FORMULATION OF DESIGN ECCENTRICITY TO REDUCE DUCTILITY DEMAND IN ASYMMETRIC BUILDINGS

A. Ghersi[1], F. Laudani[1] and P.P. Rossi[1]

Abstract

The remarkable effort of the researchers in the last decades has produced on this subject results which often appear to be contradictory and strictly related to the structural system or to the structural analysis, either static or multi-modal spatial analysis, that in the phase of design deeply affects the distribution of strength among the elements and thus the maximum ductility demand due to inelastic response analyses. Both static and multi-modal spatial analysis do not fit well the elastic and inelastic response of asymmetric buildings, but while static spatial analysis - without additional eccentricities - underestimates the displacement of both flexible and stiff edges, multi-modal spatial analysis is able to exactly catch the elastic response of the flexible edge even if it risks to further on reduce the design displacement at the stiff edge. The use of multi-modal analysis may comply with the two aims of the seismic design (no collapse under strong events, damage limitation under seismic actions having a larger probability of occurrence), if it is carried out two times separately: the first one with actual mass distribution to cover peak elastic displacements at the flexible side and the second one with a design eccentricity to fit inelastic peak displacements at the stiff side. An idealised one storey building, symmetric about one direction, has been investigated with reference to different values of the parameters influencing the inelastic response of asymmetric buildings, by using a set of thirty accelerograms selected among historical Italian seismic events in order to take account of the possibility of occurrence of earthquakes with elastic response spectrum more or less different from that of design (mean of the thirty response spectra). The study has been focused on the assessment of a formulation for the design eccentricity that reduces the maximum ductility demand (expressed as the mean value and 95% fractile of the maximum ductility demanded by the thirty accelerograms). The approach proposed and the given formulation proved to be effective in foreseeing the effects of asymmetry on systems having variable values of geometrical and mechanical parameters, thus providing a design criterion which can limit the ductility demand of asymmetric schemes without relevant increment of structural costs.

[1] Institute of "Scienza delle Costruzioni", University of Catania, Italy

Inelastic response of asymmetric buildings

Whoever analyses the wide set of papers on this subject will be probably struck by the apparent necessity of underlining both the complexity of the problem and the discrepancies among the conclusions of the researchers. In effect, while the elastic seismic behaviour is ruled by few global parameters (eccentricity between mass and stiffness centres, uncoupled lateral-torsional frequency ratio and, in a lesser way, period of vibration, shape of the response spectrum and position of mass centre with respect to the edges of the floor deck), the inelastic response seems to be influenced by location and strength of each resisting element. A considerable effort is therefore presently devoted to the standardisation of definitions and assumptions and to the identification and evaluation of the effect of every single parameter. Anyway, we believe important to put more emphasis on some concordant results of the research; a few general considerations, which can be found in most papers on this subject, may in fact constitute the basis for a retrospective analysis of the past work and for the proposition of a design approach able to limit the negative effects of asymmetry.

At first it must be noted that the conclusions of the researchers seem contradictory mainly when the attention is focused on the ductility demand, which in different papers is considered to reach the maximum at the stiff or at the flexible side and to be smaller, comparable or much greater than the one of the corresponding balanced system. On the contrary, many authors acknowledge that the inelastic displacements of the elements which constitute a spatial frame are scarcely dependent on their strength, i.e. that different structures, with elements having the same stiffness but designed so as to offer different strength, present approximately the same peak displacements. Goel and Chopra 1990 clearly state that "the element deformations of systems designed according to most building codes are not very different" and Tso and Zhu 1992 affirm that "the displacement demand is insensitive to the form of torsional provisions adopted". This is confirmed by the numerical analyses later on described; an example is provided by Figure 1, which shows the peak displacements of a structural scheme designed with different strength distributions and behaviour factors, subjected to an Italian seismic recording (Tolmezzo, Friuli, 1976).

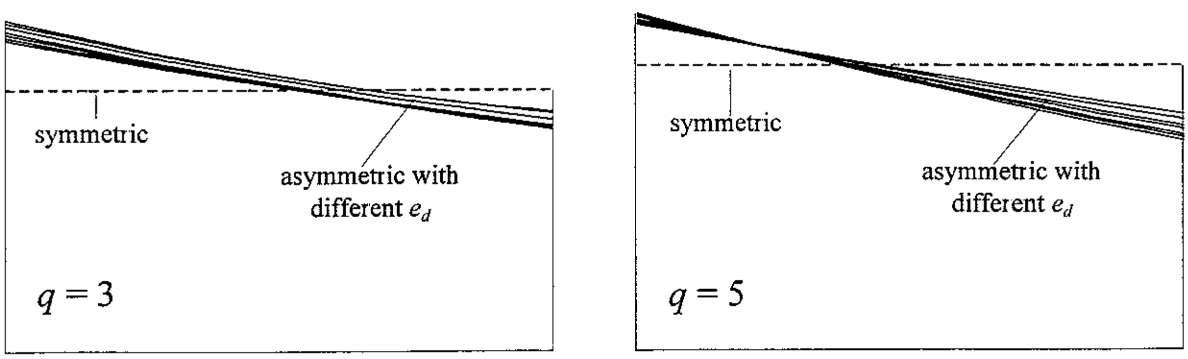

Figure 1 Peak displacements of a stiffness eccentric system subjected to Tolmezzo recording (design parameters: Ω_θ=1, T_x=T_y=1 s, γ_x=0.2, e_s=0.05 L)

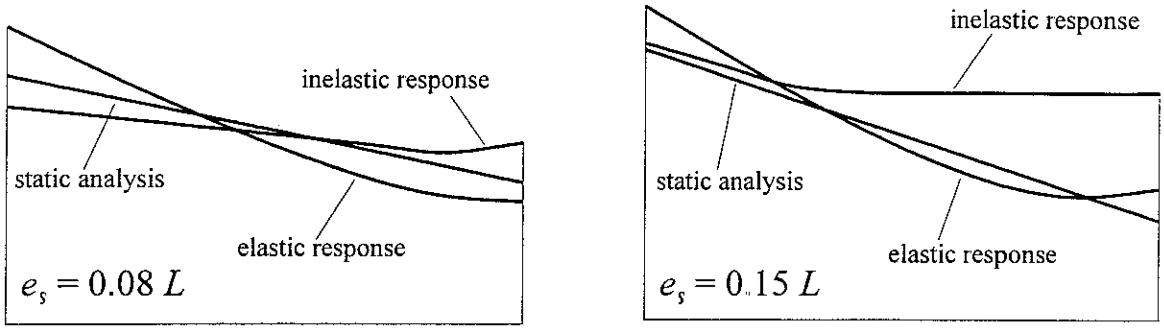

Figure 2. Peak displacements of a stiffness eccentric system subjected to El-Centro recording (design parameters: Ω_θ=1 2, T_x=T_y=1 s)

Secondly, it is often recognised that, while the elastic response of asymmetric schemes usually shows a larger rotation, if compared to the prediction of static spatial analysis, the inelastic response is much more translational. According to Goel and Chopra 1990, "yielding leads to reduced torsional deformation of medium-period and long-period systems, regardless of their stiffness eccentricity. Thus, if the system is well into the inelastic range, the effects of plan-asymmetry on system response are small". A reason for this is that within inelastic response analyses the eccentricity is not constant, because of the instantaneous variation of position of the rigidity centre due to the plasticization of the elements. A further cause is the torsional contribution of orthogonal elements which remain longer in the elastic range, mainly if the transverse seismic excitation is smaller than the one acting in the primary direction. The differences between elastic and inelastic response are illustrated by Figure 2, which shows the peak displacements of a structure subjected to El-Centro recording.

