Validation and improvement of N1 method for pushover analysis

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ABSTRACT

Many of the non-linear static methods for seismic assessment of buildings according to modern structural codes are based on the well-known N2 procedure. A more intuitive pushover procedure, N1, has recently been proposed. Its main advantage is that the explicit evaluation of an equivalent SDOF system is not required. The N1 method has been proved to provide the same accuracy as N2, but only when a lateral load distribution proportional to the first mode shape is involved. After a brief description of the main differences between the two methods, an improved version, the N1 corrected method, is presented here. It is more consistent with N2, also when constant acceleration lateral load patterns are applied. The N1 corrected method is validated according to an extensive parametric investigation of a set of case studies on steel and R/C frames, with regular and irregular mass distribution in height.

Keywords: non-linear seismic behaviour; pushover analysis; N2 method; N1 method.

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1. INTRODUCTION

Of the several non-linear static methods in the literature, the 'capacity spectrum method CSM'[1] and the 'N2 method' [2-4] have achieved growing consensus, so that their use is currently prescribed by various seismic codes (Eurocode 8 (EC8) [5] in the EU, ATC 40 [6] and FEMA 440 [7] in the US; DM2008 in Italy [8]), and they are commonly applied by many experts in seismic engineering. Differences among these methods are due to the simplifications that national standards bodies have accepted in order to make their application more straightforward.

It is well-known that the N2 method cannot be applied to the study of irregular buildings (e.g., torsionally flexible plan-asymmetric and high-rise buildings). Modified modal pushover analysis (MMPA), practical modal pushover analysis (PMPA) and N2-extended methods have thus recently been proposed. They generally combine the results of basic pushover analysis (e.g., N2) with those of standard elastic modal analysis, to take into account the influence of higher modes, in both plan and elevation [13-17].

In practice, N2 is still mainly used and more advanced methods are not yet contemplated by seismic codes, so that further study and improvement of N2 are needed.

Seismic codes normally impose the use of at least two distributions of forces to determine performance curves: one is related to the first mode of vibration and the second uses a force distribution proportional to the floor masses. The envelope of results is then examined.

All the non-linear static methods in the seismic codes present some aspects which require improvement. However, the need to define an equivalent single degree-of-freedom (SDOF) system, required by all methods which implement N2 in its original version, makes their application rather complex.

To simplify the procedure as much as possible, but maintaining it consistent with N2, Bosco *et al.* [9] proposed an alternative non-linear static procedure, called 'N1 method', for seismic assessment of structures. This method has the same theoretical background, but does not require explicit reference to the equivalent SDOF system. The abbreviation 'N1' emphasises the fact that the method is non-linear (N) and solves only '1' model of the structure, i.e., the multi degree-of-freedom (MDOF) model. It evaluates the displacement demand directly as the value provided by a standard elastic modal response spectrum analysis – RSA [10], modified to take into account the non-linear behaviour of the structure.

Conceptually, this method adopts the approach of FEMA 368 [11] and FEMA 369 [12], but introduces improvements which take into account the reduction in stiffness (and thus the increase in the period). In the classical formulation of the N2 method, this is obtained by bi-linearisation of the capacity curve.

One important advantage of N1 is that, with linear RSA, peak ground acceleration (PGA) values can be directly correlated with displacement D_c of a control point, normally assumed as the mass centre of the top floor of the building. A further (non-linear) scale for PGA to the classical relationship base-shear force V_b versus top displacement D_c can be added. This makes N1 more suitable within the modern Displacement-Based Seismic Design approach of structures [9].

The next sections provide a short summary of the state-of-the-art of the implementation of pushover methods in seismic codes, primarily focusing on the N2 and N1 methods. A similar discussion is also made by Bosco *et al*. [9]. However, it is useful to mention briefly the symbols and equations used here.

2. BRIEF DESCRIPTION OF NON-LINEAR STATIC METHODS

In spite of certain fundamental differences, all non-linear static methods are organised in two fundamental steps:

a) *Determination of the performance curve of the structure.* The *performance curve* or *capacity curve* of the structure, represented in terms of the relationship base-shear force V_b versus control point displacement D_c, is evaluated by monotonically increasing horizontal forces applied to the j-th floor of the structure (pushover analysis) until a given limit state is reached (e.g., collapse of the structure). The distribution of horizontal forces in the analysis is obtained by multiplying floor masses m_j (j=1:n) by a displacement profile:

$$F_{ij} = m_j \phi_{ij} \tag{1}$$

Subscript i refers to quantities dependent on the i-th displacement profile ϕ_{ij} adopted. Hereafter, subscript i is substituted by 1, when a force distribution proportional to the first mode shape is involved, and by the letter u, when a ‘uniform’ load distribution proportional to the floor masses (i.e., constant acceleration distribution) is applied.

Vibration period T_i of an elastic MDOF system corresponding to mode shape ϕ_{ij} is:

$$T_i = 2\pi \sqrt{\frac{m_i^*}{K_{t,i}}} \tag{2}$$

where stiffness $K_{t,i}$, the ratio of base shear over top displacement, is obtained by a set of forces F_{ij} proportional to ϕ_{ij} and m_i^* given by:

$$m_i^* = \sum_{j=1}^{n} \frac{m_j \phi_{ij}}{\phi_{in}} \tag{3}$$

It can be demonstrated [9] that mass m_i^* is related to modal mass M_i^*, corresponding to the mode shape by:

$$m_i^* = \frac{M_i^*}{\phi_{in} \Gamma_i} \tag{4}$$

where Γ_i is the modal participation factor:

$$\Gamma_i = \frac{\sum_{j=1}^{n} m_j \phi_{ij}}{\sum_{j=1}^{n} m_j \phi_{ij}^2} \geq 1 \tag{5}$$

b) *Determination of the displacement demand for a given PGA.* Each point of the capacity curve must be related to a value of PGA, in order to estimate the inelastic response of the structure under examination. This means that it is necessary to evaluate the top displacement of the actual MDOF system corresponding to a seismic input with a given PGA, i.e., the displacement demand or *Performance Point*, by means of study of an equivalent SDOF, representative of the MDOF system.

The N2 method employs an inelastic system represented by an elastic-perfectly plastic bi-linear relationship, obtained from the real capacity curve by imposing equal energy principles. It is characterised by lateral strength $V_{by,i}$ and yield displacement $D_{cy,i}$. The slope of the elastic branch, $K_{s,i} = V_{by,i} / D_{cy,i}$, is here called 'secant stiffness'.

