The Capacity Spectrum Method as a Tool for Seismic Design

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ABSTRACT: The Capacity Spectrum Method (CSM), by means of a graphical procedure, compares the capacity of the structure with the demands of earthquake ground motion on the structure. The capacity of the structure is represented by a nonlinear force-displacement curve, sometimes referred to as a pushover curve. The base shear forces and roof displacements are converted to equivalent spectral accelerations and spectral displacements, respectively, by means of coefficients that represent effective modal masses and modal participation factors. These spectral values define the capacity spectrum. The demands of the earthquake ground motion are represented by response spectra. A graphical construction that includes both capacity and demand spectra results in an intersection of the two curves that estimates the performance of the structure to the earthquake.

1 INTRODUCTION

The Capacity Spectrum Method (CSM) compares the capacity of a structure to resist lateral forces to the demands of earthquake response spectra in a graphical presentation that allows a visual evaluation of how the structure will perform when subjected to earthquake ground motion. The method is easily understandable and generally consistent with other methods that take into account the nonlinear behavior of structures subjected to strong motion earthquake ground movements.

The capacity is represented by a lateral load force-displacement diagram that takes into account the sequential yielding of structural elements as the structure is laterally displaced beyond its linear-elastic limits. This procedure is sometimes referred to as a pushover curve. The lateral load force diagram is proportioned to represent the fundamental mode of the building. The force-displacement diagram (Figure 1) is calculated in terms of lateral roof displacement (Δ_R) and total lateral force at the base of the building (V). In order to be directly comparable to demand response spectra, Δ_R and V are converted to a spectral set of coordinates (Figure 2) by using the dynamic characteristics of the fundamental mode to represent the structure as a single-degree-of-freedom structure. The procedure also allows the inclusion of higher mode effects.

The demands of the earthquake are represented by response spectra. Linear elastic response spectra, assumed at 5% damping, are modified to represent the effects of inelastic response by substituting higher damped response spectra to account for hysteretic nonlinear response of the structure.

Response spectra have traditionally been plotted with S_a (acceleration in units of gravity) vs T (period in seconds) coordinates or tripartite log coordinates. In order to more visually illustrate the relationship between accelerations and displacements, the S_a vs T coordinate system for the response spectra is converted to a set of coordinates defined by S_a and S_d. When the spectral values are plotted in this acceleration-displacement response spectrum format (ADRS), the period can be represented by lines radiating from the origin (Mahaney et al. 1993). An example of demand spectra is shown in Figure 3.

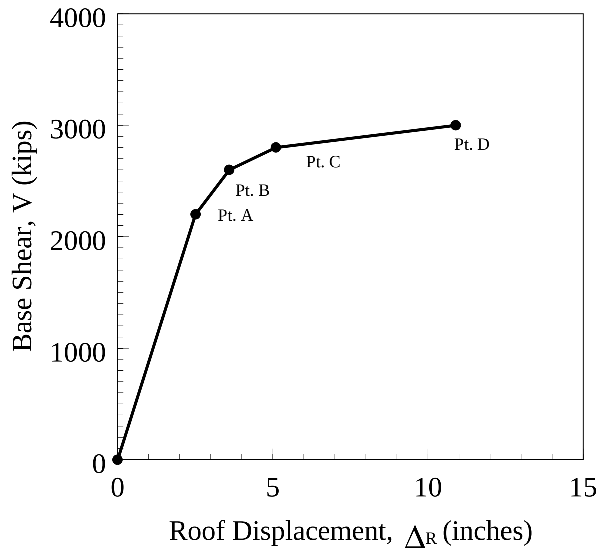

Figure 1. Capacity curve (note: V:1000 = 4.5 MN. Δ_R : 10 = 25 cm)