Design of asymmetric buildings

The two aforementioned considerations, joined together, may be considered a generalisation and a modification of the well known principle proposed in the sixties by Newmark for elastic-perfectly plastic s.d.o.f. systems. We may in this case affirm that the peak displacements of asymmetric schemes well into the inelastic range are independent of the global value of strength and of its distribution among the resisting elements, but they differ from the elastic peak displacements because of the less marked rotation. This assumption is obviously a simplification of the actual behaviour, which is valid in the mean but may be violated in single cases, like the Newmark principle itself. Nevertheless it proves to be very useful in solving the problem of the ductility demand because this one may be simply foreseen by comparing the design displacements to the peak values of elastic and inelastic analyses. When the strength of each element is assumed proportional to its stiffness, i.e. only translation is considered in design, the maximum ductility is always required at the flexible edge. Also when static spatial analysis - without additional eccentricities - is used, the elastic displacement of the flexible edge is underestimated (this is the main reason why some codes prescribe the adoption of an increased eccentricity); in this case, however, the ductility demand is greater at the stiff side, since the reduction of design displacements due to the rotation finds no correspondence in the more translational inelastic behaviour. Finally, the use of multi-modal spatial analysis is able to properly catch the elastic response of the flexible edge but risks to further reduce the design displacement (and increase the ductility demand) at the stiff edge.

A proper way to face the problem must not forget the two aims of seismic design (no collapse under strong events, damage limitation under seismic actions having a larger probability of occurrence), which in most codes are hidden by the use of an unique value of design action, provided by the elastic response spectrum divided by a coefficient (behaviour factor q in EC8). Design displacements should therefore cover peak elastic values at the flexible side and peak plastic values (divided by q) at the stiff side. The use of static spatial analysis as reference method of design, imposed by many codes and followed by nearly all researchers, requires a double effort (in terms of additional eccentricities) to solve both aspects. The mixing of these might be one of the reasons for the difficulty in interpreting the results of interesting works. Our basic assumption has therefore been that the elements' strength should be proportioned by using multi-modal spatial analysis with the actual mass distribution, for the flexible side, and with a design eccentricity (i.e. a displacement of the centre of mass towards the centre of rigidity) for the stiff side. The present paper shows how such design eccentricity is related to the elastic characteristics and to the mass distribution of the scheme, providing a thorough formulation which allows to reach the proposed goal. The design procedure, based on multi-modal spatial analysis, and the related formulation could be a strong basis for an improvement of the torsional provisions of Eurocode 8, as it is shown in a more detailed way in a companion paper (Calderoni et al. 1996). It is obviously important that the seismic code allows the designer to use static spatial analysis, but the equivalence of static to multi-modal analysis must be considered a separate problem, already solved (e.g. see Calderoni et al. 1994, 1995).

Numerical model

A preliminary step of the research has been the definition of the geometrical and elastic features of the structural model. The scheme is an idealised one-storey building with rectangular rigid deck; it is referred to the coordinate axes x and y, with origin G coincident to the geometrical centre of the deck, and it is assumed to be symmetric about the x-axis. The position of mass centre and the mass radius of gyration are assigned independently of shape and dimensions of the deck, under the hypothesis that the mass distribution may be not uniform. The main component of the seismic ground motion is considered to act along the y-direction, which is called for this reason "primary direction"; the developed procedure allows to take into account also the component acting along the x-axis ("secondary direction"), but in the present phase of the work this possibility has been neglected. The resisting elements, parallel to the axes, are assumed to have a bilinear elastic-perfectly plastic force-displacement relationship and to present no out-of-plane stiffness or strength. The general procedure developed, able to assign the stiffness of each element in such a way to obtain a required value of the global elastic parameters, is described hereafter separately for the two directions.

Elements along the primary direction (y-axis)

In order to obtain given values of location of stiffness centre and total translational and torsional stiffness of the elements oriented along this direction, a minimum number of three independent parameters is necessary. In the two-elements models, like the one used by Goel and Chopra 1990, the position of the elements must be considered variable and cannot coincide to the edge of the deck. This might have some influence on the inelastic response, because the displacement due to rotation depends on the distance from the rotation centre. More common is therefore the use of three-elements models (Tso and Zhu 1992, De Stefano et al. 1993, Chandler et al. 1996), which supply in most cases a satisfactory estimate of the inelastic response. However, the number of resisting elements might sometimes significantly affect the ductility demand; for this reason a more general automatic generation procedure has been developed, able to assign the proper stiffness to any number of elements, from three on. For a better correspondence to the actual buildings, in most analysed cases the system was constituted by eight elements in the main direction, but the effect of assuming a smaller number of elements has been investigated too.

In order to apply the procedure, according to established mathematical rules complying with the same logic, the primary model called *reference symmetric system* (RSS) is firstly defined (Figure 3). It is made up by a sub-system, called *basic system*, duplicated symmetrically with respect to the y-axis. The elements of the *basic system* are themselves symmetrical about their centre G_b, which may be considered origin of a set of local axes ξ, η.

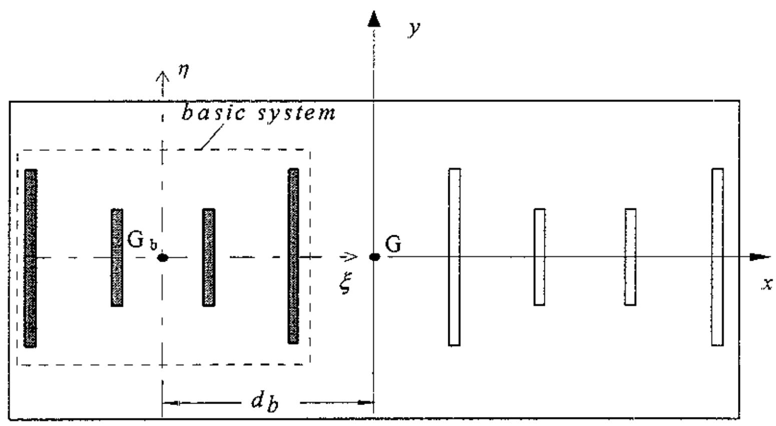

Figure 3. The *basic system* and the *reference symmetric system* (the size of the elements is drawn in proportion to their stiffness)

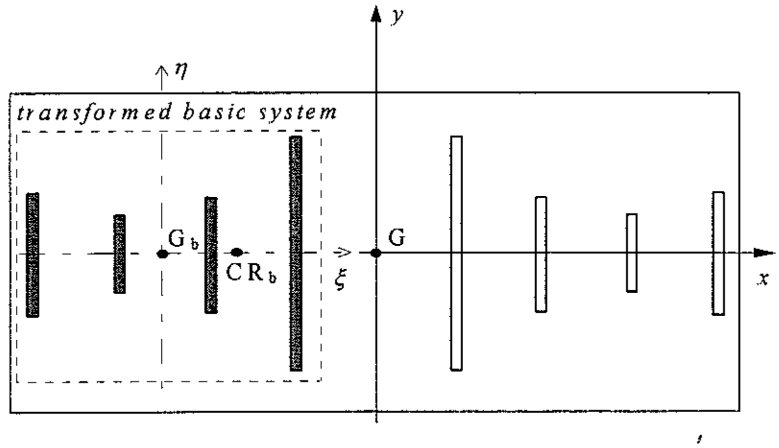

Figure 4. The *transformed basic system* and the *transformed symmetric system*

The RSS shown in the figure has an even number of elements; an odd number may be obtained by positioning the sub-systems so that the two central elements coincide and by substituting these with an unique equivalent element.