The equivalent SDOF system has a mass of m_i^* (Equation (3)) and its response parameters (force F_i^*, displacement $D_{c,i}^*$) may be obtained from the corresponding parameters of the MDOF system (base shear $V_{b,i}$, top displacement $D_{c,i}$) by the following equations:

$$F_i^* = \frac{V_{b,i}}{\phi_{in}\Gamma_i} \tag{6a}$$

$$D_{c,i}^* = \frac{D_{c,i}}{\phi_{in}\Gamma_i} \tag{6b}$$

These equations, although strictly valid only if ϕ_{ij} is a modal displacement profile, as they are not very sensitive to moderate changes in ϕ_{ij}, are used to transform the capacity curve of the MDOF system to that of a corresponding SDOF system, even when ϕ_{ij} is not a modal profile [9].

The period of the idealised SDOF system is thus:

$$T_i^* = 2\pi\sqrt{\frac{m_i^*}{K_{s,i}^*}} \tag{7}$$

in which:

$$K_{s,i}^* = \frac{F_{y,i}^*}{D_{cy,i}^*} = K_{s,i} = \frac{V_{by,i}}{D_{cy,i}} \tag{8}$$

since displacements and forces have the same constant of transformation $\phi_{in}\Gamma_i$, i.e., the SDOF and real MDOF systems have the same global stiffness.

In the real MDOF structure, the base shear due to modal forces corresponding to the modal displacement ϕ_{ij} is:

$$V_{bel,i} = m_i^* \phi_{in}\Gamma_i S_{ael}(T_i) = M_i^* S_{ael}(T_i) \tag{9}$$

and the corresponding displacement at the top floor is:

$$D_{el,i} = \phi_{in}\Gamma_i S_{del}(T_i) \tag{10}$$

where S_{ael} is pseudo-spectral acceleration and S_{del} is spectral displacement.

In the N2 method, determination of the seismic response of the SDOF system is very easy when its period is longer than transition period T_C, which separates the constant acceleration branch of the spectrum from the constant velocity branch. In this case, displacement demand $D^*_{c,i}$ of the inelastic system is equal to the displacement of the corresponding elastic structure, which may be obtained as spectral value $S_{del}(T_i^*)$. When period T_i^* is shorter than T_C, displacement $D^*_{c,i}$ is evaluated by amplifying spectral displacement $S_{del}(T_i^*)$ by a coefficient depending on force reduction factor q_i^* (ratio of elastic strength demand to yielding strength of bi-linear system), according to [4]:

$$D_{c,i}{}^* = S_{del}(T_i^*),\ldots if \ldots T_i^* \geq T_C \ldots or \ldots q_i^* \leq 1 \tag{11a}$$

$$D_{c,i}{}^* = S_{del}(T_i^*)\frac{1}{q_i^*}\left(1+\left(q_i^*-1\right)\frac{T_C}{T_i^*}\right),\ldots if \ldots T_i^* < T_C \ldots and \ldots q_i^* > 1 \tag{11b}$$

where:

$$q_i^* = \frac{S_{ael}(T_i^*)m_i^*}{F_{y,i}{}^*} \tag{12}$$

Spectral displacement $S_{del}(T_i^*)$ may be calculated by pseudo-acceleration $S_{ael}(T_i^*)$. Lastly, the displacement demand of the SDOF system is transformed back to the top displacement demand of the MDOF system by the inverse of Equation (6b):

$$D_{c,i} = \phi_{in}\Gamma_i D_{c,i}{}^* \tag{13}$$

The seismic response of the MDOF system, in terms of internal forces in members, floor displacement, plastic deformations, etc., is then assumed as that obtained by pushover analysis at top displacement $D_{c,i}$. If a response quantity attains its maximum value for a top displacement smaller than $D_{c,i}$, such a maximum must be assumed. The capacity of ductile and fragile failure mechanisms must then be checked.

3. THE N1 METHOD

Readers are referred to the work by Bosco *et al.* [9] for an exhaustive explanation of the N1 method. Its operative approach is summarised in the following steps:

a) Determination of the non-linear behaviour of the real structure. As usual in all non-linear static methods, base shear $V_{b,i}$ versus top displacement $D_{c,i}$ (subscript i indicates adopted force distribution) is determined by pushover analysis of the structure, by monotonically increasing horizontal forces until collapse. According to modern seismic codes, the analysis must be performed for at least two force distributions.

b) Idealisation of the capacity curve with a bilinear relationship. The capacity curve of the real structure is idealised within the relevant range of displacements by a bi-linear relationship characterised by a yielding point with lateral strength $V_{by,i}$ and yield displacement $D_{cy,i}$. Any of the various equivalence conditions in the literature or codes can be adopted.

c) Determination of the displacement demand corresponding to a given PGA.

c1) Determination of the elastic response of the structure. Maximum elastic displacement D_{el} of the top floor, due to a seismic event with a given PGA value, is evaluated by modal response spectrum analysis, considering the dominant vibrational mode (usually the first) or combining the contributions of the most significant modes of vibration.

c2) Correction of the elastic response of the structure. Displacement demand $D_{c,i}$ is obtained by correcting elastic displacement D_{el}, in order to take into account the difference between inelastic and elastic behaviour. According to the bi-linear relationship determined at point *b)*, effective period $T_{e,i}$ of the inelastic structure is given by:

$$T_{e,i} = T_1 \sqrt{\frac{K_{t,1}}{K_{s,i}}} \tag{14}$$

To evaluate inelastic displacement $D_{c,i}$, elastic displacement D_{el} is multiplied by the ratio between the spectral displacement corresponding to $T_{e,i}$ and that corresponding to fundamental period T_1. A suitable correction must be assumed for structures with T_1 shorter than T_C, depending on coefficient $R_{\mu,i}$, i.e., the ratio between elastic strength demand V_{bel} (obtained from elastic spectral analysis) and the yielding strength of structure $V_{by,i}$. The following equations are therefore obtained:

$$D_{c,i} = D_{el} \frac{S_{del}(T_{e,i})}{S_{del}(T_1)}, \ldots if \ldots T_{e,i} \geq T_C \ldots or \ldots R_{\mu} \leq 1 \tag{15a}$$

$$D_{c,i} = D_{el} \frac{S_{del}(T_{e,i})}{S_{del}(T_1)} \frac{1}{R_{\mu,i}} \left[1 + \left(R_{\mu,i} - 1 \right) \frac{T_C}{T_{e,i}} \right], \ldots if \ldots T_{e,i} < T_C \ldots and \ldots R_{\mu,i} > 1 \tag{15b}$$

in which reduction factor $R_{\mu,i}$ is calculated as follows:

$$R_{\mu,i} = \frac{V_{bel} \dfrac{S_{ael}(T_{e,i})}{S_{ael}(T_1)}}{V_{by,i}} \tag{16}$$

4. COMPARISON OF N1 AND N2 METHODS

An ample parametric investigation was carried out to validate the N1 method, considering a set of 10 steel frames and 12 R/C frames with rigid connections, designed with regular and irregular mass distribution in height. This enables us to compare the results from the two methods and to propose an operative approach to correct N1 in order to obtain the same results given by N2, even when a force distribution proportional to the floor masses is applied.

