of the higher modes of vibration of the structure are negligible and the displacement shape of the first mode of vibration can be used as the vector $\mathbf{\Phi}$. However, it is recommended that the analysis is repeated by two displacement profiles that bound the actual seismic response of the structure (e.g. constant and linear). Alternative procedures (adaptive load patterns [10–13], modal pushover analysis [14, 15], adaptive displacement pattern [16], etc.) have been suggested in order to obtain an improved correspondence of static versus dynamic inelastic response.

b. Second step: *evaluation of the displacement demand for a given PGA*. In order to judge the inelastic response of the structure under examination, it is necessary to relate each point of the capacity curve to a value of PGA. This means that it is necessary to evaluate the top displacement of the actual multi degree-of-freedom (MDOF) system corresponding to a seismic input having a given PGA, i.e. the so-called *displacement demand*.

Both the previously mentioned theoretical methods obtain this by: (1) individuating a SDOF system equivalent to the structure under examination; (2) calculating the displacement demand of this system; and (3) finally converting it into the displacement demand of the actual MDOF system. Note that the name (N2) of the method proposed by Fajfar underlines this aspect: evaluation of the nonlinear (N) response of the structure by two (2) different numerical models (the MDOF and SDOF systems).

The period T of an elastic MDOF system, vibrating according to a modal displacement profile $\mathbf{\Phi}$, is

$$T = 2\pi\sqrt{\frac{m^*}{K}} \tag{2}$$

where the stiffness K is the ratio of base shear over top displacement (obtained by a set of forces F_i proportional to $\mathbf{\Phi}$) and m^* is given by

$$m^* = \frac{\sum m_i \Phi_i}{\Phi_n} \tag{3}$$

The mass m^* is related to the effective modal mass M^* corresponding to the considered mode shape by the equation

$$m^* = \frac{M^*}{\Phi_n \Gamma} \tag{4}$$

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where Γ is the modal participation factor

$$\Gamma = \frac{\sum m_i \Phi_i}{\sum m_i \Phi_i^2} \tag{5}$$

The equivalent SDOF system has a mass equal to m^* and its response parameters (force F^* and displacement D^*) may be obtained from the corresponding parameters of the MDOF system (base shear V_b and top displacement D) by means of the equations

$$F^* = \frac{V_b}{\Phi_n \Gamma} \tag{6a}$$

$$D^* = \frac{D}{\Phi_n \Gamma} \tag{6b}$$

These equations, although strictly true only if $\mathbf{\Phi}$ is a modal displacement profile, are not very sensitive to moderate changes in $\mathbf{\Phi}$; they are thus used to transform the capacity curve of the MDOF system to the capacity curve of a corresponding SDOF system also when $\mathbf{\Phi}$ is not a modal displacement profile.

The capacity curve of the SDOF system is then idealized within the relevant range of displacements by a bi-linear relationship characterized by a lateral strength F_y^* and a yield displacement D_y^*. Different equivalence conditions have been suggested in the literature or by codes, as pointed out below. The post-yield slope is assumed null by Fajfar in the last version of his method [3]. The slope of the elastic branch, equal to the ratio $K_s = F_y^*/D_y^*$, is called hereinafter 'secant stiffness'. Note that K_s depends on the maximum displacement demanded by the earthquake, which is not known *a priori*; thus, it may be necessary to make use of an iterative procedure. The period of the idealized SDOF system is

$$T^* = 2\pi\sqrt{\frac{m^*}{K_s}} \tag{7}$$

In the method proposed by Freeman the displacement demand of the SDOF system is provided by an elastic response spectrum with a fictitious damping ratio, larger than that of the actual structure in order to take into account the hysteretic energy dissipation.

In the N2 method the displacement demand D_{req}^* of the inelastic SDOF system is related to the displacement of the corresponding elastic structure D_{el}^*, which may be obtained as the spectral value $S_{\mathrm{de}}(T^*)$. In general, being R_μ the force reduction factor (ratio of the elastic strength demand to the actual strength of the bi-linear system) and μ the ductility demand, it follows that $D_{\mathrm{req}}^* = \mu D_y^*$, $D_{\mathrm{el}}^* = R_\mu D_y^*$, i.e. $D_{\mathrm{req}}^* = \mu D_{\mathrm{el}}^*/R_\mu$. Furthermore, according to Vidic *et al.* [17] it is possible to assume

$$R_\mu = \mu \quad \text{when } T^* \geqslant T_C \tag{8a}$$

$$R_\mu = (\mu - 1)\frac{T^*}{T_C} + 1 \quad \text{when } T^* < T_C \tag{8b}$$

being T_C the transition period that separates the constant acceleration branch of the spectrum from the constant velocity branch. Therefore, the displacement D_{req}^* can be evaluated by amplifying the spectral displacement $S_{\mathrm{de}}(T^*)$ by a coefficient depending on the force reduction factor R_μ,

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according to the following equations:

$$D^*_{\mathrm{req}} = S_{\mathrm{de}}(T^*), \quad T^* \geqslant T_C \tag{9a}$$

$$D^*_{\mathrm{req}} = \frac{1}{R_\mu}\left[1 + (R_\mu - 1)\frac{T_C}{T^*}\right] S_{\mathrm{de}}(T^*), \quad T^* < T_C \tag{9b}$$

The spectral displacement $S_{\mathrm{de}}(T^*)$ may be calculated by the pseudo-acceleration $S_{\mathrm{ae}}(T^*)$ as

$$S_{\mathrm{de}}(T^*) = \frac{T^{*2}}{4\pi^2} S_{\mathrm{ae}}(T^*) \tag{10}$$

Finally, the displacement demand of the SDOF system is transformed back to the top displacement demand of the MDOF system by the inverse of Equation (6b)

$$D_{\mathrm{req}} = \Phi_n \Gamma D^*_{\mathrm{req}} \tag{11}$$

The seismic response of the MDOF system, in terms of member internal forces, floor displacement, plastic deformations, etc. is then assumed as that obtained by pushover analysis at a top displacement equal to D_{req}. If any response quantity is larger for a top displacement smaller than D_{req}, this maximum value has to be used instead.