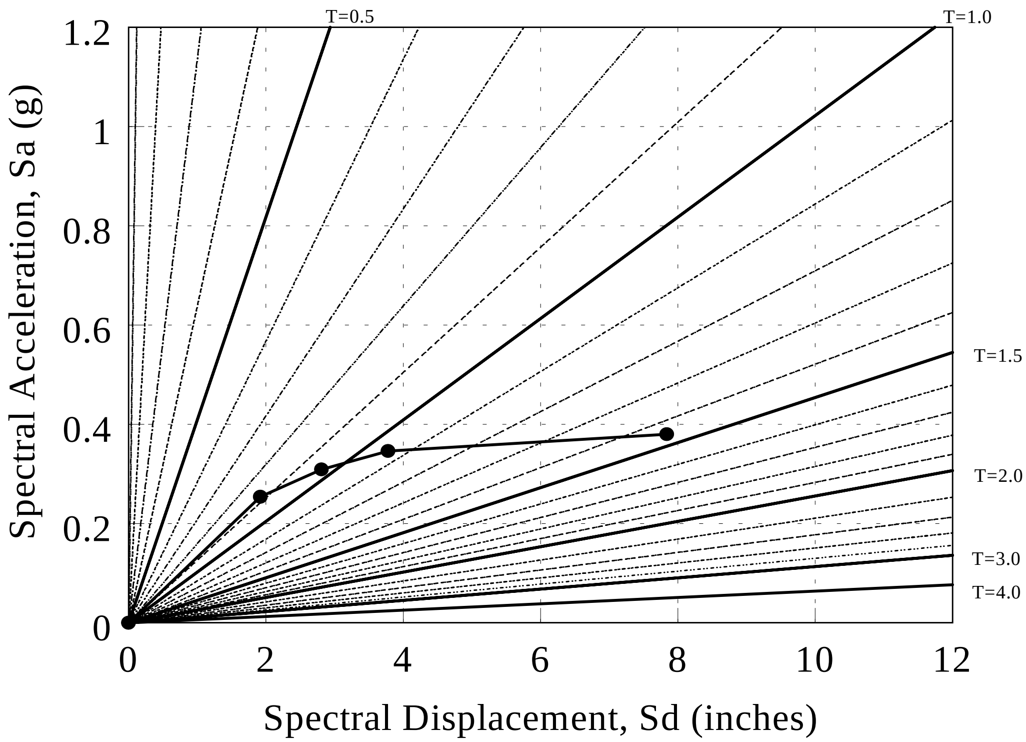

Figure 2. Capacity spectrum (points on curve correspond to points A through D in Figure 1). Note: S_a is in units of gravity. Δ_R : 10 = 4.5 MN.

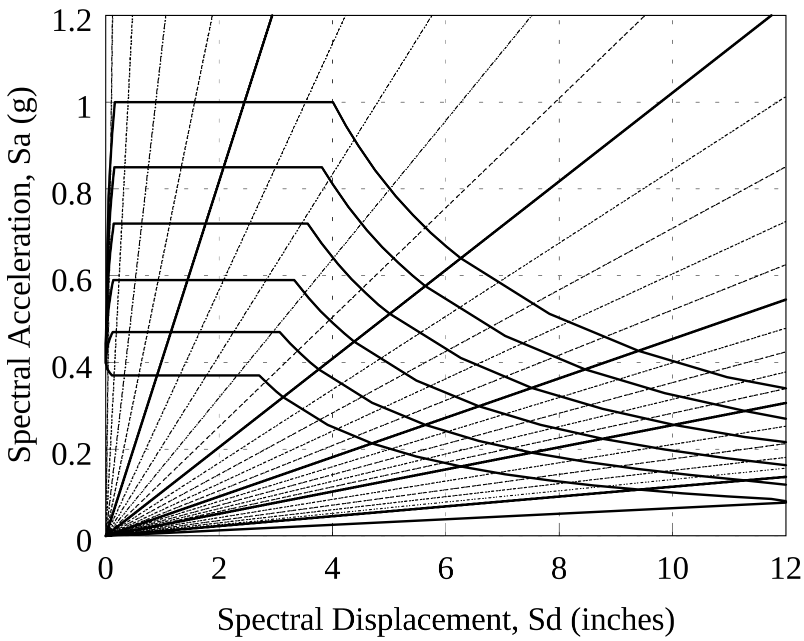

Figure 3. Family of demand response spectra in ADRS format for values of damping, β_{eff} = 5%, 8%, 12%, 18%, 26%, and 40%. Note: S_d : 10 = 25 cm.