The set of frames covers a wide range of the structural parameters which influence their seismic responses and estimation by non-linear static methods. The fundamental period of the frames is in the range 0.40-2.40s, thus including cases in which both equal displacement and equal energy rules must be taken into account.

4.1. Characteristics of analysed frames

All the analysed frames were designed according to the method of Ghersi *et al.* [18] and Marino *et al.* [19]. This method allows the frames to achieve collapse by means of a global mechanism, in accordance with the capacity design criteria of modern seismic codes for frame structures (i.e., flexural strength of columns greater than that of converging beams, so that plastic hinges form in the latter, and the shear-over strength of members exceeds flexural strength). The frames were characterised by various span lengths (from 4.0 to 5.5 m), inter-storey heights (from 3.0 to 3.5 m), number of spans (1, 2, 3, 5 and 6), number of storeys (2, 3, 4 or 6), sizes of steel and R/C sections, and extent of gravity loads in seismic combination. The irregularity in elevation of some frames was introduced by varying gravity loads on the floors.

The steel frames were designed with HEB180 and HEB160 profiles (wide-flange shapes available in Europe) for columns, and IPE140 and IPE120 profiles for beams, made of S355 grade steel (characteristic yielding stress f_{yk}=355 MPa, elastic modulus E_s=210 GPa). The main geometric characteristics are listed in Table 1 and shown in Fig. 1 (see footnotes to tables for names assigned to frames).

Table 1. Geometries and design loads of steel frames.

STEEL FRAMES:	Span length [m]	Inter-storey height [m]	Gravity loads in seismic combination [kN/m]
1_S3x4_R	4	3	Ro=8; F=11,5
2_S5x2_R	4	3	Ro=11,5; F=11,5
3_S5x4_I	v4/5	v3,5/3	Ro=10; F=v5/20
4_S5x4_I	v4/5	v3,5/3	Ro=4; F=v2/8
5_S5x4_R	v4/5	3	Ro=3; F=4
6_S1x3_R	4	3	Ro =8; F=10
7_S1x3_I	4	3	Ro=20; F=v5/10
8_S1x4_R	5,5	3	Ro=8; F=10
9_S3x4_R	4	3	Ro=25; F=28
10_S5x4_R	v4/5	3	Ro=4; F=5,5

Sixj, i=n. of spans, j=n. of storeys, R=Regular, I=Irregular, Ro=Roof, F=Floors, v=variable.

Several geometries of R/C frames were examined, but columns and beam sizes and their steel reinforcements were unchanged. All the columns had a rectangular section of 400 x

300mm and were reinforced with 10#16 longitudinal bars; all beams had a rectangular section of 300 (height) x 200mm (width) and were reinforced with 3#20 longitudinal bottom bars and 4#20 longitudinal top bars (Fig. 2). The materials were C35/45 concrete (f_{ck} =35 MPa, E_c =30 GPa) and steel grade B450C (f_{yk} =450 MPa, E_s =210 GPa). The main geometric characteristics of the frames are shown in Table 2 and Fig. 3.

Table 2. Geometries and design loads of R/C frames.

R/C FRAMES:	Span length [m]	Inter-storey height [m]	Gravity loads in seismic combination [kN/m]
11_RC1x3_R	5,5	3	Ro=28;F=30
12_RC1x3_R	5,5	3	Ro =14;F =15
13_RC6x3_R	5,5	3	Ro =28;F =30
14_RC6x3_R	5,5	3	Ro =14;F =15
15_RC5/2x4_I	5,5	v3,5/3	Ro =10;F =v10/30
16_RC3x3_R	5,5	3	Ro =28;F =30
17_RC3x3_R	5,5	3	Ro =14;F =15
18_RC3x3_I	5,5	3	Ro =20;F =v10/40
19_RC3x3_I	5,5	3	Ro =10;F =v5/20
20_RC2x6_R	5,5	3	Ro =14;F =15
21_RC2x6_I	5,5	3	Ro =10;F =v20/50
22_RC3x3_I	5,5	3	Ro =7;F =v4/10

RCixj, i=n. of spans, j=n. of storeys. R=Regular, I=Irregular, Ro=Roof, F=Floors, v=variable.