2.1. Nonlinear static methods in seismic codes

Most of the nonlinear static methods stipulated by recent codes for the evaluation of the seismic response of structures are substantially based on the N2 method. The differences that may be found among these methods are due to the various simplifications that the national standards bodies have accepted or not in order to make their use more straightforward.

Eurocode 8 [5] imposes, as a general provision, to evaluate top displacement demand 'from the elastic response spectrum in terms of the displacement of an equivalent SDOF system'. More in detail, in the Annex B (informative) it faithfully follows Fajfar formulation [3] of N2 method and defines a bi-linear elastic-perfectly plastic idealization of the capacity curve of the SDOF system by imposing that: (1) the yield value F^*_y is equal to the strength corresponding to the target displacement (the ultimate strength provided by pushover analysis may be used as a starting value for an iterative procedure) and (2) the yield displacement D^*_y is chosen in such a way that the areas under the actual and the idealized curves are equal. The displacement demand D^*_{req} is given by formulations substantially coincident with Equation (9), in which the reduction factor is named as q_u (instead of R_μ) and expressly defined as

$$q_u = \frac{S_{\mathrm{ae}}(T^*) m^*}{F^*_y} \tag{12}$$

The top displacement demand of the MDOF system is then obtained by Equation (11). A nearly identical approach is followed by OPCM 3431 [9].

The nonlinear static method proposed by the prestandard for the seismic rehabilitation of existing buildings FEMA 356 [6] does not explicitly define an equivalent SDOF system. This method requires the direct idealization of the capacity curve of the structure into a bi-linear relationship (with hardening) by imposing that: (1) the actual and the idealized curves cross for a base-shear force equal to 60% of the yield value $V_{b,y}$; (2) the post-yield segment passes through the actual curve at the target displacement; and (3) the areas under the two curves are the same. The effective

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period T_e of the structure (corresponding to the secant stiffness) is calculated by the equation

$$T_e = T_1 \sqrt{\frac{K_i}{K_s^h}} \tag{13}$$

where T_1 is the elastic fundamental period provided by modal analysis, K_i is the elastic lateral stiffness of the structure corresponding to the used force distribution and K_s^h is the secant stiffness of the bi-linear relationship with hardening. The displacement demand of the structure is then evaluated by multiplying the spectral displacement corresponding to period T_e by coefficients C_0 and C_1, where C_0 is the first modal participation factor and C_1 a modification factor given by

$$C_1 = 1, \quad T_e \geqslant T_C \tag{14a}$$

$$C_1 = \frac{1}{R_\mu}\left[1+(R_\mu - 1)\frac{T_C}{T_e}\right], \quad T_e < T_C \tag{14b}$$

The reduction factor R_μ is

$$R_\mu = \frac{S_{\mathrm{ae}}(T_e) M_1^*}{V_{b,y}} \tag{15}$$

where M_1^* is the effective modal mass calculated for the fundamental mode of vibration. Other modification factors, provided by FEMA 356 to take into account stiffness and strength degradation and P-Δ effect, are not relevant for the discussion. It may easily be noted that the use of response spectrum values together with coefficients C_0 and C_1 leads to a global formulation analogous to that provided by Equations (9) and (11), confirming that also this approach implicitly utilizes the equivalent SDOF system. More specifically, when a vector $\mathbf{\Phi}$ proportional to the first mode of vibration is used the results differ from those provided by N2 method only because of the different bi-linearization criterion (which usually provides larger values of K_s with respect to those obtained by an elastic-perfectly plastic model). Other differences arise when the assumed vector $\mathbf{\Phi}$ is not proportional to the first mode of vibration, because mass m^* and participation factor Γ considered in the N2 method differ from the corresponding values used in FEMA 356, obtained by the first mode of vibration, and because of the use of K_i in Equation (13) (while K_1, stiffness related to the first mode of vibration, would be more proper).

Finally, it is also worth mentioning that the 'NEHRP recommended provisions for seismic regulations for new buildings and other structures' (called FEMA 368 [7]). The nonlinear static method stipulated by this document excels among the other available methods for its simplicity in application with respect to the traditional formulation of the N2 method, because it does not require any idealization of the pushover curve by a bi-linear relationship or the definition of any equivalent SDOF system. Independently of the force distribution used, the top displacement demand of the structure is determined by the elastic top displacement, corresponding to the dominant mode of vibration (usually the first one) or evaluated by combining the elastic modal values of the structure (FEMA 369 [18], Commentary to FEMA 368), eventually increased to account for differences between elastic and plastic response. In particular, if the fundamental period T_1 is smaller than T_C the elastic displacement has to be amplified multiplying it by the coefficient

$$C_i = \frac{(1 - T_C/T_1)}{R_\mu} + \frac{T_C}{T_1} \tag{16}$$

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being R_μ the ratio of the base shear given by the elastic response spectrum analysis to the strength of the structure. Note that the coefficient C_i may be rewritten in the form of the coefficient C_1 stipulated by FEMA 356 (Equation (14b)). Therefore, the difference between C_i and the coefficient used within the traditional formulation of the N2 method (Equation (9b)) is that the period T_1 is used instead of T^*. Note that the approach presented in the new version of NEHRP recommendations (FEMA 450 [19]) is similar to that contained in FEMA 356.