2 PROCEDURE

The procedure can be summarized by the following:

Capacity curve: Estimate or calculate the capacity curve in terms of roof displacement, Δ_R, and base shear, V, (i.e. total lateral force at base).

Dynamic characteristics: Estimate or calculate modal vibrational characteristics such as periods of vibration, mode shapes, modal participation factors, and effective modal mass ratios.

Capacity Spectrum: Convert the V vs Δ_R capacity curve to a S_a vs S_d capacity spectrum by use of dynamic characteristics..

Response Spectra: Obtain or calculate response spectra for several levels of damping, including the 5-percent damped spectrum.

Graphical Solution: Plot capacity spectrum and family of damped response spectra on an ADRS format (i.e. S_a vs S_d coordinates with period T lines radiating from origin). The intersection of the capacity spectrum with the appropriately damped response spectrum represents the estimated demands of the earthquake on the structure.

2.1 *Capacity Curve - Pushover*

The capacity curve is determined by statically loading the structure with realistic gravity loads combined with a set of lateral forces to calculate the roof displacement Δ_R and base shear V that defines first significant yielding of structural elements. The yielding elements are then relaxed to form plastic hinges and incremental lateral loading is applied until a nonlinear static capacity curve is created. The curve is created by superposition of each increment of displacement and includes tracking displacements at each story (ATC 1982). This procedure is sometimes referred to as the pushover analysis.

There are several levels of sophistication that may be used for the pushover analysis, ranging from applying lateral forces to each story in proportion to the standard code procedure to applying lateral story forces as masses times acceleration in proportion to the first mode shape of the elastic model of the structure. For added sophistication, at each increment beyond yielding, the forces may be adjusted to be consistent with the changing deflected shape. For tall buildings the effects of the higher modes of vibration may be considered (Paret et al. 1996).

It is assumed that the structure can take a number of cycles along the capacity curve and behave in a hysteretic manner. The stiffness is assumed to reduce to an equivalent global secant modulus measured to the maximum excursion along the capacity curve for each cycle of motion.

It should be noted that the capacity curve need not be exact in order to be useful. A reasonable approximation of the elastic limit and the inelastic limit will give a general idea of how the building will respond to various earthquake demands. This was the basis of the original rapid evaluation procedure (Freeman et al. 1975). As the pushover analyses become more detailed, it is useful to denote yielding and cracking benchmarks along the capacity curve. The pushover should be continued to the largest displacement practicable until degradation of the overall system occurs or limits of structural stability occur. In cases where a target displacement is set as a goal, it is generally worthwhile to push a little further to establish a better confidence level.

2.2 *Dynamic Characteristics*

The conversion of the Δ_R vs V capacity curve to the S_d vs S_a capacity spectrum can be accomplished by knowing the dynamic characteristics of the structure in terms of period (T) mode shape (φ_x) and lumped floor mass (m_x). A single degree of freedom (SDOF) system is used to represent a translational vibrational mode of the structure. This system has an effective mass equal to αM, where α is the effective mass ratio and M is the total mass of the structure. This system also has a roof participation factor ($PF\varphi_R$) that gives the ratio of the roof displacement (Δ_R) to the displacement of the mass (S_d) of the SDOF system. The value of α can be calulated as follows:

$$\alpha = (\Sigma m_x\varphi_x)^2 \div \Sigma m_x \Sigma m_x\varphi_x^2$$

For most multistory buildings this can be estimated as equal to 0.80. Thus, $S_a = V \div \alpha Mg$ can be estimated as $S_a = (V/W) \div 0.80$.