A *transformed symmetric system* (TSS), with a given torsional stiffness, is obtained by applying a linear transformation to the stiffness of the elements of the *basic system,* which is modified proportionally to the distance of the elements from G_b and to a parameter β_1 (Figure 4). The stiffness of the generic element of the *transformed basic system* is therefore

$$k'_{ib} = k_{ib}\,(1+\beta_1\,\xi_{ib}) \tag{1}$$

It may be easily demonstrated that such transformation do not change the total translational stiffness and the torsional stiffness about G_b. It is in fact

$$\sum_{i=1}^{n_b} k'_{ib} = \sum_{i=1}^{n_b}\left(k_{ib} + \beta_1\,k_{ib}\,\xi_{ib}\right) = \sum_{i=1}^{n_b} k_{ib} + \beta_1 \sum_{i=1}^{n_b} k_{ib}\,\xi_{ib} = K_b \tag{2}$$

$$\sum_{i=1}^{n_b} k'_{ib}\xi_{ib}^2 = \sum_{i=1}^{n_b}\left(k_{ib} + \beta_1\,k_{ib}\,\xi_{ib}\right)\xi_{ib}^2 = \sum_{i=1}^{n_b} k_{ib}\xi_{ib}^2 + \beta_1 \sum_{i=1}^{n_b} k_{ib}\,\xi_{ib}^3 = K_{\theta b} \tag{3}$$

being $\sum_{i=1}^{n_b} k_{ib}\,\xi_{ib} = \sum_{i=1}^{n_b} k_{ib}\,\xi_{ib}^3 = 0$ because of the symmetry of the *basic system.*

The first moment of the new distribution about G_b is

$$\sum_{i=1}^{n_b} k'_{ib}\,\xi_{ib} = \sum_{i=1}^{n_b}\left(k_{ib} + \beta_1\,k_{ib}\,\xi_{ib}\right)\xi_{ib} = \sum_{i=1}^{n_b} k_{ib}\xi_{ib} + \beta_1 \sum_{i=1}^{n_b} k_{ib}\xi_{ib}{}^2 = \beta_1\,K_{\theta b} \tag{4}$$

and the abscissa of the centre of rigidity of the *transformed basic system* is therefore

$$\xi'_{CRb} = \frac{\sum_{i=1}^{n_b} k'_{ib}\xi_{ib}}{\sum_{i=1}^{n_b} k'_{ib}} = \frac{\beta_1\,K_{\theta b}}{K_b} \tag{5}$$

The torsional stiffness of the *transformed basic system* about its centre of rigidity and that of the TSS about G can finally be expressed as

$$K'_{\theta b} = K_{\theta b} - K_b\,\xi'^2_{CRb} \tag{6}$$

$$\begin{aligned} K_{\theta Gy} &= 2\left[K'_{\theta b} + K_b\left(d_b - \xi'_{CRb}\right)^2\right] = \\ &= 2\left[K_{\theta b} + K_b\,d_b^2 - 2\,K_b\,d_b\,\xi'_{CRb}\right] = 2K_{\theta b} + 2K_b\,d_b{}^2 - 4\beta_1\,K_{\theta b}\,d_b \end{aligned} \tag{7}$$

It is therefore possible to obtain a TSS having a given translational and torsional stiffness simply by selecting a whatsoever *basic system* with K_b=0.5 K_y and evaluating the coefficient β_1 as

$$\beta_1 = \frac{2\,K_{\theta b} + 2\,K_b\,d_b^{\,2} - K_{\theta Gy}}{4\,K_{\theta b}\,d_b} \tag{8}$$

An *asymmetric system* may be obtained from TSS by assigning a mass centre not coincident with the geometric centre G. Such system is usually called *mass eccentric system* (MES) and its corresponding balanced system is the same TSS. As alternative, a further linear transformation may be applied to the whole TSS, by modifying the stiffness of each element proportionally to its distance from G and to a parameter β_2 (Figure 5). Once again the transformation let the translational stiffness and the torsional stiffness about G unchanged, while the abscissa of the centre of rigidity is related to β_2 by an expression analogous to Equation (5), which can be inverted giving

$$\beta_2 = \frac{K_y\;x_{CR}}{K_{\theta Gy}} \tag{9}$$

If the mass centre is coincident to G the *asymmetric system* so generated is called *stiffness eccentric system* (SES). The corresponding balanced system is obtained by moving CM to CR.

The distinction between MES and SES is considered fundamental by some authors (e.g. Goel and Chopra 1990), while others note that most actual systems are contemporaneously mass and stiffness eccentric (Tso and Zhu 1992). Our opinion is that the ruling parameter is not the type of model (MES or SES) but the position of mass centre with respect to the edges of the deck; this parameter proved to have some importance, although minor, in the elastic analyses (Calderoni et al. 1994) and it seems logic that an analogous influence may be found in the inelastic behaviour. Nevertheless the present research has been focused separately to MES and SES models and the results finally obtained show that the effect of such distinction on the design eccentricity, although perceptible, is not relevant.

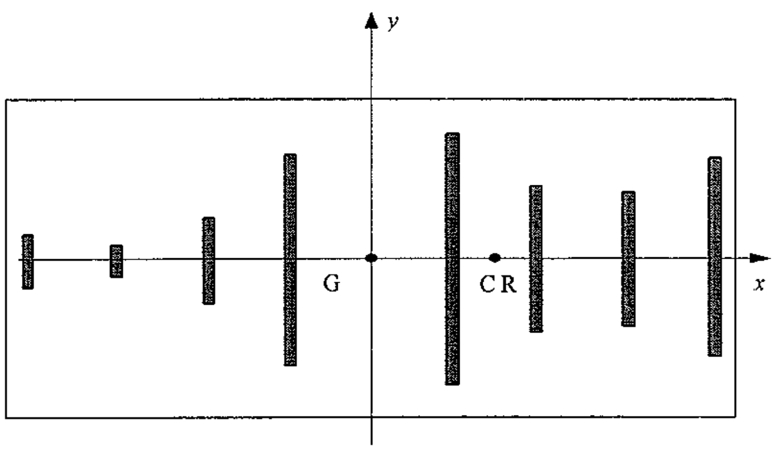

Figure 5 The *asymmetric system* (SES) obtained by means of the last transformation

Elements along the secondary direction (x-axis)

The presence of elements oriented along the secondary direction contributes to reduce the rotation in the inelastic range, in particular when the transversal component of the seismic ground motion is small or it is totally neglected in the analysis. However, the analyses carried on aim at evaluating two limit behaviours, with increased and reduced rotation, given respectively by the elastic and the inelastic response. The absence or the early plasticization of the secondary elements, although actually possible, can limit the reduction of inelastic rotation, which is not safe for our purpose. For this reason the utilised model has elements in the x-direction able to provide a translational stiffness (equal to the one in the y-direction) and a torsional stiffness (in most cases 1/5 of the total torsional stiffness, although other values have been assumed too, in order to evaluate the influence of this parameter). A number of three elements, located symmetrically along the x-direction has been fixed; their stiffness, necessary to comply with the above requirements, is

$$k_{1x} = k_{3x} = \frac{K_{\theta x}}{2\, y_{1x}^2} \qquad\qquad k_{2x} = K_x - 2\, k_{1x} \tag{10}$$

Seismic ground motion

It is well known that a proper selection of the input ground motion has a great importance in every response analysis. When one or few seismic recordings are used, large differences in response are to be expected. To overcome this problem, a probabilistic approach has been used, i.e. each structural scheme has been subjected to a set of accelerograms and statistical information has been extracted by the set of the results. Thirty historical Italian accelerograms having different characteristics (duration, peak ground acceleration and elastic response spectrum) have been selected in order to constitute a representative set of national accelerograms (Table 1). In order to homogenise them, the recordings have been scaled so that the elastic response spectrum of each of them presents an equal value of the area subtended between 0.5 and 3 seconds and the mean elastic response spectrum of the whole set has a given value (0.35 g) in correspondence of the period of 1 second. The mean elastic response spectrum so obtained (Figure 6) sufficiently recalls the elastic spectrum imposed by EC8 for firm soil in areas characterised by expected peak ground acceleration of 0.35 g.