3. OPERATIVE APPROACH FOR NONLINEAR STATIC ANALYSIS

3.1. The proposed approach

It has been already pointed out that linear force-based methods underlie a reference to SDOF systems, which is not explicitly recalled in their application. The authors believe that also nonlinear static methods may be performed without an explicit reference to a SDOF system. Indeed, the displacement demand may be directly evaluated by the values provided by an elastic analysis (lateral forces or modal response spectrum analysis), modified so as to take adequately into account the nonlinear behavior. Skipping the definition of the equivalent SDOF system, this requires the analysis of just one structural model. This operative approach, which will be called 'N1 method' hereinafter, is summarized in the following steps:

a. Determination of the nonlinear behavior of the real structure. As for all nonlinear static methods, the base shear V_b versus top displacement D relationship is determined by a pushover analysis of the structure.

b. Determination of the displacement demand corresponding to a prefixed PGA.

b1. Determination of the elastic response of the structure. Assuming that the structural behavior remains elastic, the modal response spectrum analysis—or the equivalent lateral forces analysis when appropriate—provides the strength demand $V_{b,\mathrm{el}}$ and the maximum displacement D_{el} of the top floor due to the seismic event for the prefixed PGA. For the sake of simplicity, hereinafter we assume that the first mode is clearly predominant and properly describes the response of the structure.

b2. Determination of the displacement demand by the elastic displacement. The displacement demand D_{req} is obtained by correcting the elastic displacement D_{el}, in order to take into account the difference between inelastic and elastic behavior (this one characterized by the elastic stiffness K_1, ratio of the base shear to the top displacement provided by the elastic analysis). First, the bi-linear (elastic-perfectly plastic) relationship equivalent to the performance curve of the actual structure is determined by the usual criteria proposed in the literature; here, the bi-linear idealization stipulated by EC8 is adopted. The difference between K_1 and K_s is accounted by evaluating the effective period T_e[‖]

$$T_e = T_1\sqrt{\frac{K_1}{K_s}} \tag{17}$$

[‖]If the elastic displacement has been evaluated by means of equivalent lateral forces analysis, the first period T_1 can be determined by approximate formulations, as Rayleigh's method.

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and multiplying the displacement D_{el} by the ratio of the spectral displacements corresponding to the period T_e and the fundamental period T_1, respectively.

A second correction accounts for the potential increase of the displacement due to the yielding of the structure. This correction, necessary for periods shorter than T_C, depends on the coefficient R_μ, ratio of the elastic strength demand to the maximum strength of the structure. The following equations are therefore obtained:

$$D_{\mathrm{req}} = D_{\mathrm{el}} \frac{S_{\mathrm{de}}(T_e)}{S_{\mathrm{de}}(T_1)} \quad \text{if } T_e \geqslant T_C \text{ or } R_\mu \leqslant 1 \tag{18a}$$

$$D_{\mathrm{req}} = D_{\mathrm{el}} \frac{S_{\mathrm{de}}(T_e)}{S_{\mathrm{de}}(T_1)} \frac{1}{R_\mu} \left[1 + (R_\mu - 1)\frac{T_C}{T_e}\right] \quad \text{if } T_e < T_C \text{ and } R_\mu > 1 \tag{18b}$$

Note that, analogously to all the other methods, R_μ is related to the bi-linear idealization of the performance curve in the relevant range of displacements. Indeed, the strength demand is obtained by multiplying $V_{b,\mathrm{el}}$ by the spectral accelerations ratio $S_{\mathrm{ae}}(T_e)/S_{\mathrm{ae}}(T_1)$, while the maximum strength is the yield value in the bi-linear relationship.

3.2. Theoretical comparison of N1 and N2 methods

It can be demonstrated that, when a distribution of the horizontal forces along the height proportional to the first mode of vibration of the structure is used (i.e. $\mathbf{\Phi} = \mathbf{\Phi}_1$), N2 and N1 methods provide identical results. In fact, in such a case the mass m^* coincides with the mass m_1^* associated with the first mode of vibration and the periods T^* and T_e are identical. Furthermore, being

$$D_{\mathrm{el}} = \Phi_n \Gamma_1 S_{\mathrm{de}}(T_1) \tag{19}$$

and the modal participation factor of the first mode of vibration Γ_1 equal to the coefficient Γ given by Equation (5), Equations (18a), (18b) and (19) are perfectly equivalent to (9a), (9b) and (11).

Instead, when the assumed vector $\mathbf{\Phi}$ is not proportional to the first mode of vibration, the mass m^* and the coefficient Γ considered by the N2 method are different from m_1^* and Γ_1 considered by the N1 method and, therefore, the two methods do not provide identical results.

3.3. Evaluation of PGA corresponding to a prefixed limit state

Although static nonlinear methods are mainly used to determine the top displacement demand of a structure for a given PGA, they can be also applied to obtain the PGA corresponding to every point (D, V_b) of the performance curve. It is thus possible to have a global vision of the structural behavior, individuating the values of PGA corresponding to different limit states. With reference to N2 method, Fajfar and Dolšek [20] proposed to plot seismic intensity versus top displacement, naming IN2 (Incremental N2) the obtained curves. As an alternative, we propose to maintain the classical $V_b - D$ performance curve, adding a further (nonlinear) scale for PGA. In this way it is possible to see contemporaneously base shear, top displacement and seismic acceleration.

The procedure to relate the points of the performance curve to the corresponding PGA is quite simple. First of all it may be noted that, when the elastic top displacement $\overline{D}_{\mathrm{el}}$ and the base-shear force $\overline{V}_{b,\mathrm{el}}$ have been evaluated by modal spectrum response analysis for an arbitrarily chosen value $\overline{a}_g$ of PGA, the values D_{el} and $V_{b,\mathrm{el}}$ corresponding to any other value a_g of PGA can be obtained by a scaling. Furthermore, when a point (D, V_b) of the performance curve of the structure is selected, the relevant range of displacement is defined ($D_{\mathrm{req}} = D$) and the bi-linear idealization of the performance curve as well as the period T_e are consequently determined. At the same time, also the ductility μ is known and the force reduction factor R_μ may be evaluated by Equations (8a) and (8b). Then, the corresponding displacement D_{el} is obtained by inverting Equations (18a) and (18b). Finally, the PGA corresponding to D_{el} is evaluated by multiplying it by the ratio $\overline{a}_g/\overline{D}_{\mathrm{el}}$, thus obtaining

$$a_g=\overline{a}_g\frac{D}{\overline{D}_{\mathrm{el}}}\frac{S_{\mathrm{de}}(T_1)}{S_{\mathrm{de}}(T_e)} \quad \text{if } T_e\geqslant T_C \text{ or } R_\mu\leqslant 1 \tag{20a}$$

$$a_g=\overline{a}_g\frac{D}{\overline{D}_{\mathrm{el}}}\frac{S_{\mathrm{de}}(T_1)}{S_{\mathrm{de}}(T_e)}\frac{R_\mu}{1+(R_\mu-1)T_C/T_e} \quad \text{if } T_e<T_C \text{ and } R_\mu>1 \tag{20b}$$