The value of $PF\varphi_R = (\Sigma m\varphi \div \Sigma m\varphi^2)\ \varphi_R$. This can be estimated at 1.4, thus, $S_d = \Delta_R \div 1.4$.

2.3 *Capacity Spectrum*

The capacity spectrum can now be plotted by calculating S_a and S_d from the above equations. The secant period at each point along the curve can be calculated by the following: $T = 2\pi\ (S_d \div S_a g)^{1/2}$.

After the capacity spectrum has been plotted, it is useful to approximate an equivalent bilinear capacity representation that establishes an effective yield point (μ=1) and an effective peak inelastic limit. Points of displacement ductility ratios (μ) can be marked along the post-elastic line that will be useful in a later phase of the CSM. This process is shown in Figure 4. Six points on the bilinear curve represent μ = 1.0, 1.5, 2.0, 2.5, 3.0, and 3.7. The value 3.7 represents the inelastic limit of the pushover analysis.

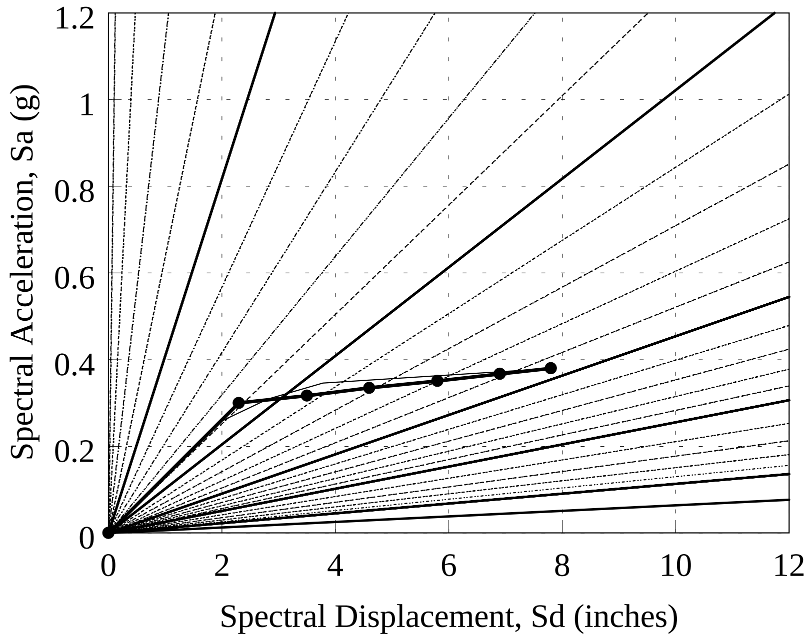

Figure 4. Bilinear idealized Figure 2 capacity spectrum. Note: S_d : 10 = 25cm.

2.4 *Demand Curves - Response Spectra*

The demand curve is represented by earthquake response spectra (Fig. 3). It is presented at various levels of damping. For example, the 5 percent damped response spectrum (top curve in Figure 3) is generally used to represent the demand when the structure is responding linearly-elastic. Higher damped response spectra are used to represent inelastic response spectra to account of hysteretic nonlinear response of the structure. In Figure 3, damped spectra are also shown for β = 8%, 12%, 18%, 26%, and 40% of critical damping. These higher damped spectra may be associated with global displacement ductilities ranging from μ = 1.25 for β = 8% to μ greater than 4.0 for β = 40%. The relationship between μ and β_{eff} is dependent on the slope of the bilinear line and the stability of the cyclic hysteretic loops of the structural system under repeated cycles of loading (ATC 1996, Newmark & Hall 1982, Priestley et al. 1996, WJE 1996).

2.5 *Graphical Solution*

When both the capacity spectrum and the demand response spectrum are defined with the same set of coordinates, they can be plotted together.

The Capacity Spectrum Method can be summarized as follows: If the capacity curve can extend through the envelope of the demand curve, the building survives the earthquake. The intersection of the capacity and appropriately damped demand curve represents the inelastic response of the structure.