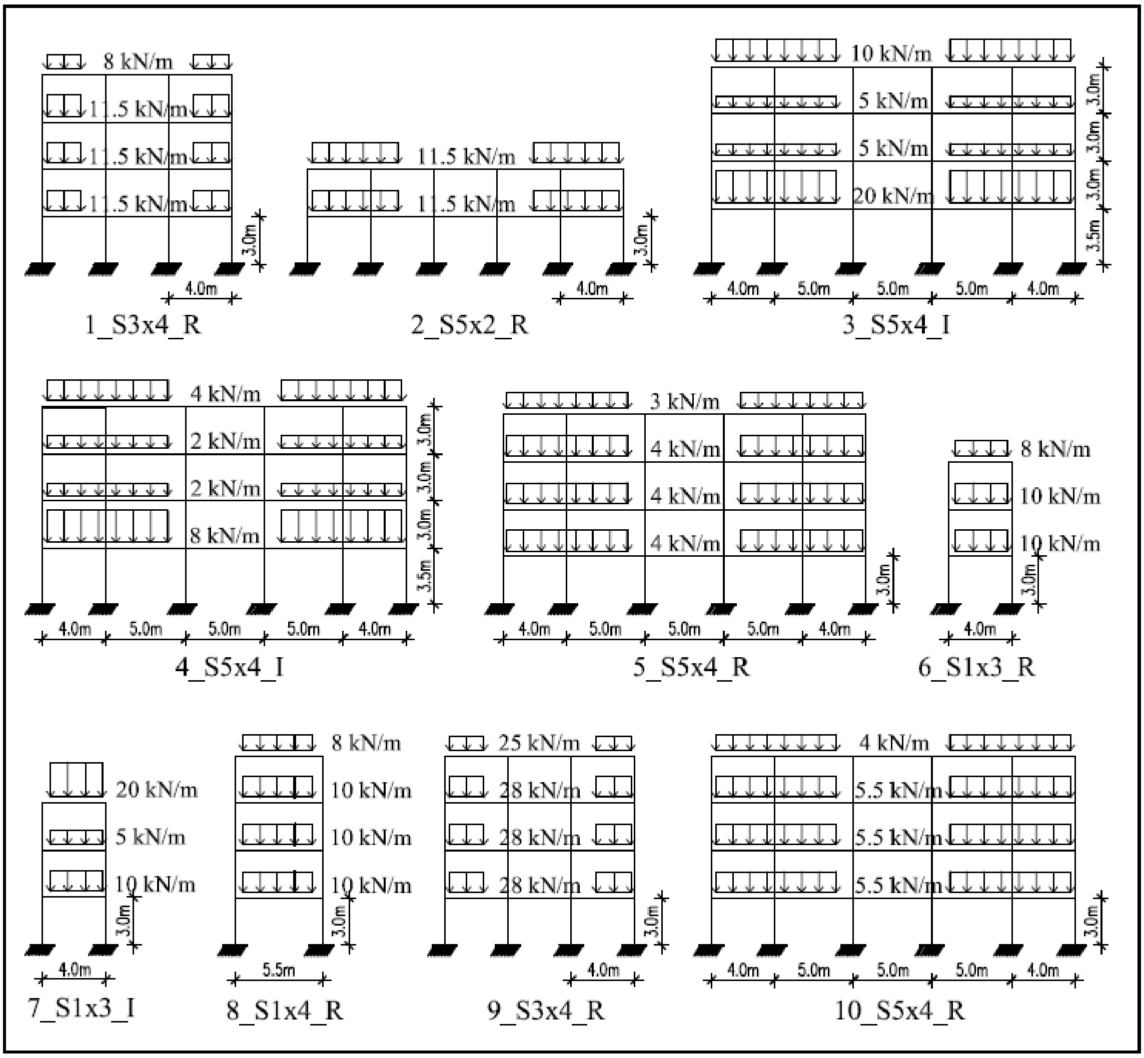

Fig. 1. Geometries and design loads of steel frames.

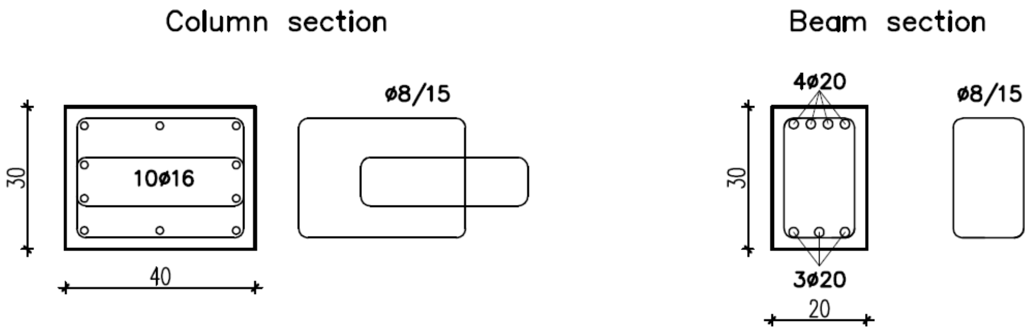

Fig. 2. Dimensions and reinforcement of columns and beams of R/C frames.

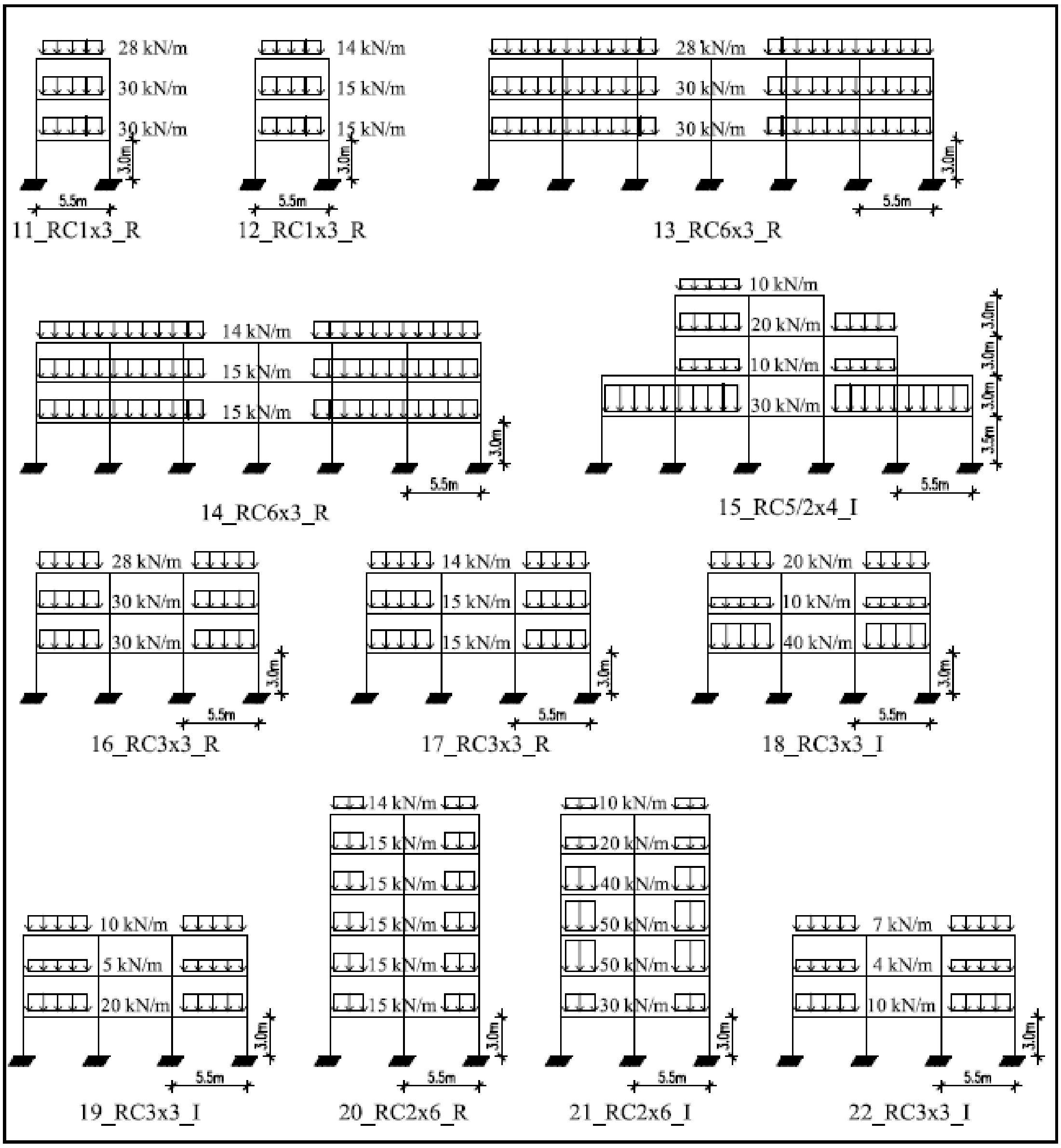

Fig. 3. Geometries and design loads of R/C frames.

