EFFECTS OF IN-ELEVATION IRREGULARITY ON THE ELASTIC SEISMIC RESPONSE OF IN-PLAN ASYMMETRIC BUILDINGS

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ABSTRACT

In the eighties Chopra demonstrated that the dynamic behaviour of a special class of in plan irregular buildings, named *regularly asymmetric systems*, may be obtained by coupling the results of a modal planar analysis to the normalised torsional response of a single-storey system. As a result of this observation the design procedures developed using single-storey systems (i.e. the simplified analysis of torsional effects proposed in many codes) may be used for multi-storey structures belonging to this special class. In spite of the fact that real structures are generally non regularly asymmetric systems designers apply approximated methods of analysis basing on their intuition. In order to define clear limits in the application of simplified methods of analysis, two parameters of irregularity are defined in the paper and their correlation with the maximum error due to the application of the aforementioned methods is investigated.

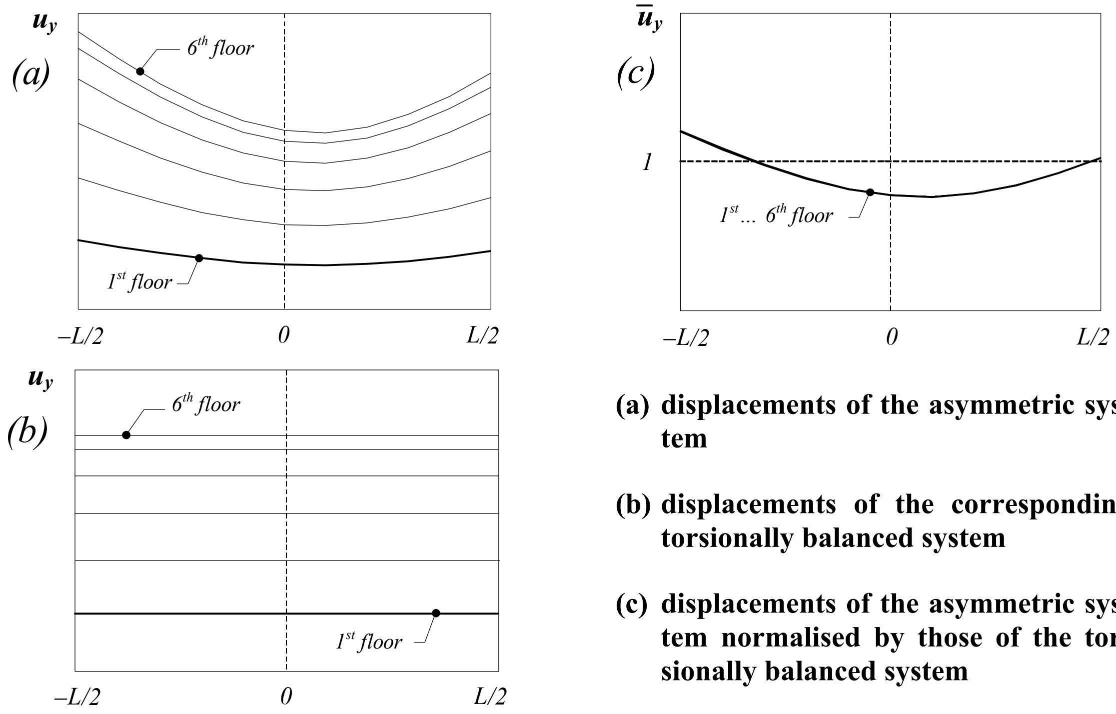
INTRODUCTION

The use of static analysis has been firstly conceived for plane frames, basing on the observation that, for such schemes, the distribution of forces corresponding to the first vibration mode is not so much different from a triangular one. Much more questionable is the application of static analysis to three-dimensional schemes, the dynamic response of which presents torsional rotations inconsistent with plane models. The analysis of the modal response of single-storey spatial schemes suggested a simple way for safely evaluating the displacements of the deck: the use of eccentricities in the application of static forces, calibrated in such a way as to catch the dynamic increase of torsional rotation [2], [4]. Simplified formulations of these eccentricities have been included in most seismic codes [5], [11], [12]. In the eighties Chopra demonstrated the validity of this approach for a particular class of multi-storey buildings, named "regularly asymmetric". Their dynamic behaviour may be exactly described by coupling the results of a modal