3.4. Examples

The nonlinear static analysis has been performed, according to the different approaches analyzed in the paper, for a wide set of frames described in detail in the next section. An example of the different shape of pushover curves and of their bi-linear idealization is shown in Figure 1, which refers to two frames (named 1ADH and 9EFL and characterized by period of vibration $T_1=0.486$ s and $T_1=1.948$ s, respectively) pushed by forces proportional to the first mode of vibration until the achievement of the collapse prevention limit state. The frame 1ADH rapidly reaches the collapse mechanism after the first yielding. Therefore, the pushover curve is not very different from its bi-linear idealizations (with or without hardening) and the secant stiffness is almost equal to the initial stiffness K_i. Instead, the frame 9EFL reaches collapse after the development of a large number of plastic hinges. As a consequence, the stiffness degradation of this frame is much more gradual and the secant stiffness is strongly influenced by the bi-linearization criterion used, i.e. K_s is much smaller than K_i, whereas K_s^h is close to K_i.

The correspondence between points of the performance curve and PGA has been evaluated according to the procedure described in Section 3.3. It may be noted that the relationship between

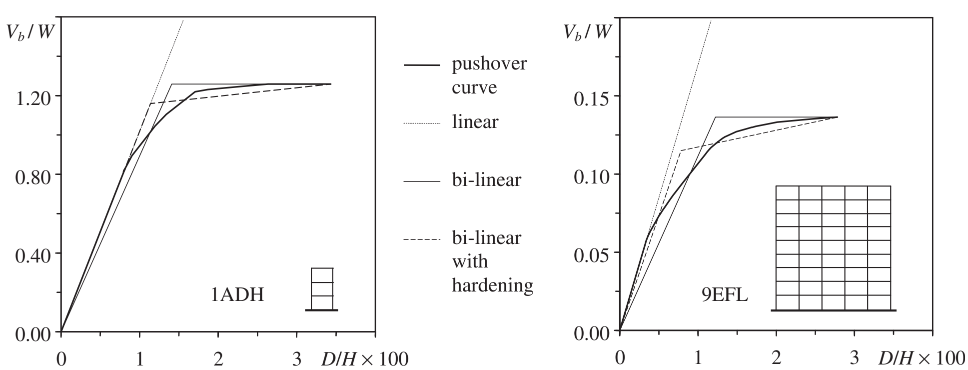

Figure 1. Pushover curves and corresponding bi-linear idealizations of the frames 1ADH and 9EFL (normalized with respect to the total weight W and the height H of the frame) obtained by a force distribution proportional to the first mode of vibration.

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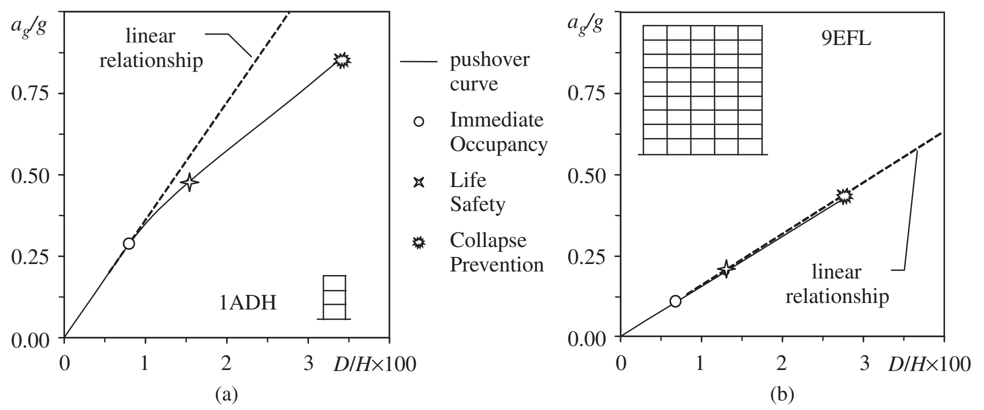

Figure 2. Normalized roof displacement and PGA biunivocal relationship obtained by the N1 method using a force distribution proportional to the first mode of vibration.

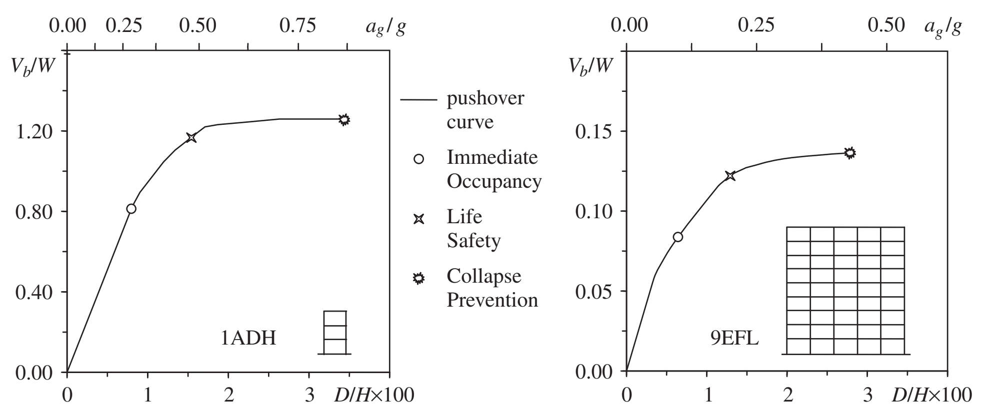

Figure 3. Performance curve obtained by a force distribution proportional to the first mode of vibration with a double scale for abscissa axis (D/H and PGA) for the frames 1ADH and 9EFL.

a_g and D may be approximately linear (Figure 2(b)) or not (Figure 2(a)), depending on the fundamental period of the structure. The performance curve is represented in Figure 3 with a double scale for abscissas. This representation is very useful because it describes the increase of yielding with PGA and at the same time it allows to check several performances objectives as required by modern seismic codes. Indeed, if the points corresponding to the achievement of the selected performances objectives (for instance immediate occupancy IO, life safety LS and collapse prevention CP) are highlighted, the graphs immediately provide the corresponding values of PGA, which have to be compared with those specified by the code. Alternatively, the roof displacements of the structure, corresponding to the values of PGA stipulated by the code, may be determined and compared with those corresponding to the selected performance objectives.