Table 1 Reference code, origin and component of the thirty selected accelerograms

Ref. code	Recording	Comp.	Ref. code	Recording	Comp.	Ref. code	Recording	Comp.
32	Codroipo	ew	168	Forgaria	ew	621	Bagnoli I.	ew
32	Codroipo	ns	168	Forgaria	ns	621	Bagnoli I.	ns
38	Tolmezzo	ns	169	San Rocco	ew	627	Merc. S. Sev.	ew
143	Buia	ew	169	San Rocco	ns	627	Merc. S. Sev.	ns
143	Buia	ns	177	Buia	ew	636	Calitri	ew
152	Forgaria	ew	301	Patti	ew	636	Calitri	ns
152	Forgaria	ns	301	Patti	ns	643	Rionero	ew
153	San Rocco	ew	302	Naso	ew	643	Rionero	ns
156	Buia	ew	302	Naso	ns	644	Bisaccia	ew
156	Buia	ns	360	Cascia	ew	644	Bisaccia	ns

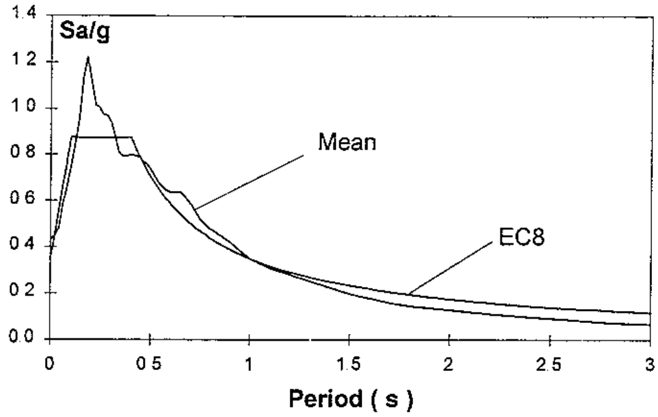

Figure 6 Mean elastic response spectrum of the set of accelerograms

Evaluation of design eccentricity

The design procedure we are proposing consists in assigning the elements' strength by means of two multi-modal spatial analyses: the first one with the mass centre located in its actual position; the second one with the mass centre displaced of a quantity e_d (design eccentricity) towards the centre of rigidity. In order to identify the best value of e_d each system, having given geometrical and inertial characteristics, has been designed many times with design eccentricity ranging from 0 to 1.5 e_s (being e_s the eccentricity between mass and stiffness centre), using as design spectrum the mean elastic response spectrum of the selected ground motions divided by a fixed value of the behaviour factor q. The resisting schemes thus obtained have been subjected to the set of accelerograms. In parallel, the corresponding *balanced system*, in which the mass centre has been displaced to coincide with the stiffness centre in order to obtain a purely translational behaviour, has been designed and subjected to the ground motions. Among the output data, the attention has been focused on the largest peak ductility demand among all elements: the value required by each seismic event has been normalised by the corresponding value of the *balanced system* and a global estimate is provided by the mean value $d_{0.50}$ and by the 95% fractile $d_{0.95}$ of the normalised ductility demand of the thirty accelerograms. The numerical analyses show that, when no design eccentricity is used, the maximum ductility is always demanded by the element at the stiff edge; both parameters increase in a non linear way with the stiffness eccentricity of the model (e.g. see Figure 7), reaching values which can be very high depending on the characteristics of the scheme. The use of design eccentricity strongly reduces this effect, although the reduction has a limit because value of e_d greater than e_s have a minor effect; in this last case, indeed, the maximum ductility is often demanded by central elements, the strength of which is not increased by the use of design eccentricity. From the relation of $d_{0.50}$ and $d_{0.95}$ versus e_d it is possible to define the value of e_d necessary to limit the ductility demand to a given value (e.g. see Figure 8). In the performed analyses the limit $d_{0.95}$=1.3 has been primarily imposed, but other values have been used too in order to analyse their influence on e_d.

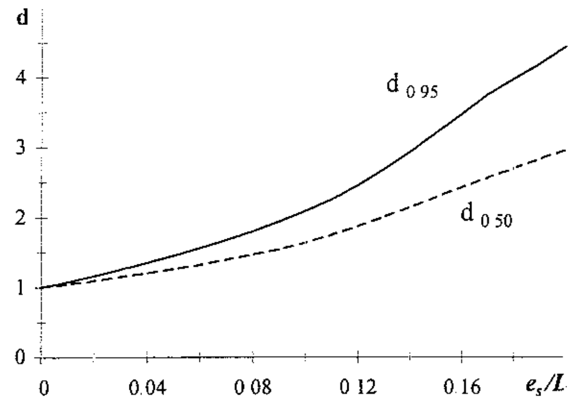

Figure 7. Normalised ductility demand versus stiffness eccentricity (design parameters: SES, Ω_θ=1, γ_x=0 2, T_y=1 s, q=5)

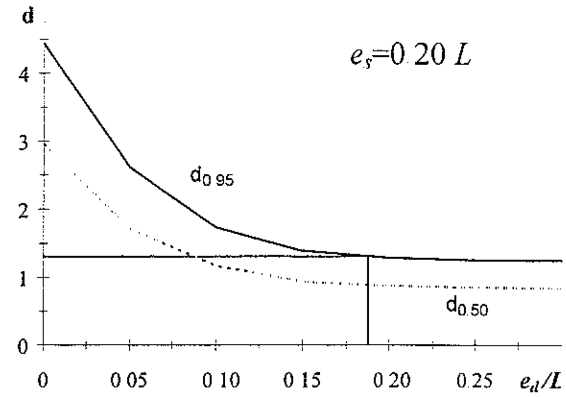

Figure 8. Normalised ductility demand versus design eccentricity (design parameters: SES, Ω_θ=1, γ_x=0.2, T_y=1 s, q=5)

Numerical analyses

The above described procedure allows to evaluate the optimum value of e_d for a scheme with assigned elastic and inertial characteristics. In order to find a general formulation, able to provide safe values of the design eccentricity in all actual situations, we investigated the influence of the position of mass centre (i.e. of the type of model, MES or SES) and that of the parameters e_s (stiffness eccentricity), Ω_θ (uncoupled lateral-torsional frequency ratio), T_y (uncoupled translational period) and q (behaviour factor). In all the numerical analyses we assumed dimensions of the rigid deck L=29.50 m and B=12.50 m, total mass corresponding to 1 t/m^2, mass radius of gyration =0.312 L. In most cases the *basic system* is defined by n_b=4, d_b=8.25 m, ξ_3=2.00 m, ξ_4=6.50 m, k_3=0.075 K_y, k_4=0.175 K_y and the rate of torsional stiffness due to the orthogonal elements is γ_x=0.2, although these data have been changed in a few cases in order to check the effect of the number of resisting elements and of the contribution given by the orthogonal elements.

For every assigned value of the above parameters, the automatic generation procedure defines the stiffness of each element. The total stiffness of the resisting elements oriented along the y-direction and the torsional stiffness of all elements about CR are given by

$$K_y = m\left(\frac{2\pi}{T_y}\right)^2 \qquad\qquad K_\theta = \Omega_\theta^2\, r_m^2\, K_y \tag{11}$$

while the rates of the torsional stiffness due to the elements along the y-axis evaluated about CR and G are respectively

$$K_{\theta y} = K_\theta\,(1-\gamma_x) \qquad\qquad K_{\theta G y} = K_{\theta y} + K_y\, x_{CR}^{\;2} \tag{12}$$

These values allow to evaluate the parameters β_1 and β_2 by means of Equations (8) and (9).