planar analysis to the normalised torsional response of a single-storey system [1]. Thus it is also possible the use of static analysis, provided that static forces are applied with eccentricities defined for the single-storey coupled system. Unfortunately, very few buildings strictly respect the conditions required for being defined regularly asymmetric, because structural properties (as the location of centre of mass C_M, centre of rigidity C_R and the radius of gyration of mass r_m and stiffness r_k) often vary from one floor to another. Nevertheless, static analysis is commonly applied to buildings that present some irregularities, basing on the intuition that a small non-correspondence to the theoretical requirements cannot dramatically modify the behaviour of a building. Intuition is, doubtlessly, a powerful tool of the human kind but, if we accept this, which is the limit? How can we define a level of irregularity that makes reliable the use of static analysis? This paper tries to answer these questions, proposing a parameter that numerically defines in-elevation irregularity and showing how it is related to the errors committed in the evaluation of the seismic behaviour by means of simplified analyses.

ANALYSIS OF REGULARLY ASYMMETRIC SYSTEMS

In regularly asymmetric systems the in-plan distribution of the maximum displacements at each floor, evaluated by means of spatial modal analysis, describes proportional curves (Fig. 1a). The ratio of these displacements over those of the corresponding balanced system (Fig. 1b) gives the same curve at each floor (normalised displacements, Fig. 1c). Consequently a single-storey system that describes the effects of lateral-torsional coupling exists [1], [10]. Planar modal analysis of the balanced system, corrected by means of the normalised response of the corresponding single-storey asymmetric system, can be utilised, obtaining the same results of the spatial modal analysis.



(a) displacements of the asymmetric system

(b) displacements of the corresponding torsionally balanced system

(c) displacements of the asymmetric system normalised by those of the torsionally balanced system

Figure 1. Application of modal analysis to a multi-storey regularly asymmetric system (Ω_θ=1.0; T_y=1 s; e_s=0.10 L)

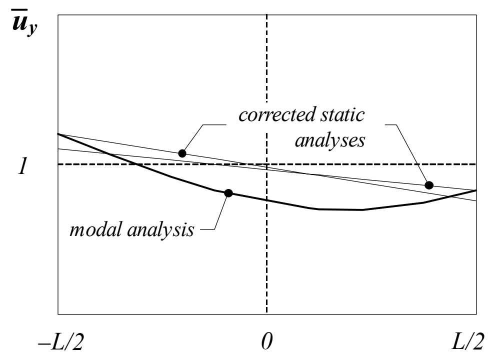
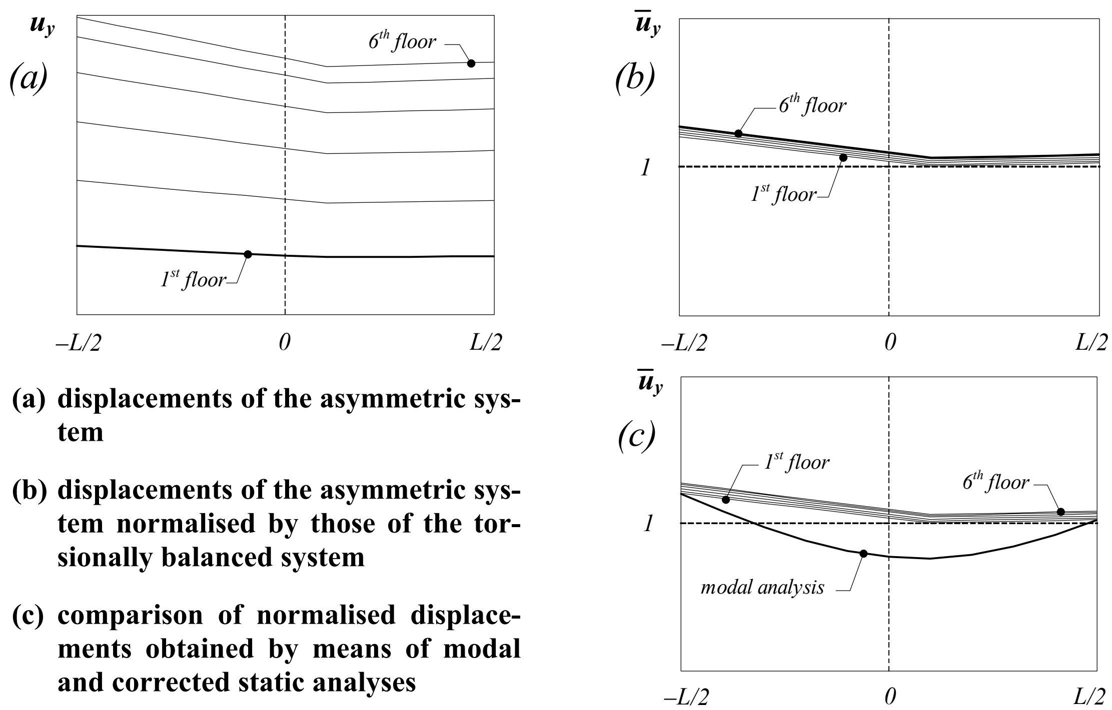