4.2. *Validation of N1 method*

The top displacement demand (*Performance Point*) of the frames was determined by applying both N2 (EC8) and N1 methods and comparing the results. According to common practice, pushover analyses involve two distributions of horizontal forces, one proportional to the first mode shape (‘*modal pushover*’, i=1) and one proportional to the floor masses (‘*uniform pushover*’, i=u).

The seismic action considered in all analyses was characterised by the elastic response acceleration and displacement spectra of EC8, shown in Fig. 4 (soil class D, topographic class T1, reference return period of 475 years, PGA = 0.39g). The same figure also shows the range of periods for the frames.

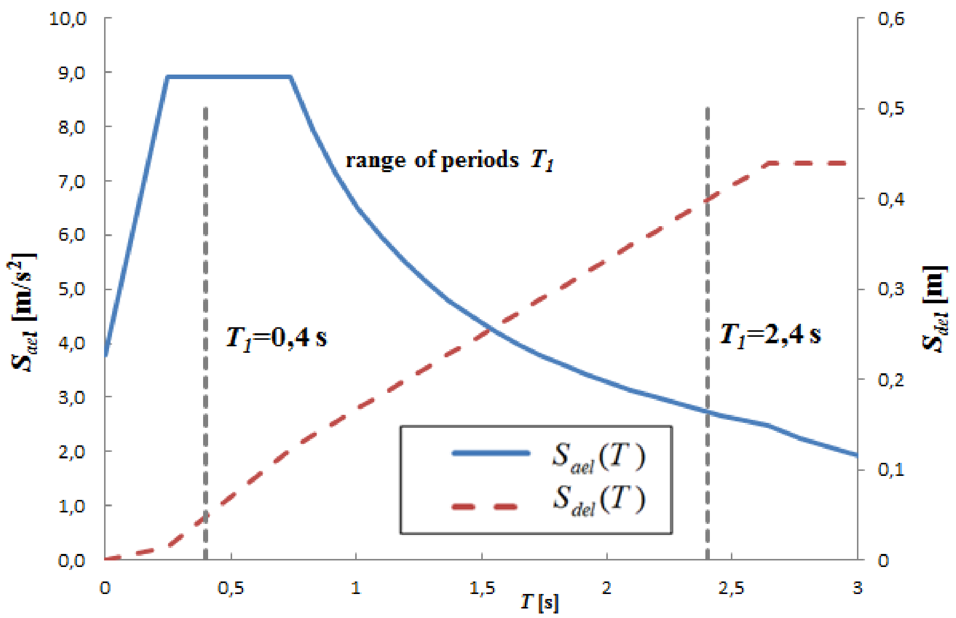

Fig. 4. Elastic response spectra according to EC8.

Non-linear analysis of frames was carried out with MidasGen rel. 7.4.1 structural software [20]. A fibre approach was adopted [21-24] to reproduce the non-linear behaviour of members, thus keeping account of the M-N interaction in evaluating the structural response.

The capacity curves to estimate the *Performance Point* with both non-linear static methods were determined for each frame and lateral load distribution with the same mathematical model, to ensure good comparability of results.

4.2.1. Modal pushover analysis

'*Modal pushover*' analyses (i=1) were first carried out on all frames applying a force distribution proportional to the first mode shape. Top displacement $D_{c,1}$ (*Performance Point*) of the frames was determined with both N2 (EC8) and N1 methods with the same bi-linearisation curve criterion. The results are compared in Fig. 5: those obtained with N2 are assumed as reference values (X-axis) and those with N1 are shown on the Y-axis. All the values clearly lie almost exactly on the bisector, thus demonstrating that, for the considered load profile, both methods give the same results for both regular and irregular frames, independently of their T_1 period.

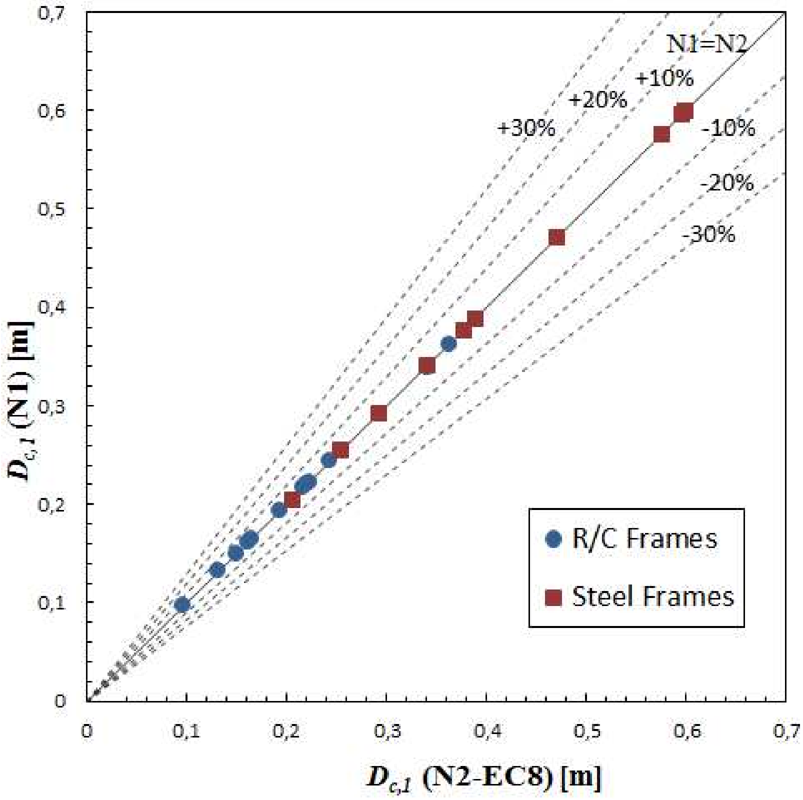

Fig. 5. Comparison between top displacement demands determined with N1 and N2, with modal lateral force distribution.

These results confirm those of Bosco *et al.* [9] according to their analyses of 108 steel frames. The equivalence of the two methods in the modal pushover case can easily be explained by considering that period T_1^* of the equivalent SDOF system of N2 coincides perfectly with period $T_{e,1}$ of N1, provided that the same bi-linearisation criterion for the capacity curve is used.