To illustrate the Capacity Spectrum Method in the ADRS format the idealized capacity curve of Figure 4 is superimposed on the response spectra of Figure 3. This example is included in

ATC-40 (ATC 1996) showing three procedures. The procedure shown in Figure 5 is a simplified version of those shown in ATC-40.

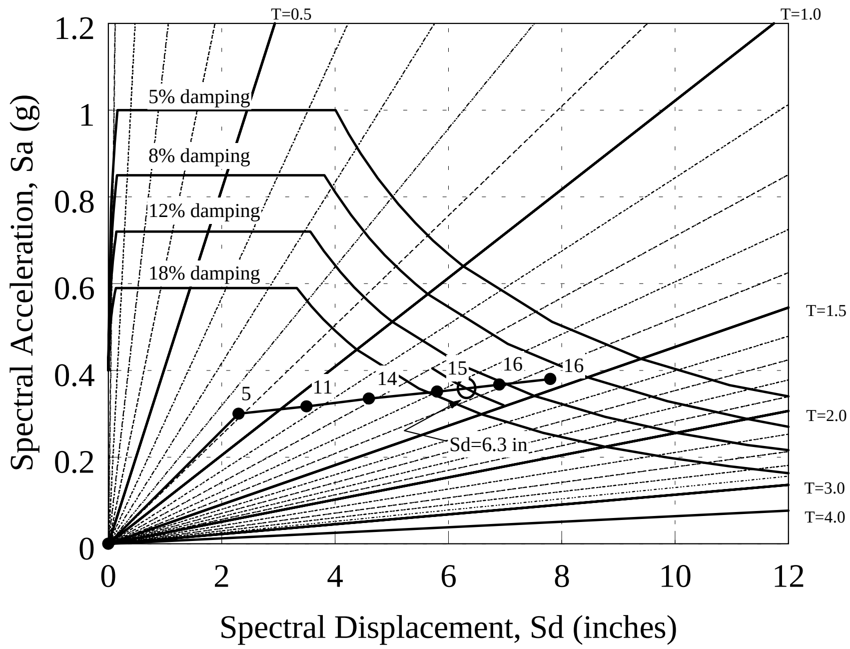

Figure 5. CSM graphical solution. Note S_d : 10 = 25 cm.

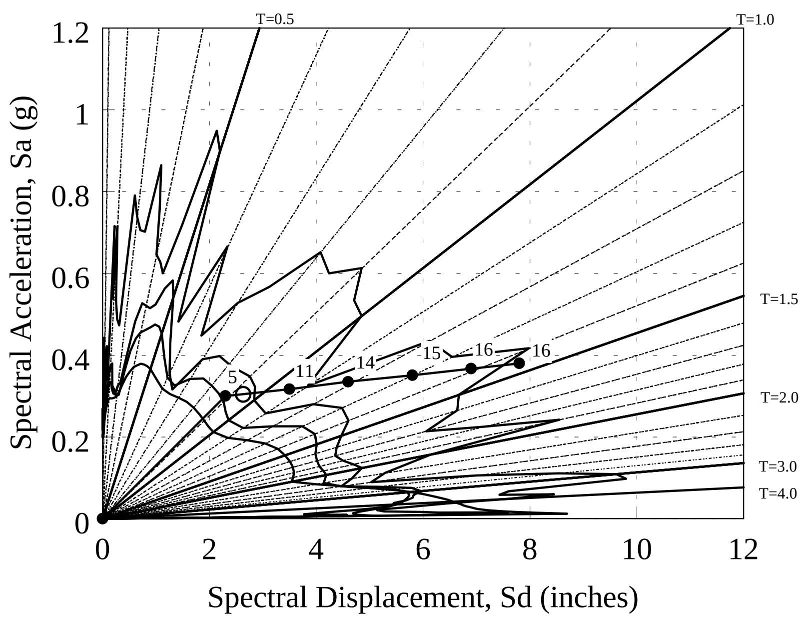

Figure 6. CSM for idealized capacity (Fig. 4) and Loma Prieta Arguello Drive recorded motion. Response spectra for 0%, 5%, 10%, and 20% damping. Note S_d : 10 = 25 cm.