Figure 2. Application of corrected static analyses to a one-storey asymmetric system (Ω_θ=1.0; T_y=1 s; e_s=0.10 L)

The maximum modal displacements of both flexible and stiff side of the single-storey system may be evaluated by means of two static analyses (Fig. 2) with appropriate eccentricities e_{d1} and e_{d2} [2], [4]. Using the same eccentricities in the static analysis of the three-dimensional scheme, we obtain displacements of the regularly asymmetric structure (Fig. 3a) that, when normalised with respect to those represented in Fig. 1b, give curves that slightly differ from one floor to another (Fig. 3b). Consequently, the corrected static analysis is conservative in the upper floors, while in the lower floors it gives values closer to those of the modal analysis (Fig. 3c). These small differences are connected to the different distribution of forces applied in the case of modal or static analysis. Note that the comparison has been carried on using the same base shear in the two approaches, as some seismic codes suggest, in order to avoid larger differences



(a) displacements of the asymmetric system

(b) displacements of the asymmetric system normalised by those of the torsionally balanced system

(c) comparison of normalised displacements obtained by means of modal and corrected static analyses

Figure 3. Application of corrected static analysis to a multi-storey regularly asymmetric system (Ω_θ=1.0; T_y=1 s; e_s=0.10 L)

ANALYSIS OF NON REGULARLY ASYMMETRIC SYSTEMS

Equivalent Single Storey System

Every regularly asymmetric building is characterised by some properties, which are necessary for defining the correspondent single-storey system: it has an elastic axis; the mass centres of all the floors are lined up in vertical; the radius of gyration of stiffness and that of masses do not vary along the height [1], [10]. Unfortunately, in most actual buildings the in-plan distribution of stiffness varies from one storey to another. As a consequence, they do not have an elastic axis and different positions of generalised centres of rigidity, twist and shear centres may be evaluated, depending on the distribution of the horizontal actions used for the calculation [2], [4], [6], [9], [13]. In order to individuate a correspondent single-storey system, it is possible to refer to the optimum torsion axis proposed by Anasthassiadis et al. [7], defined as the vertical line that joins the points of the floors where the equivalent seismic forces must be applied in order to minimise the sum of the squares of the deck rotations. Such an axis coincides with the elastic axis when this one exists; furthermore, its position is only in a minor way influenced by the distribution of the horizontal forces, differently from what observed for other reference points (particularly the centre of rigidity) [6], [9]. Regarding the radius of gyration of stiffness, in a regularly asymmetric system it is unique and may be evaluated by the following expression,

$$r_k = e_l \sqrt{\frac{u_{F,i}}{\theta_{M,i}\ \ e_l} - \left(\frac{\theta_{F,i}}{\theta_{M,i}}\right)^2} \tag{1}$$

where u_{Fi}, θ_{Fi} are displacement and rotation of the deck of the ith floor produced by a set of horizontal forces F and θ_{Mi} is the rotation of the same deck produced by a set of torsional couples M obtained multiplying F by an eccentricity e_1. In irregular buildings, this expression gives a different value at each floor. We suggest assuming, as radius of gyration of stiffness of the correspondent single-storey system, the mean of the values provided by the Eq.(1) at all the floors. Finally, if the mass centres do not lie on a vertical axis or their radii of gyration vary along the height, we may use the mean value of these quantities in order to characterise the correspondent single-storey system. It is thus possible to evaluate the eccentricities e_{d1} and e_{d2} necessary to perform a *corrected static analysis*. Or, as an alternative, we are able to perform a planar modal analysis and to correct it by means of the normalised response of the single-storey asymmetric system (*corrected planar modal analysis*).

Parameters for Measuring the Irregularity along the Height

With the above-mentioned assumptions, it is possible to evaluate structural displacements and internal actions by means of simplified analyses also for irregular schemes. Anyway, it is necessary to know the entity of the errors produced by the use of approximate methods and to relate it to simple parameters that take into account the level of irregularity.