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4. VALIDATION OF NONLINEAR STATIC METHODS

In order to assess the effectiveness of the considered methods, a wide parametrical investigation is carried out with reference to a set of 108 steel frames with rigid connections. The validation involves two stages. First, the displacement demand of the considered frames determined by the FEMA 356, FEMA 368 and N1 methods is compared with that obtained by the N2 (Eurocode 8) method. Second, the displacement demand obtained by the FEMA 356, FEMA 368, N1 and N2 (EC8) methods is compared with the actual maximum displacements of the frames determined by nonlinear dynamic analysis. Three values of PGA are considered: 0.14, 0.35 and $0.53g$ in order to examine the response of structures that undergo different levels of plastic deformation. In Italy, OPCM 3431 [9] specifies these values to represent seismic events having probability of exceedance of 50, 10 and 2% in 50 years, respectively, in high seismicity regions.

Both pushover and nonlinear dynamic analyses are carried out by the DRAIN-2DX computer program [21, 22]. A member-by-member modeling with plastic hinges assigned at member ends is adopted. A Rayleigh viscous damping is used and set at 5% for the first two modes of vibration. Strain hardening and geometrical nonlinearity are not considered. According to EC8, nominal dead loads plus quasi-permanent live loads are assumed as initial gravity loads in the analyses. The elastic spectrum proposed by EC8 for ground type D is used for nonlinear static methods; a set of ten artificial accelerograms, generated by the SIMQKE computer program [23] and compatible with the above elastic response spectrum, is used for nonlinear dynamic analysis.

4.1. Analyzed frames

All the analyzed steel frames are designed by the method proposed by Neri [24] and described by Gioncu and Mazzolani in [25] and by Ghersi *et al.* in [26]. This design method allows the frames to achieve collapse by a global mechanism. Each frame is regular, i.e. it has equal span length, floor masses and inter-story heights. Wide-flange shapes available in Europe and steel grade with yield stress $f_y = 275\,\text{MPa}$ are used for beams and columns. The frames are different for the number of stories (3, 6 and 9), the number of spans (1, 3 and 5), the size of cross-section used for beams (from IPE220 to IPE330), the amount of gravity loads (high or low) and the span length (4.50 and 5.50 m). 'High value' refers to the gravity loads providing a design bending moment in the non-seismic combination equal to the flexural strength of the beam; 'low value' refers to 50% of the previous value. Each frame is identified in the paper by a code which points out its main characteristics, i.e. the number of stories and spans, the cross-section used for beams, the amount of gravity loads and the span length, in accordance with the notation reported in Table I; for instance the code '5CFH' identifies a six-story frame with three spans 4.50 m long and IPE270 beams which sustain high gravity loads.

The set of frames above described covers a wide range of the structural features that influence seismic response and its estimation by nonlinear static methods. Indeed, the results provided by these methods strongly depend on the fundamental period of the frame and on the bi-linear idealization of the pushover curve. The fundamental periods of the considered frames range from 0.46–2.20 s, thus including cases in which the equal displacement rule may or may not apply.

4.2. Comparison among nonlinear static methods

The top displacement demand of the frames described in the previous section is determined by the N2 (EC8), FEMA 356, FEMA 368 and N1 methods. According to common practice, the pushover

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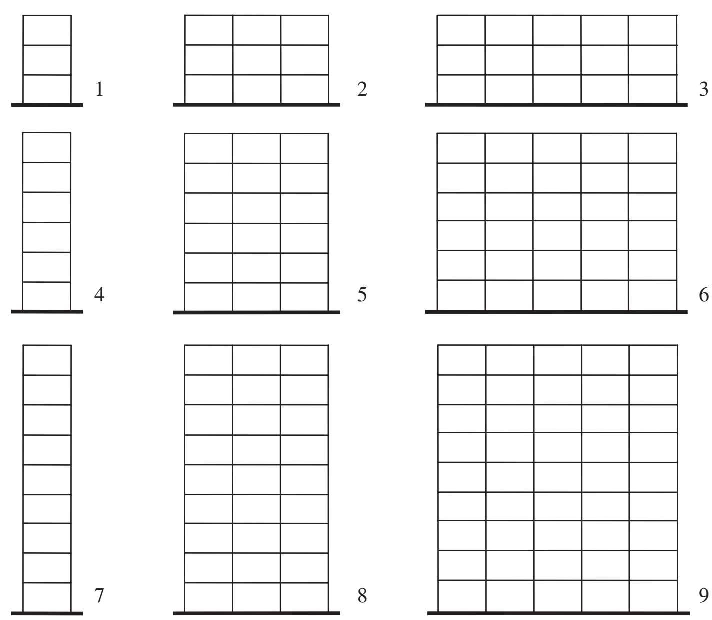
Table I. Characteristics of the analyzed frames.

GEOMETRICAL SCHEME



BEAM CROSS-SECTIONS					
	A	B	C	D	E
	IPE 220	IPE 240	IPE 270	IPE 300	IPE 330

VERTICAL LOAD			SPAN LENGTH		
q	F	D	L	H	L
	high	low		4.50 m	5.50 m

analysis is performed twice considering two force distributions, horizontal forces proportional to the first mode of vibration and proportional to the masses; hereinafter the distributions are called 'modal' and 'constant' force distribution.