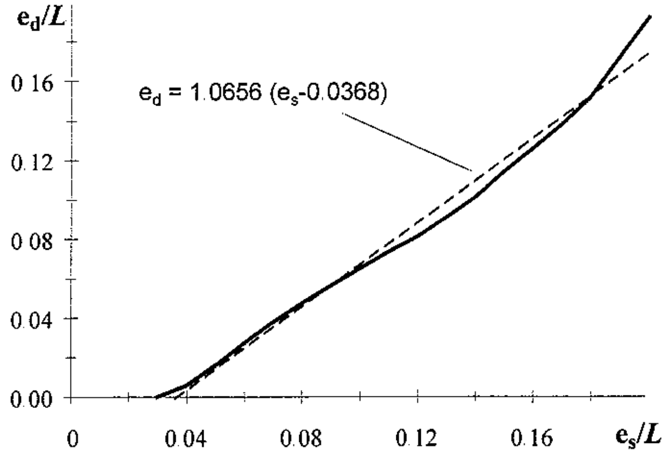

Figure 9. Design eccentricity (necessary to obtain $d_{0.95}$=1.3) versus stiffness eccentricity (design parameters: SES, Ω_θ=1, γ_x=0.2, T_y=1 s, q=5)

As a first step of the study we examined the influence of e_s on e_d. In all examined cases the relation between these two parameters is about linear (e.g. see Figure 9) and can be approximated by a straight line having equation

$$e_d = k\,(e_s - e_r) \tag{13}$$

The second step consisted therefore in the search of a relation among the parameters k, e_r and the elastic characteristics of the scheme. Starting from a basic case (Ω_θ=1, T_y=1 s, q=5) each parameter has been separately varied, in the following range: Ω_θ = 0.6 to 1.6; T_y = 0.4 s to 1.8 s; q = 1.5 to 5. Figure 10 shows the relation of k and e_r versus Ω_θ for two different values of $d_{0.95}$ (1.2 and 1.3). It may be first of all noted that the value of $d_{0.95}$ influences e_r while it has a very small effect on k; this situation has been identically found also in the analyses concerning the variation of T_y and q. Furthermore, it is apparent that e_r grows up with Ω_θ in an approximately linear way, with a slope which depends on the required value of $d_{0.95}$ but it is independent of the type of system. On the contrary k is nearly constant for SES and slightly decreasing as Ω_θ increases for MES. Figure 11 and 12 show the relation of k and e_r versus q and T_y respectively. The parameter e_r is practically independent of q, while k is once again decreasing as q increases, but without perceivable differences between SES and MES. On the contrary, the effect of T_y, although not negligible, do not show a clear tendency. Finally the results obtained by varying γ_x from 0.001 to 0.4 and by changing the number of resisting elements are not reported in any figure, because they are really scarcely relevant.

From the above described results we propose to express the parameters k and e_r by means of the following equations

$$k = 2.25 - 0.5\,\Omega_\theta - 0.1\,q \tag{14}$$

$$e_r = \frac{\Delta d}{3}\left(\Omega_\theta - 0.7\right) L \quad \geq 0.01\,L \tag{15}$$

being $\Delta d = d_{0.95} - 1$.

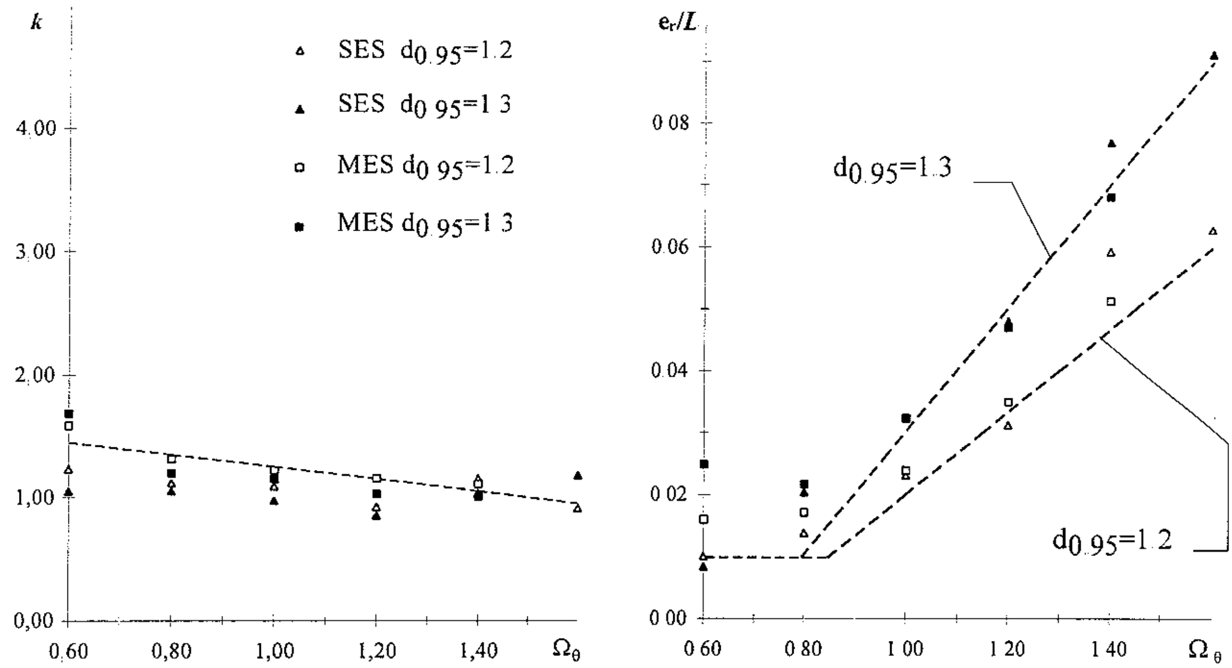

Figure 10 Values of k and e_r versus Ω_θ (constant parameters: γ_x=0.2, T_y=1 s, q=5)

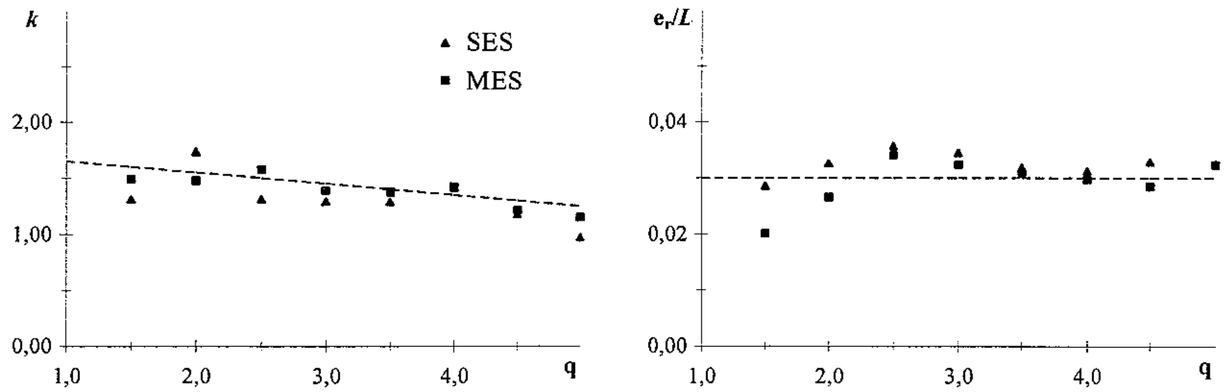

Figure 11. Values of k and e_r versus q (constant parameters: Ω_θ=1, γ_x=0.2, T_y=1 s)