4.2.2. Uniform pushover analysis

'*Uniform pushover*' analyses (i=u) were then carried out on all frames applying lateral force distribution proportional to the floor masses. The result of lateral forces being lower than in the modal pushover case, the frames showed stiffer behaviour and therefore had smaller values of $D_{c,u}$.

The displacement demands obtained with the two methods are compared in Fig. 6 and Tables 3 and 4.

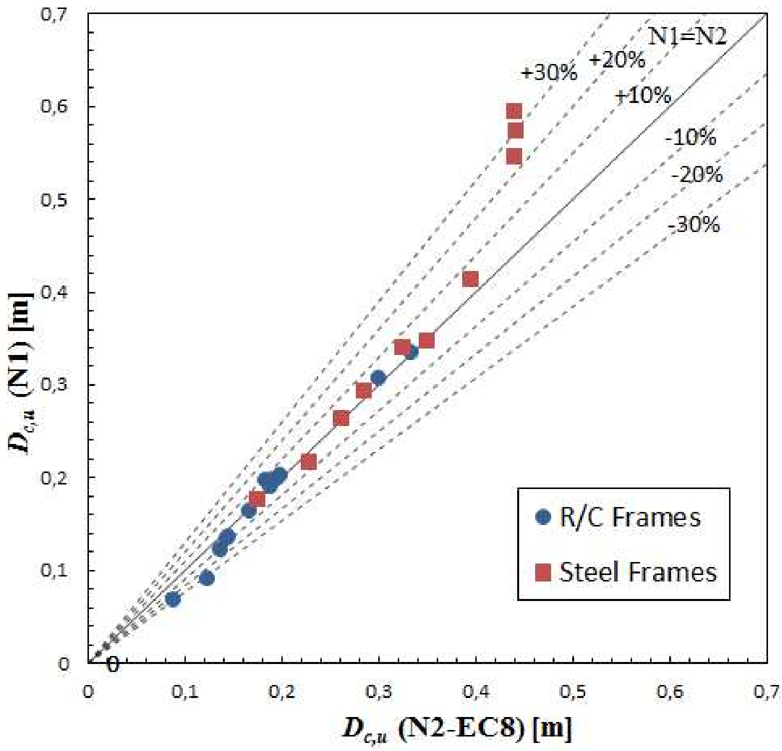

Fig. 6. Comparison between top displacement demands with N1 and N2, with uniform lateral force distribution.

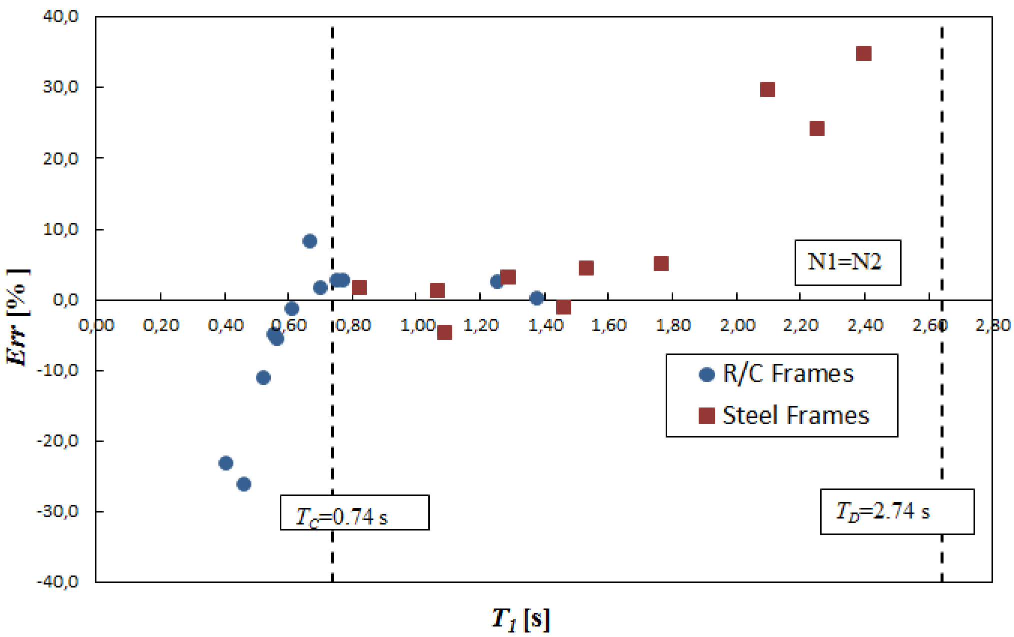

Fig. 7. Percentage difference between top displacement demands determined with N1 and N2, with uniform lateral force distribution.

Table 3. Comparison between N2 (EC8) and N1 for steel frames: ‘uniform pushover’ analysis.

STEEL FRAMES:	T_1 [s]	T_u^* [s]	$T_{e,u}$ [s]	$D_{c,u}$ (N2-EC8) [m]	$D_{c,u}$ (N1) [m]	*Err* [%]
1_S3x4_R	1,29	1,70	1,32	0,284	0,293	3,2
2_S5x2_R	1,07	1,57	1,31	0,261	0,264	1,3
3_S5x4_I	2,25	3,31	2,41	0,439	0,545	24,1
4_S5x4_I	1,46	2,11	1,54	0,351	0,347	-1,1
5_S5x4_R	1,53	1,95	1,54	0,325	0,339	4,4
6_S1x3_R	0,83	1,04	0,81	0,174	0,177	1,6
7_S1x3_I	1,09	1,36	1,15	0,228	0,217	-4,8
8_S1x4_R	2,40	4,15	3,14	0,441	0,594	34,8
9_S3x4_R	2,10	3,55	2,78	0,441	0,573	29,7
10_S5x4_I	1,77	2,37	1,87	0,394	0,414	5,0

Table 4. Comparison between N2 (EC8) and N1 for R/C frames: ‘uniform pushover’ analysis.

R/C FRAMES:	T_1 [s]	T_u^* [s]	$T_{e,u}$ [s]	$D_{c,u}$ (N2-EC8) [m]	$D_{c,u}$ (N1) [m]	*Err* [%]
11_RC1x3_R	0,70	1,12	0,87	0,187	0,191	1,7
12_RC1x3_R	0,52	0,82	0,64	0,137	0,121	-11,2
13_RC6x3_R	0,77	1,16	0,92	0,194	0,199	2,7
14_RC6x3_R	0,57	0,85	0,67	0,143	0,135	-5,6
15_RC5/2x4_I	0,67	1,09	0,75	0,183	0,198	8,3
16_RC3x3_R	0,76	1,18	0,94	0,197	0,202	2,6
17_RC3x3_R	0,56	0,86	0,68	0,144	0,137	-4,9
18_RC3x3_I	0,61	1,01	0,73	0,167	0,165	-1,3
19_RC3x3_I	0,46	0,74	0,54	0,123	0,091	-26,1
20_RC2x6_R	1,26	1,80	1,42	0,299	0,306	2,4
21_RC2x6_I	1,38	2,01	1,59	0,334	0,335	0,2
22_RC3x3_I	0,40	0,63	0,48	0,089	0,068	-23,3

In the case of ‘*uniform pushover*’ analysis, when a constant force distribution is applied, N1 and N2 yield quite different displacement demands.