The top displacement demands of the frames obtained by the considered methods using the modal force distribution are compared in Figure 4. In the graphs, the results obtained by the N2 method are assumed as reference values and reported on the X-axis, whereas the results obtained by the FEMA 356 method (a), the FEMA 368 method (b), and the N1 method (c) are reported on the Y-axis. In these figures, dots lying along the bisector denote that the relevant method (FEMA 356, FEMA 368 or N1 method) provides the same results obtained by the N2 method, whereas dots that are below or above the bisector denote that the relevant method is less or more conservative than the N2 method, respectively. Before discussing the results, we should remember that in this case the mass m^* and the factor Γ used by the N2 method are equal to the mass m_1^* and the modal

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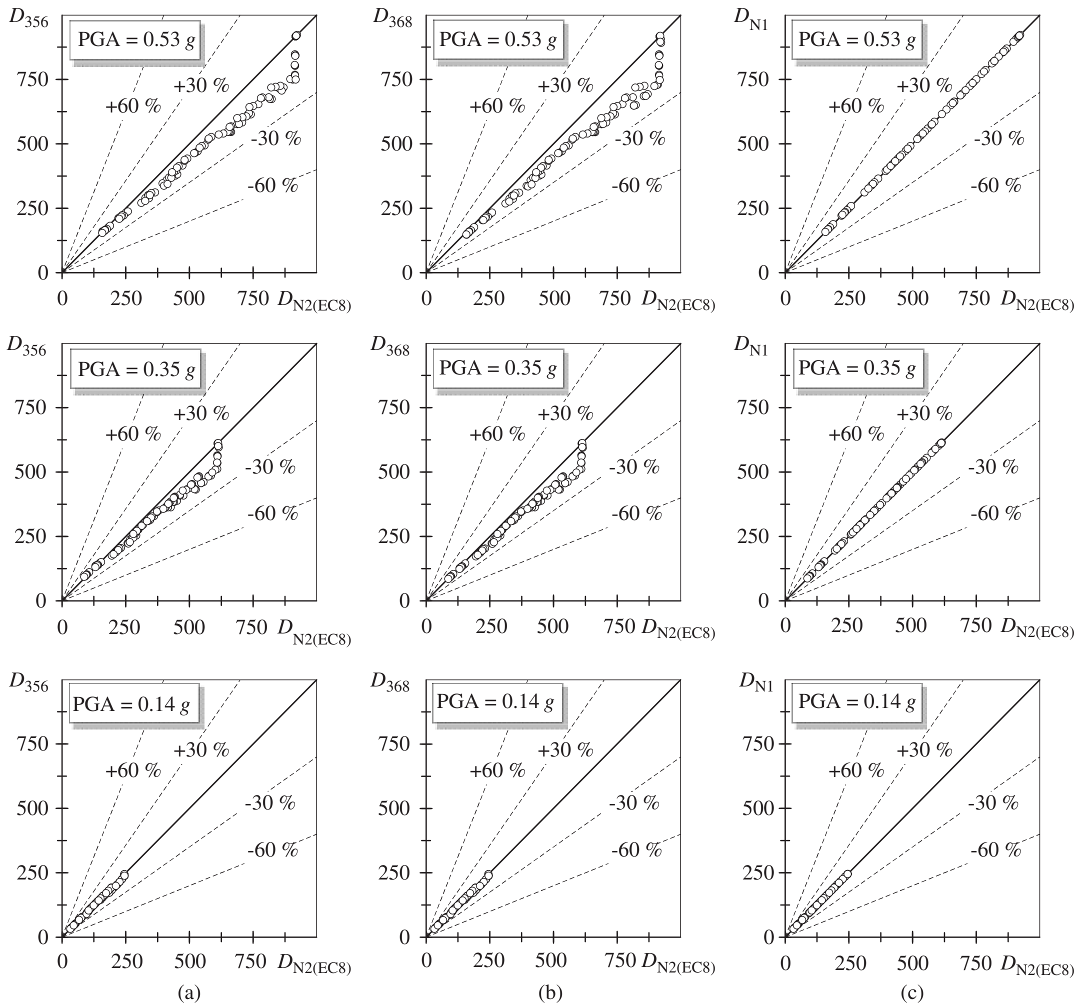

Figure 4. Comparison between top displacement demands (mm) determined by the analyzed methods considering a force distribution proportional to the first mode of vibration.

participation factor Γ_1 used by the other methods; therefore, the differences among the methods are only due to the assumed value of stiffness. The top displacement demands obtained by FEMA 356 and FEMA 368 are nearly the same, because the stiffness considered by the two methods (K_s^h and K_1, respectively), and therefore the relevant periods, are very similar. It may be further noted that these methods underestimate the displacement demand of the analyzed frames with respect to the N2 method, owing to the fact that the more the frames are into the inelastic range, the more the stiffness K_s used by N2 (EC8) method is smaller than that used by FEMA 356 and FEMA 368. On the contrary, the N1 method uses the same stiffness K_s of the N2 (EC8) method, and consequently provides the same displacement demand. This is confirmed by Figure 4(c) where all the dots lie along the bisector, regardless of the value of the PGA.

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The same comparison for the results obtained by the constant force distribution is shown in Figure 5. In this case FEMA 356, FEMA 368 and N1 methods consider similar effective periods, because they refer to the same value of mass, m_1^*, and the square root of stiffness ratios in Equations (13) and (16) are both close to unity, and thus provide similar results for all the analyzed frames. On the other hand, the N2 method uses a mass m^* equal to M (total mass of the system) and larger than m_1^*, thus providing a period T^* and a spectral displacement $S_{\mathrm{de}}(T^*)$ always larger than the values obtained by the other methods. However, the displacement demand is also proportional to the factor Γ, which is equal to one in N2 method and equal to $\Gamma_1 > 1$ in the FEMA 356, FEMA 368 and N1 methods. For this reason, the differences between the displacement demand obtained by N2 method and those provided by the other methods are generally small. Only for structures with long period, i.e. belonging to the constant displacement branch of the response spectrum, the elongation of the period T^* with respect to T_e and T_1 does not produce any increase in the displacement demand, while the difference between Γ and Γ_1 makes the displacement demand evaluated by the other methods larger than that obtained by the N2 method. On the contrary, for frames with T_1 smaller than T_C, the N1 method slightly underestimates the displacement demand with respect to the N2 (EC8) method. Indeed, in this range of periods the spectral displacement is proportional to the square of the period and the effect of the difference between m^* and m_1^*, which makes $S_{\mathrm{de}}(T^*)$ larger than $S_{\mathrm{de}}(T_e)$, prevails over the effect of the difference between Γ and Γ_1. For these frames, further small differences between the displacement demands are due to the fact that all the methods provide different values of R_μ.