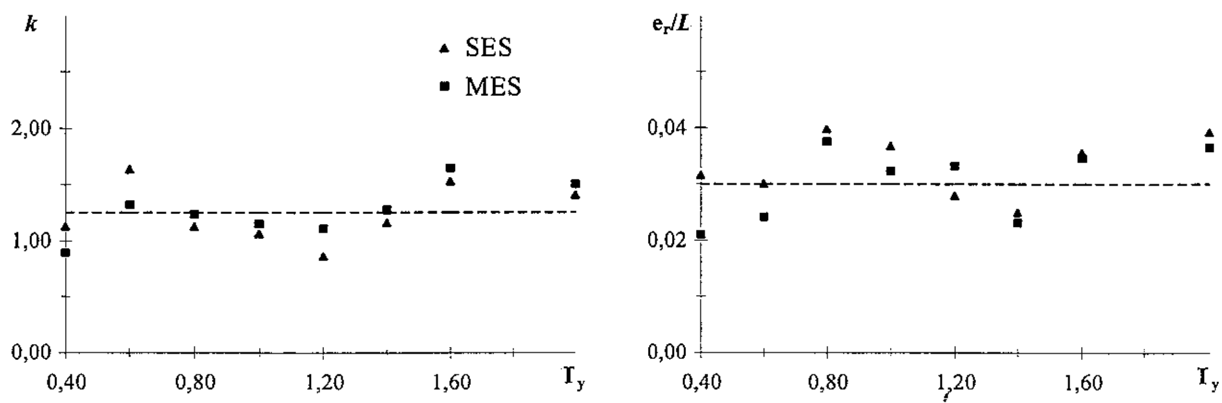

Figure 12. Values of k and e_r versus T_y (constant parameters: Ω_θ=1, γ_x=0.2, q=5)

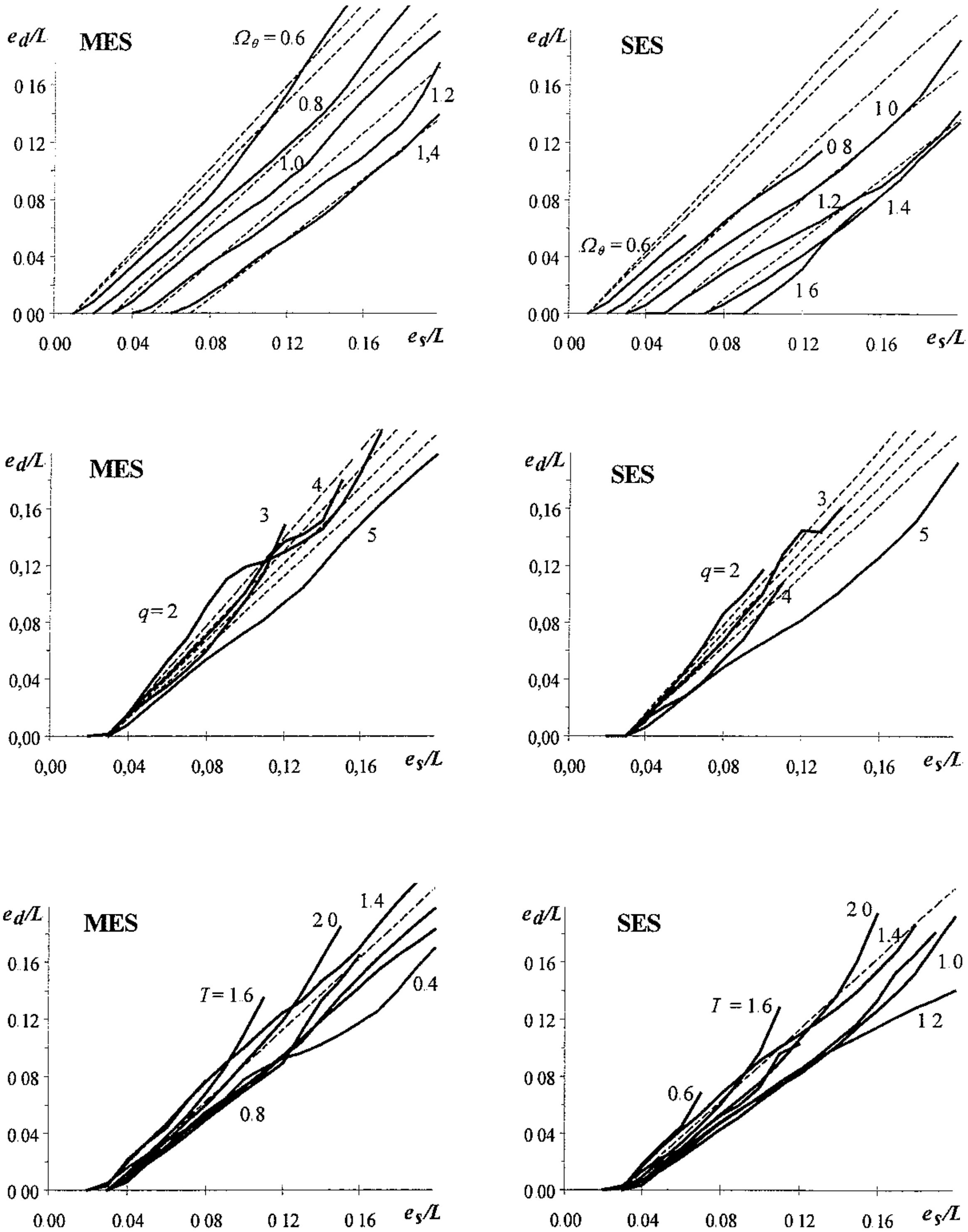

Figure 13. Calculated and proposed values of design eccentricity (necessary to obtain $d_{0.95}$=1.3) plotted versus stiffness eccentricity