Matching the studies of Bosco *et al.* [9], Fig. 7 shows that, for structures with periods T_1 between T_C and T_D, the differences between the two methods are negligible, whereas for structures with longer periods, N1 overestimates the displacement demand with respect to N2 by as much as +30%. In contrast, for structures with periods shorter than T_C, N1 underestimates displacement by up to -30%.

Note that, in the case of ‘*uniform pushover*’ analysis, T_u^* does not coincide with its respective $T_{e,u}$, but the two differ by a factor α estimated as follows:

$$T_{e,u} = T_1\sqrt{\frac{K_{t,1}}{K_{s,u}}} = \alpha T_u^* \tag{17}$$

in which:

$$T_u^* = 2\pi\sqrt{\frac{m_u^*}{K_{s,u}^*}} = 2\pi\sqrt{\frac{M}{K_{s,u}}} \quad (18)$$

$$\alpha = \sqrt{\frac{m_1^*}{M}} < 1 \quad (19)$$

In deriving Equation (18), when uniform force distribution is applied, m_u^* is equal to *M*, the total mass of the real structure and the modal participation factor Γ_u is 1.

As already reported by Bosco *et al.* [9], the differences between the methods (Figs. 6 and 7; Tables 3 and 4) are due to the following contrasting effects:

- -N1 refers to mass m_1^*, but N2 uses mass $m_u^* = M > m_1^*$, thus providing period T_u^* and spectral displacement $S_{del}(T_u^*)$ which are always larger than that obtained with N1;
- -the displacement demand is proportional to factor $\phi_{in}\Gamma_i$, which is 1 in N2 and $\phi_{1n}\Gamma_1 > 1$ in N1;
- -$R_{\mu,u}$ differs from q_u^*; consequently, for structures with periods $T_{e,u} < T_u^*$, the coefficients of displacement correction for equal energy rules change;

These contrasting effects are compensated as the period of the structure varies. That is, for a structure with periods $T_{e,u}$ and T_u^* between T_C and T_D, the difference between the displacement demands of N2 and N1 is generally small, since the spectral displacement is linearly proportional to the period and therefore $S_{del}(T_u^*)/S_{del}(T_{e,u}) \cong T_u^*/T_{e,u} > 1$. However, this effect is almost completely compensated, since $\phi_{un}\Gamma_u = 1 < \phi_{1n}\Gamma_1$ and $D_{c,u}$ (N2-EC8) from Equation (13) is almost equal to $D_{c,u}$ (N1) (Equation (15a)). For structures with long periods, i.e., belonging to the constant displacement branch of the response spectrum, elongation of period T_u^* with respect to $T_{e,u}$ and T_1 does not produce any increase in the displacement demand. However, the difference between $\phi_{un}\Gamma_u$ and $\phi_{1n}\Gamma_1$ increases the displacement demand evaluated by N1 with respect to N2. This difference is greater if both $T_{e,u}$ and T_u^* are longer than T_D, and decreases only if T_u^* is longer than T_D. For structures with periods shorter than T_C, the opposite result is obtained. In this range, the spectral displacement is proportional to the square of the period and the effects of the difference between m_u^* and m_1^*, which makes $S_{del}(T_u^*)$ larger than $S_{del}(T_{e,u})$, prevails over those of the difference between $\phi_{un}\Gamma_u$ and $\phi_{1n}\Gamma_1$. This difference is greater if both $T_{e,u}$ and T_u^* are shorter than T_C and decreases only if $T_{e,u}$ is shorter than T_C. For these frames, further small differences between the displacement demand arise from the fact that different values of force reduction factors $R_{\mu,u}$ (Equation (16)) and q_u^* (Equation (12)) are obtained.

4.3. Conclusions: assessment of reliability of N1

Tests on a sample of 22 two-dimensional frames allowed the following conclusions to be drawn:

- in all the studied frames, N1 as formulated by Bosco *et al*. [9] and N2 yield the same displacement demand when a force distribution corresponding to the first mode shape is applied (‘*modal pushover*’ analysis). This evidence, which can be theoretically proved only for regular frames dominated by main mode shape [9], was also confirmed for irregular frames;
- N1 and N2 provide clear-cut differences for short- and long-period frames when a force distribution proportional to the floor masses is used (‘*uniform pushover’* analysis). These differences are highlighted in the previous section and it is therefore necessary to introduce the appropriate corrections to make N1 consistent with N2.

5. PROPOSAL FOR AN N1 CORRECTED METHOD

According to the above results, an improved version of N1 is now proposed with the aim of obtaining results consistent with N2 even in the case of ‘*uniform pushover*’ analysis.

As already discussed in section 4, when a force distribution proportional to the floor masses is applied, the effective period $T_{e,u}$ of N1 is always shorter than the respective period of the idealised SDOF system T_u^* of N2 by a factor α (see Equation (19)). According to Equation (4) and by means of easy calculations, it can be demonstrated that:

$$\alpha = \frac{1}{\phi_{1n}\Gamma_1}\left(\frac{1}{\phi_{1n}\Gamma_1} + \frac{1}{\phi_{1n}\Gamma_1^2\{\phi_1\}^T[M]\{r\}}\sum_{i=2}^{n}\Gamma_i\{\phi_i\}^T[M]\{r\}\right)^{-\frac{1}{2}} = \frac{1}{\phi_{1n}\Gamma_1}\beta \tag{20}$$

where Γ_i are the modal participation factors of the higher modes already defined in Equation (5), and $\{r\}$ is the influence vector for seismic excitation.

It is easy to verify that, if eigenvectors $\{\phi_i\}$ are normalised with respect to mass matrix $[M]$, the expression of coefficient β can be reduced to:

$$\beta = \left(\frac{1}{\phi_{1n}\Gamma_1} + \frac{1}{\phi_{1n}\Gamma_1^3}\sum_{i=2}^{n}\Gamma_i^2\right)^{-\frac{1}{2}} \tag{21}$$

where $\beta \cong 1$ in structures where the first mode shape is dominant, but it is also close to 1 in irregular frames, as Table 5 shows.