4.3. Effectiveness of nonlinear static methods for the estimation of dynamic response

As shown in Section 4.2, different methods provide slightly different displacement demand. In particular, the proposed N1 approach and the N2 (EC8) method coincide when a force distribution correspondent to the first modal shape is used, while they present not negligible differences for short and long period frames when other distributions are used. However, it should be noted that every method does not describe the ‘reality’, but only provides an ‘estimate’ of the actual dynamic response of frames subjected to earthquake excitation. Therefore, the effectiveness of the methods in estimating the seismic response must be judged by comparing the scatters between the displacement demands provided by each method and that determined by nonlinear dynamic analysis. Here, this comparison is made in terms of top displacement, top story drift and first story drift demand, with reference to FEMA 356, FEMA 368, N1 and N2 (EC8) methods.

For each response parameter, the maximum value obtained by the modal and constant force distributions in the considered displacement range is assumed as estimate of the actual dynamic response. The comparison between the results obtained by the methods and those obtained by nonlinear dynamic analysis is reported in Figure 6. Results obtained by FEMA 368 method are not reported because they are substantially coincident with those provided by FEMA 356. The response obtained by the dynamic analysis, expressed as the mean of the values obtained for the ten considered accelerograms, is reported on the X-axis, whereas the results obtained by the different methods of nonlinear static analysis are reported on the Y-axis.

The maximum top displacement demand of all the considered frames is obtained by the modal force distribution and in this case, as previously demonstrated, the N2 and N1 methods are equivalent, while the FEMA 356 and FEMA 368 methods provide slightly smaller displacements (Figure 4). Nevertheless, Figure 6(a) shows that the estimate of the top displacement demand of the analyzed frames is conservative with respect to the results of dynamic analysis and the accuracy is

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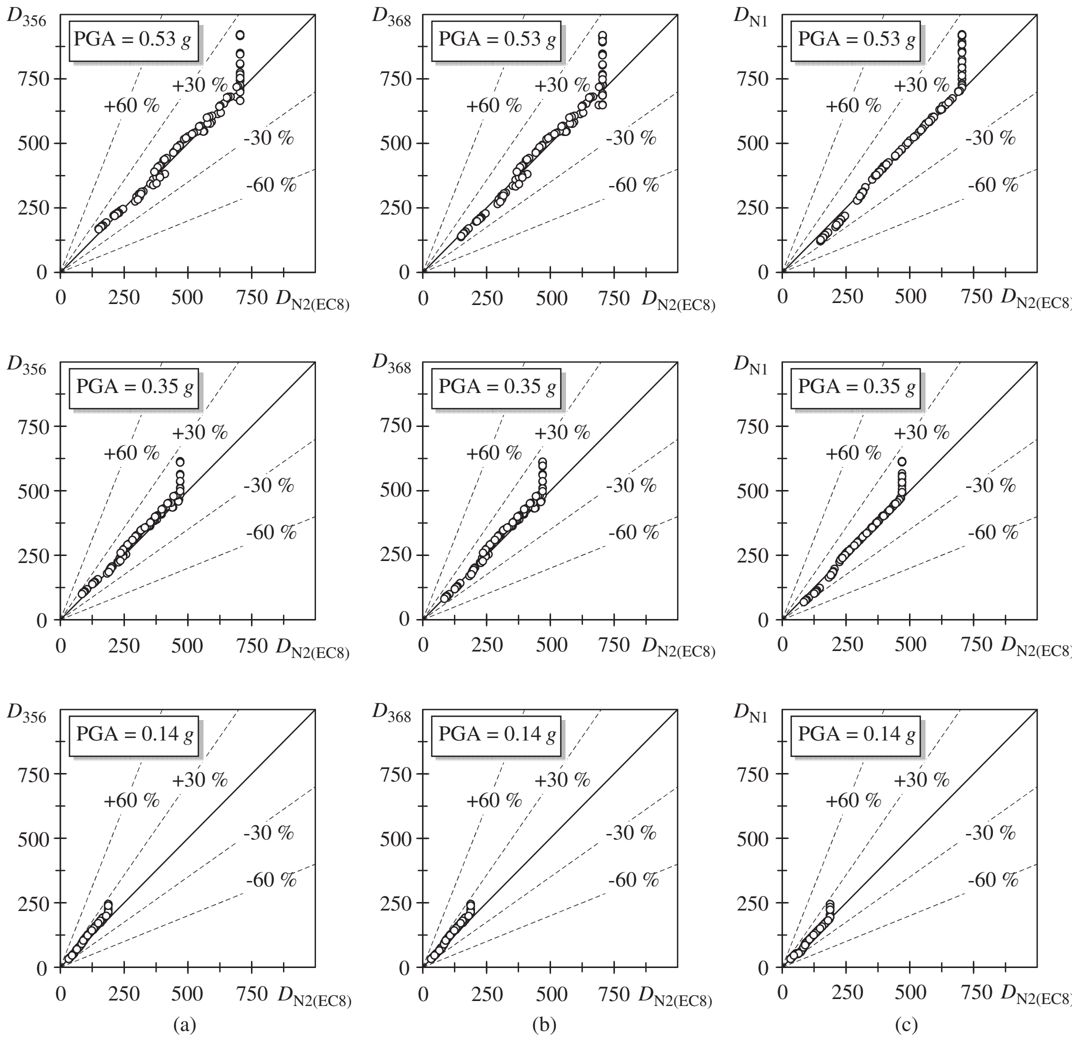

Figure 5. Comparison between top displacement demands (mm) determined by the analyzed methods considering a force distribution proportional to the floor masses.

similar for all the considered methods. The largest differences may be observed when the frames are well excited into inelastic range; e.g. for PGA $=0.35g$ and $0.53g$ the demand of some long period frames evaluated by the nonlinear static methods is about 50% larger.