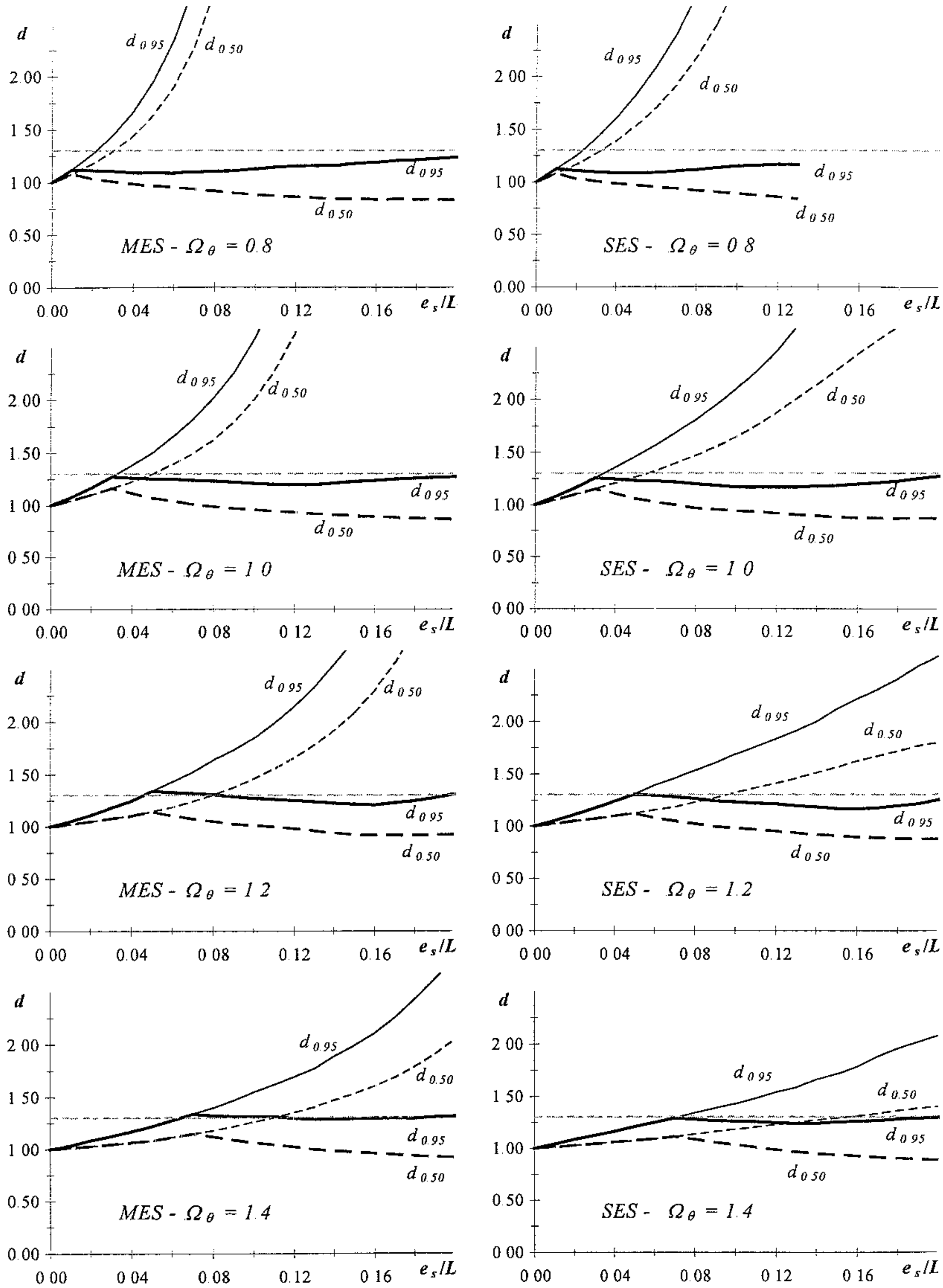

Figure 14. Ductility demand of schemes designed with and without the proposed approach and formulation of design eccentricity, with Δd=0.3 (constant parameters: γ_x=0.2, T_y=1 s, q=5)

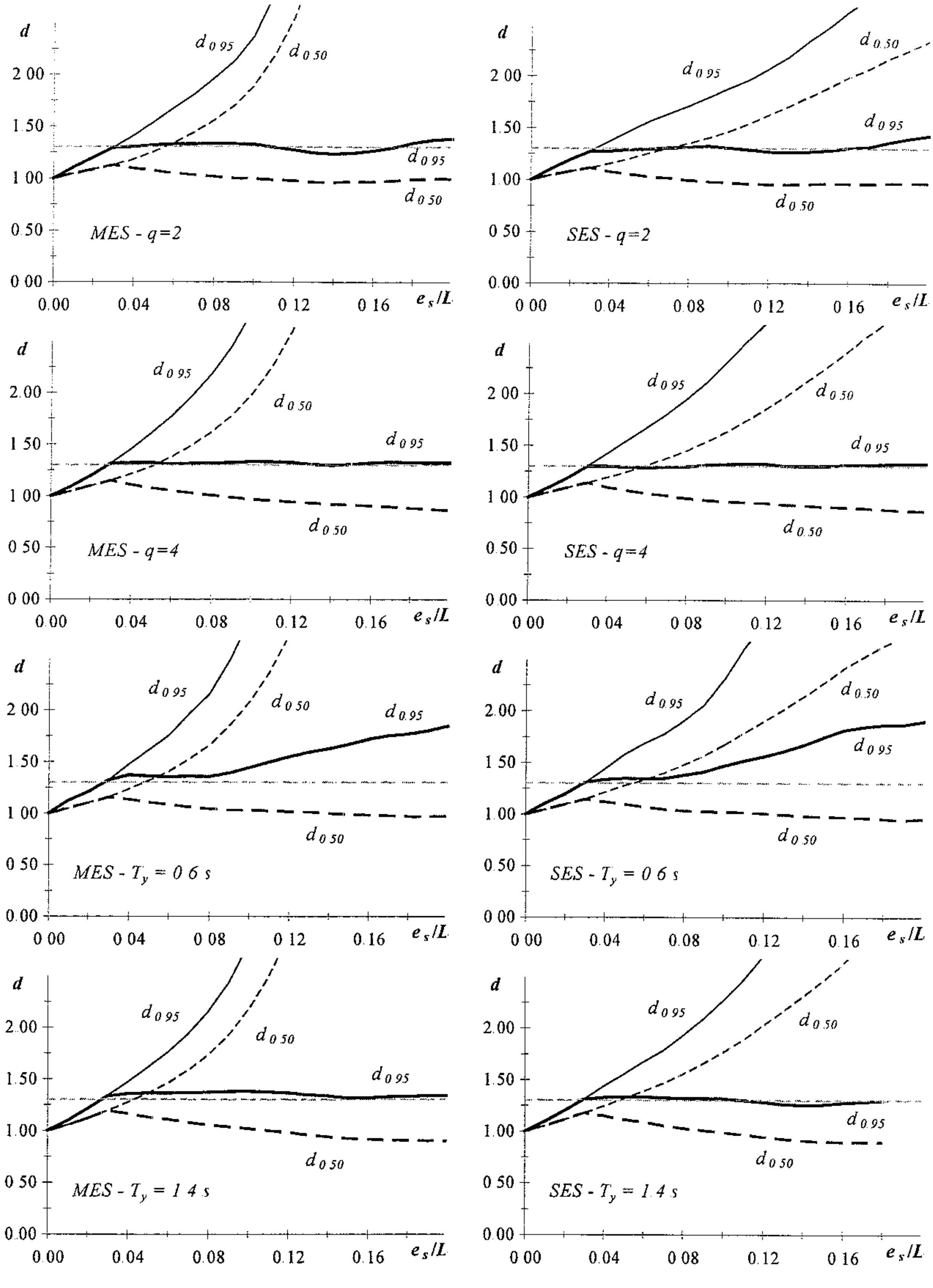

Figure 15. Ductility demand of schemes designed with and without the proposed approach and formulation of design eccentricity, with Δd=0.3 (constant parameters: Ω_θ=1, γ_x=0.2)

The dependence of k on Ω_θ and q is truly not so strong, and a possible simpler alternative could be to give it a constant value k=1.25. The values provided by Equations (14) and (15) are shown as dashed lines in Figures 10 to 12, confirming the effectiveness of the formulations.

Figure 13 compares the values of design eccentricity given by Equation (13) to those evaluated by means of the numerical analyses. In nearly all cases the proposed values are greater than those numerically calculated, showing the safety of the afore mentioned formulation. In some cases, in particular for SES, this safety is higher, but still acceptable, and only in very few situations, mainly for particular values of T_y, the numerical results are slightly larger than those given by the formula. This is confirmed by Figures 14 and 15, which plot the actual values of ductility demand for schemes designed with and without the proposed approach and formulation (with Δd=0.3). It is apparent that the goal of limiting the ductility demand has been perfectly achieved, with a partial exception for very rigid schemes (T_y=0.6 s). It must be furthermore noted that, having imposed the value 1.3 to the 95% fractile of the normalised ductility demand, the mean value of the normalized ductility ($d_{0.50}$) is always close to 1, even in the case of stiff schemes, showing that the ductility demand of asymmetric structures designed according to the proposed rules and parameters is always coincident, in the mean, to that of the corresponding balanced schemes. We therefore propose to use $d_{0.95}$=1.3 and Δd=0.3 in Equation (15), i.e. to assume

$$e_r = 0.1\left(\Omega_\theta - 0.7\right) L \quad \geq 0.01\, L \tag{16}$$

Some consideration may be also given to the location of the centre of strength. If the torsional contribution of the orthogonal elements is neglected, it coincides with the mass centre. Both the contribution of orthogonal elements and the use of design eccentricity move it toward the stiffness centre. In particular, the proposed values of e_d shifts the strength centre to a position not far from the mid-way between the mass and stiffness centres, position which many researchers have suggested to be optimal for a good response of the structure (e.g. see Chandler et al. 1996 and De Stefano et al. 1993).