Table 5. Values of coefficients α and β for frames.

STEEL FRAMES:	α [-]	β [-]	R/C FRAMES:	α [-]	β [-]
1_S3x4_R	0,77	1,04	11_RC1x3_R	0,78	1,01
2_S5x2_R	0,84	1,01	12_RC1x3_R	0,78	1,02
3_S5x4_I	0,73	0,99	13_RC6x3_R	0,79	1,02
4_S5x4_I	0,73	0,99	14_RC6x3_R	0,79	1,02
5_S5x4_R	0,79	1,05	15_RC5/2x4_I	0,69	1,09
6_S1x3_R	0,78	1,03	16_RC3x3_R	0,79	1,02
7_S1x3_I	0,84	0,96	17_RC3x3_R	0,79	1,02
8_S1x4_R	0,76	1,02	18_RC3x3_I	0,73	0,99
9_S3x4_R	0,78	1,02	19_RC3x3_I	0,73	1,00
10_S5x4_I	0,79	1,05	20_RC2x6_R	0,79	1,03
			21_RC2x6_I	0,79	1,00
			22_RC3x3_I	0,76	1,00

Equation (20) estimates the differences between the two methods.

Since coefficient α (Equation (20)) can easily be evaluated by modal response spectrum analysis, bypassing the definition of the SDOF system, Equations (14) and (16) can be corrected and Equation (15a) reformulated if the following form is used in '*uniform pushover*' analyses:

$$T_{e,u}{}^{cor} = \frac{1}{\alpha} T_{e,u} \tag{22}$$

$$R_{\mu,u}{}^{cor} = \frac{M}{M_1^*} \frac{V_{bel} \dfrac{S_{ael}(T_{e,u}{}^{cor})}{S_{ael}(T_1)}}{V_{by,u}} \tag{23}$$

clearly:

$$\frac{M}{M_1^*} = \phi_{1n} \Gamma_1 \frac{1}{\beta^2} \tag{24}$$

giving:

$$D_{c,u}{}^{cor} = \frac{D_{el}}{\phi_{1n}\Gamma_1} \frac{S_{del}(T_{e,u}{}^{cor})}{S_{del}(T_1)} \tag{25a}$$

$$if \ldots T_{e,u}{}^{cor} \geq T_C \ldots or \ldots R_{\mu,u}{}^{cor} \leq 1$$

$$D_{c,u}{}^{cor} = \frac{D_{el}}{\phi_{1n}\Gamma_1} \frac{S_{del}(T_{e,u}{}^{cor})}{S_{del}(T_1)} \frac{1}{R_{\mu,u}{}^{cor}} \left[1 + \left(R_{\mu,u}{}^{cor} - 1 \right) \frac{T_C}{T_{e,u}{}^{cor}} \right] \tag{25b}$$

$$if \ldots T_{e,u}{}^{cor} < T_C \ldots and \ldots R_{\mu,u}{}^{cor} > 1$$

Fig. 8 compares the top displacement demands for uniform lateral load distribution obtained with the proposed *N1 corrected method* with those of the N2 (EC8) method.

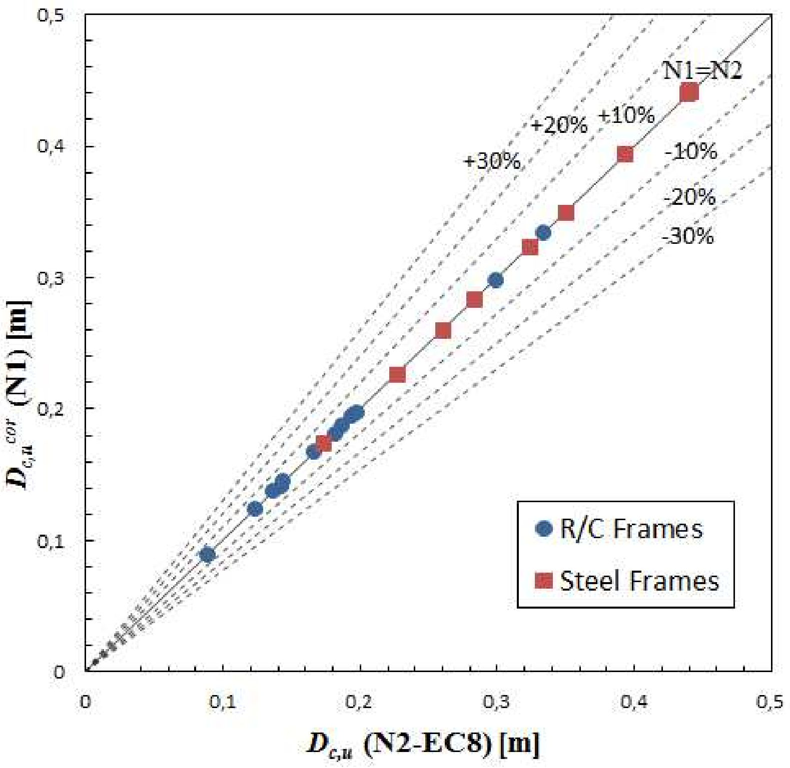

Fig. 8. Comparison between top displacement demands with uniform lateral load distribution with N1 corrected and N2.

Clearly, with this proposal, the results from the N1 corrected and N2 (EC8) methods also coincide when a force distribution proportional to the floor masses is used, for all regular and irregular frames. In its corrected format, N1 can be applied in the same compliance conditions as N2, i.e., in plan-regular buildings with dominating first mode, for which $\beta \cong 1$.

6. SUMMARY AND CONCLUSIONS

Bosco *et al.*[9] proposed the N1 method as an alternative to the well-known N2 method to assess the seismic response of existing and new structures by non-linear static analysis. N1 has the merit of being simpler in practical application, because it does not require definition of the equivalent SDOF system as an intermediate step.

The two methods provide the same results in the case of modal patterns of lateral forces, but they lead to different displacement demands for short- and long-period structures in the case of uniform patterns. These discrepancies were analysed allowing us to extend and improve N1. In the present work, the N1 corrected method is presented and validated with reference to a set of 22 steel and R/C frames, with regular and irregular mass distribution in height. This analytical correction was facilitated by introducing coefficient α, calculated in closed form from the results of modal analysis. N1 thus becomes equivalent to N2 and can be used for practical applications in the same conditions (i.e., in plan-regular buildings with dominating first mode). Like N2, N1 can also be extended to study of irregular or high-rise buildings in which the influence of higher modes (in both plan and elevation) can be evaluated by standard elastic modal response spectrum analysis. Further in-depth study of the N1 corrected method is needed and should be the focus of future research.

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