Similar considerations apply to the top story drift, because the maximum value is again obtained by the modal force distribution. Anyway in this case the nonlinear static methods may significantly underestimate the top story drift demand, mainly for lower values of PGA because of the influence of higher modes of vibration (Figure 6(b)). On the contrary, the maximum first story drift demand is always given by the constant force distribution. For this reason, the results obtained by FEMA, N1 and N2 (EC8) methods do not coincide (as already shown in Figure 5), but they are very close to each other for all the considered frames and nearly always conservative (Figure 6(c)).

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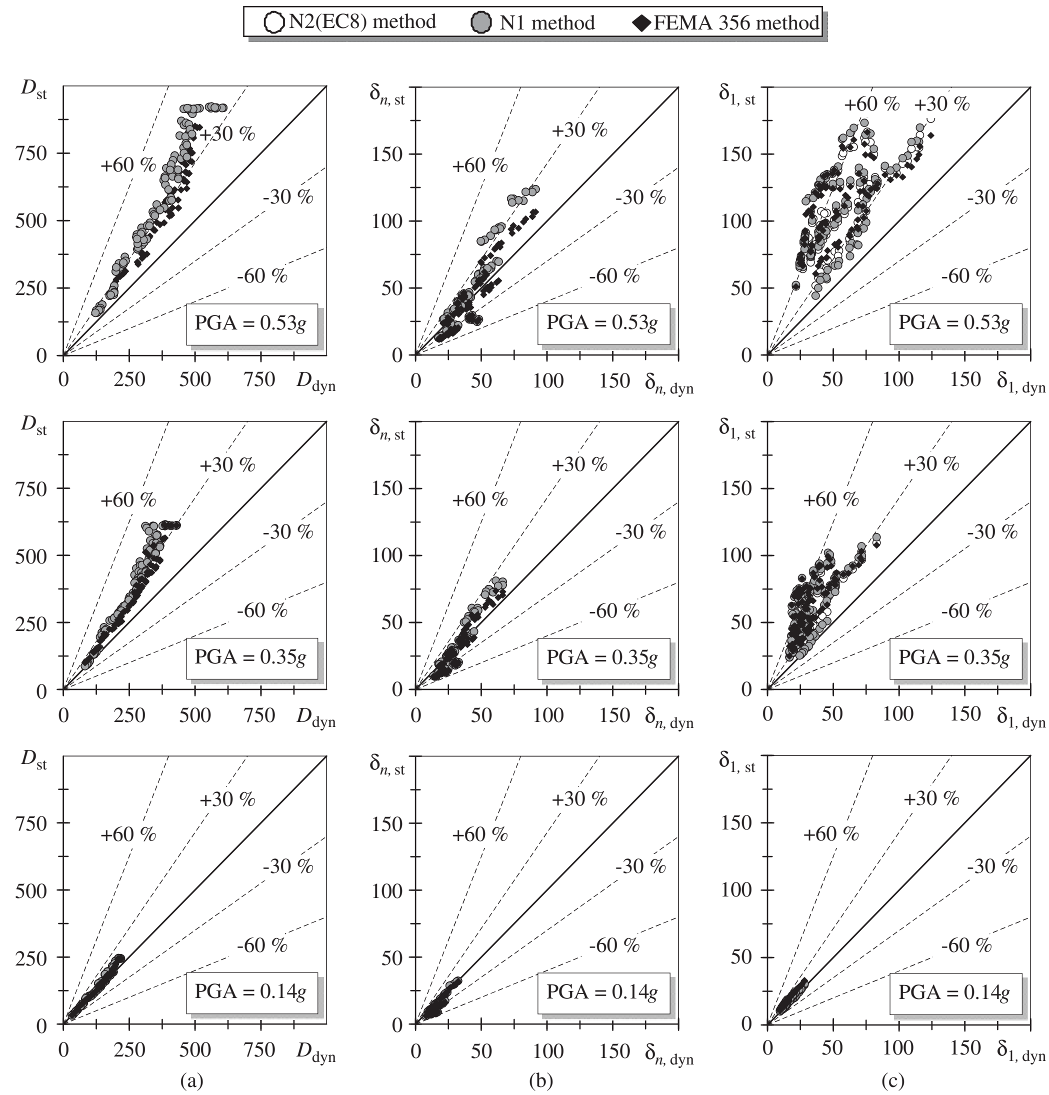

Figure 6. Comparison between displacement demands (mm) determined by the N1, N2-EC8 and FEMA 356 nonlinear static methods ($D_{\mathrm{st}}, \delta_{i,\mathrm{st}}$) and nonlinear dynamic analysis ($D_{\mathrm{dyn}}, \delta_{i,\mathrm{dyn}}$): (a) top displacement; (b) top story drift; and (c) first story drift.

5. CONCLUSIONS

In this paper, different methods for performing nonlinear static analysis (proposed by researchers or by seismic codes) are examined and compared, showing the conceptual differences and evidencing the aspects that, sometime, may lead to different results. An operative approach, which does not require to explicitly consider a SDOF system, called N1 method is proposed. Furthermore, it is suggested to add to the performance curve a further scale that indicates the PGA and, thus, allows

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a thorough vision of the structural behavior with reference to several performance objectives, as now required by the most recent seismic codes. Finally, the effectiveness of all the methods is confirmed by the comparison between the results they provide and those obtained by nonlinear dynamic analysis, with reference to a large number of frames covering a wide range of the structural features that influence seismic response.

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