The demand response spectra for β = 5%, 8%, 12%, and 18% from Figure 4 are also shown in Figure 5. Note that the elastic limit (i.e., point marked 5) of the capacity curve does not reach the 5 percent demand curve; therefore, the elastic demand exceeds the elastic capacity and the structure will displace into the inelastic range. Also note that the inelastic capacity at the 4^{th} point, (β_{eff} = 15 percent) is less than the demands of the β_{eff} = 15 percent demand curve as interpolated between the 12% and 18% demand curves and the capacity at the 5^{th} point (β_{eff} = 16%) is greater than the demands at the 16 percent demand curve. This tells us that the response will be somewhere between the 15% and 16% damped points. Thus, the common intersection can be estimated at about S_d = 6.3 (16 cm) at β_{eff} = 15.5% damping. In other words, when the sample structure is subjected to the sample earthquake, β_{eff} = 15½%, and S_a = 0.36g. This can be translated back to a roof displacement of 22 cm ($\Delta_R = S_d \times PF\varphi_R$) and a base shear coefficient of 0.28 ($V/W = \alpha\ S_a$). The important observation is that the inelastic capacity limit is reasonable beyond the demand and that the displacement ductility demands are about three.

Generally, the design response spectra are smooth in shape (e.g. Fig. 3), such as those in building codes; however, response spectra derived from actual earthquake records are irregular and contain spikes at predominant response periods. These spikes tend to fade away at higher damping values. An example is shown in the ADRS format in Figure 6. In the CSM solution for the sample building, the structure just barely exceeds yield. The demand S_d lies between 5% and 10% damping at about 6cm.

3 TOOL FOR SEISMIC DESIGN

The Capacity Spectrum Method (CSM) was originally developed as a rapid evaluation method for a pilot seismic risk project of the Puget Sound Naval Shipyard for the U.S. Navy (Freeman et al. 1975). It was later used as a procedure to correlate earthquake ground motion with observed building performance (Freeman 1978 & ATC 1982) and was further developed for the TriServices "Seismic Design Guidelines for Essential Buildings" (Army 1986) as part of the two-level approach to seismic design. CSM has been reformatted and updated for the U.S. Postal Service (ATC 1991), "Standards of Seismic Safety for Existing Federally Owned or Leased Buildings" (NIST 1994), a proposed revised edition of the TriServices manual on dynamic analysis (WJE 1996) and the State of California (ATC 1996).

The CSM and the ADRS format have been shown to be a useful tool evaluating existing buildings for seismic performance, verifying designs of new construction for performance goals, and correlating observed damage with recorded earthquake motion. The CSM can be used as a rapid evaluation procedure to obtain rough estimates for large inventories of buildings (Freeman, et al., 1975) or as a detailed procedure for new (Army 1986, Freeman 1987 & WJE 1996) and existing buildings (Army 1988 & ATC 1996). The procedure appears to be compatible with other approximate inelastic design and evaluation methods (Freeman 1995).

4 SUMMARY AND CONCLUSIONS

The CSM is applicable to a variety of uses such as a rapid evaluation technique for a large inventory of buildings, a design verification procedure for new construction of individual buildings, an evaluation procedure for an existing structure to identify damage states, and a procedure to correlate damage states of buildings to amplitudes of ground motion. The procedure has been successfully used to correlate recorded motion and observed performance for buildings that have been subjected to various earthquake ground motions, such as those from the San Fernando (1971), Loma Prieta (1989), or Northridge (1994) earthquakes. The CSM stands up well when compared to other procedures such as the equal displacement method and the secant methods and has the added advantage of giving the engineer the opportunity to visualize the relationship between demand and capacity. Differences between the various methodologies have more to do with unknowns in material behavior and quantification of energy dissipation than in the methods of analysis.

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