The mean increase of strength due to the proposed approach is small, comparable to that obtained by complying with the prescriptions of seismic codes. The global overstrength, i.e. the ratio of the total strength of the resisting elements in the primary direction over the strength of those of the corresponding balanced system, is plotted in Figure 16. It is apparent the influence of the uncoupled lateral-torsional frequency ratio: when the scheme is torsionally stiff it is necessary just a small overstrength even in the case of relevant eccentricities, while torsionally flexible structures require a large overstrength also for small eccentricities. In the same figure is plotted, as a term of comparison, the overstrength obtained by satisfying a clause given by some codes (like UBC), which require not to reduce the strength of the elements when a spatial analysis is performed; this is equivalent to consider a design eccentricity equal to the stiffness eccentricity (e_d=e_s). In this case the overstrength does not depend in a substantial way on Ω_θ (the curve shown is referred to Ω_θ=1, but it is nearly coincident with those calculated for different values of this parameter). A comparison with the previously described curves shows that such a provision is acceptable in the case Ω_θ=1, but it is insufficient for torsionally flexible structures and excessive for torsionally stiff schemes. The introduction of this prescription in seismic codes is probably a good solution to the problem of limiting ductility demand in asymmetric buildings, because of its simplicity, but it should be connected to explicit limitations to avoid torsional flexibility; at the same time the use of a more exact approach, like the one proposed in this paper, should be allowed.

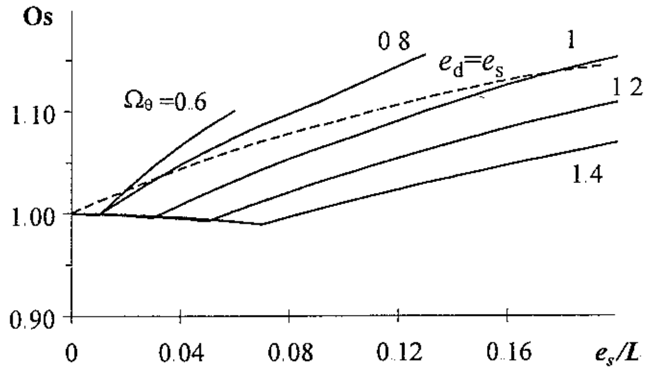

Figure 16. Overstrength of systems designed according to the proposed procedure, compared to the one obtained by using $e_d=e_s$ (constant parameters: SES, γ_x=0.2, T_y=1 s, q=5)

Conclusions

The proposed approach (use of two multi-modal analyses, the first one with the mass centre in its nominal position and the second one with the centre displaced of a quantity, named design eccentricity, towards the stiffness centre) appears to be a powerful tool in order to overpass the problems connected to asymmetry, by limiting the ductility demand without relevant increment of structural costs. The formulation of design eccentricity, here proposed, has been tested for a wide set of values of geometrical and inertial parameters, proving a large effectiveness. The given formula, or any simplification of it, may therefore constitute a good basis for an improvement of the European seismic code.

References

Calderoni B., Ghersi A., Mazzolani F.M. (1994) A new approach to the problem of in-plan regularity in seismic design of buildings. *Proceedings of 10th European Conference on Earthquake Engineering*, Vienna, vol.2, pp.843-848.

Calderoni B., Ghersi A., Mazzolani F.M. (1995) Critical analysis of torsional provisions in seismic codes. *Proceedings of Seventh Canadian Conference on Earthquake Engineering*, Montreal, Canada.

Calderoni B., Ghersi A., Mazzolani F.M. (1996) Critical analysis of EC8 approach to face the problem of structural regularity. *Proceedings of European Workshop on the Seismic Behaviour of Asymmetric and Set-Back Structures*, Anacapri, Italy.

Chandler A.M., Duan X.N., Rutenberg A. (1996) Seismic torsional response: assumptions, controversies and research progress. *European Earthquake Engineering*, n.1, pp.37-51.

De Stefano M., Faella G., Ramasco R. (1993) Inelastic response and design criteria of plan-wise asymmetric systems. *Earthquake Engineering and Structural Dynamics*, vol.22, pp.245-259.

Goel R.K., Chopra A.K. (1990) Inelastic seismic response of one-story, asymmetric-plan systems. *Report no. UCB/EERC-90/14, Earthquake Engineering Research Center*, Berkeley, California.

Tso W.K., Zhu T.J. (1992) Design of torsionally unbalanced structural systems based on code provisions. I: ductility demand. *Earthquake Engineering and Structural Dynamics*, vol.21, pp.609-627.

Notation

L, B dimensions of the deck along the x and y-direction

G geometrical centre of the deck

m mass of the deck

CM mass centre

r_m mass radius of gyration about the mass centre

k_{jx} stiffness of the j[th] element parallel to x-axis

k_{iy} stiffness of the i[th] element parallel to y-axis

y_j , x_i distance of the above elements from the x and y-axis respectively

n_x , n_y number of resisting elements parallel to the x and y-axis

K_x , K_y total lateral stiffness of the elements parallel to the x and y-axis

$$\boldsymbol{K}_x = \sum_{j=1}^{n_x} \boldsymbol{k}_{jx} \qquad \boldsymbol{K}_y = \sum_{i=1}^{n_y} \boldsymbol{k}_{iy}$$

$K_{\theta Gy}$ torsional stiffness of the elements parallel to the y-axis about G

$$K_{\theta Gy} = \sum_{i=1}^{n_y} k_{iy}\, x_i^2$$

CR centre of rigidity

x_{CR} , y_{CR} coordinates of CR

e_s stiffness eccentricity, i.e. distance between CR and CM

$K_{\theta x}$, $K_{\theta y}$ torsional stiffness of the elements parallel to the x and y-axis about CR

$$K_{\theta x} = \sum_{j=1}^{n_x} k_{jx} (y_j - y_{CR})^2 \qquad K_{\theta y} = \sum_{i=1}^{n_y} k_{iy} (x_i - x_{CR})^2$$

K_θ total torsional stiffness about CR

$$K_\theta = K_{\theta x} + K_{\theta y}$$

r_k stiffness radius of gyration about CR

$$r_k = \sqrt{\frac{K_\theta}{K_y}}$$

γ_x rate of torsional stiffness due to the elements parallel to x-axis

$$\gamma_x = \frac{K_{\theta x}}{K_\theta}$$

T_x, ω_x uncoupled translational period and frequency along the x-direction

$$T_x = 2\pi\sqrt{\frac{m}{K_x}} \qquad \omega_x = \frac{2\pi}{T_x} = \sqrt{\frac{K_x}{m}}$$

T_y, ω_y uncoupled translational period and frequency along the y-direction

$$T_y = 2\pi\sqrt{\frac{m}{K_y}} \qquad \omega_y = \frac{2\pi}{T_y} = \sqrt{\frac{K_y}{m}}$$

T_θ, ω_θ uncoupled torsional period and frequency

$$T_\theta = 2\pi\sqrt{\frac{m r_m^2}{K_\theta}} \qquad \omega_\theta = \frac{2\pi}{T_\theta} = \sqrt{\frac{K_\theta}{m r_m^2}}$$

Ω_θ uncoupled lateral-torsional frequency ratio

$$\Omega_\theta = \frac{\omega_\theta}{\omega_y} = \frac{r_k}{r_m}$$

G_b centre of the *basic system*

d_b distance between G and G_b

k_{ib} stiffness of the i^th^ element of the *basic system*

ξ_{ib} abscissa of the above element

n_b number of elements of the *basic system*

K_b total lateral stiffness of the elements of the *basic system*

$$K_b = \sum_{i=1}^{n_b} k_{ib} = \frac{1}{2} K_y$$

$K_{\theta b}$ total torsional stiffness of the elements of the *basic system* about G_b

$$K_{\theta b} = \sum_{i=1}^{n_b} k_{ib}\, \xi_{ib}^2$$

k'_{ib} stiffness of the i^th^ element of the *transformed basic system*

CR_b centre of rigidity of the *transformed basic system*

ξ'_{CRb} abscissa of the rigidity centre of the *transformed basic system*

$K'_{\theta b}$ total torsional stiffness of the elements of the *transformed basic system* about their stiffness centre