In order to find a measure of the irregularity of a structure along the height, two parameters have been defined with the aim of take into account two different aspects. The first parameter comes from the definition of optimum torsion axis, which aims at minimising the sum of the squares of the deck rotations produced by a distribution of lateral

forces. Noting that this sum is null for regularly asymmetric schemes, for which optimum torsion axis and elastic axis coincide, and greater than zero for irregular schemes, we may assume as measure of non-regularity the parameter:

$$\Theta_1 = \frac{1}{N}\sqrt{\sum_{i=1}^{N}\theta_i^{\,2}} \tag{2}$$

where θ_i is the deck rotation caused by a distribution of forces F applied to the optimum torsion axis and N is the number of storeys of the building.

The effectiveness of the parameter Θ_1 is limited by the fact that it is not able to properly catch the effect of the variation of radius of gyration of stiffness along the height. E.g., it is null for mass-eccentric buildings having stiffness centres lined along the symmetry axis but presenting at the same time relevant variation of torsional stiffness at different floors [9], [10]. In order to overcome this, we may use a second parameter Θ_2, which takes into account the vertical irregularity caused by the variation of the radius of gyration of stiffness. In single-storey systems the translation u_F of the corresponding balanced systems produced by a force F and the rotation θ_M induced by a couple $M = F \cdot e_1$ may be easily calculated by means of the following expressions:

$$u_F = \frac{F}{K} \tag{3}$$

$$\theta_M = \frac{F\, e_1}{K\, r_k^{\,2}} \tag{4}$$

being K the translational stiffness of the system. The deck rotation is thus given by the expression:

$$\theta_M = \frac{u_F\, e_1}{r_k^{\,2}} \tag{5}$$

The previous equation, which expresses the rotation of the deck θ_M in function of the displacement u_F, is valid for regularly asymmetric systems but not for non regularly asymmetric systems. Such observation suggests a second parameter able to quantify the vertical irregularity of the buildings.

In fact, with reference to an actual multi-storey system, restrained the deck rotations, the displacements $u_{F,i}$ of the floors due to a distribution of forces F_i may be evaluated. Therefore the rotations of deck of the actual building, caused by the application of a distribution of couples $F_i\, e_1$, may be evaluated by means of the Eq. (5). The radius of gyration of stiffness is calculated according to the already-cited formula valid for regular asymmetric systems (Equation 1). Only if the analysed building is a regularly asymmetric system, the obtained rotations would be equal to those evaluated applying the couples $F_i\, e_1$ on the spatial model. At the aim of defining a structural parameter able to quantify the non regularity along the height the difference between the rotation $\theta_{M,i}$ produced in the examined multi-storey systems by a considered distribution of couples and that evaluated by means of the Eq. (5) is calculated at each floor. The sum of the squares of

such differences has been assumed as measure of vertical irregularity and the parameter Θ_2 ha been defined as:

$$\Theta_2 = \frac{\sqrt{\sum_{i=1}^{N}\left(\theta_{M,i} - \frac{u_{F,i} e_s}{r_{k\,med,i}^2}\right)^2}}{N} \tag{6}$$

where the radius of gyration of stiffness $r_{k\,med,i}$ is the mean value between those obtained in the $i+1,\ldots,N$ floors.

Both indices Θ_1 and Θ_2 are equal to zero in regularly asymmetric systems and greater than zero in non regularly asymmetric systems.

NUMERICAL MODELS

In order to validate the use of static analysis for irregular structures, two typologies of asymmetric buildings have been considered in the present paper. Irregularity has been introduced by means of random but limited modifications of the stiffness of the resisting elements of two regularly asymmetric schemes. The structure of the reference systems (Fig. 4) presents shear-type frames symmetrically disposed with respect to the *x* and *y*-axes (four in the *y*-direction and two in the *x*-direction). All the *y*-direction frames are equal each other in the first scheme, which generate a set of irregular buildings named "class A", while in the second scheme one external frame has rigidity double than the others (giving irregular buildings of "class B"). Both schemes have six storeys and present *x*-direction frames equal each other, mass and radius of gyration of mass with the same value at all the floors, centres of mass disposed along a vertical axis.

Starting from these two basic systems, two sets of 1500 irregular buildings have been generated, by randomly imposing:

- the number of resisting elements to be modified, in the range one to six;
- the storey and position of the elements to be varied (more than one modification may occur at the same element);
- the entity Δ of the variation of the lateral rigidity of each element, in the range $-50\ \% \leq \Delta \leq 400\ \%$.

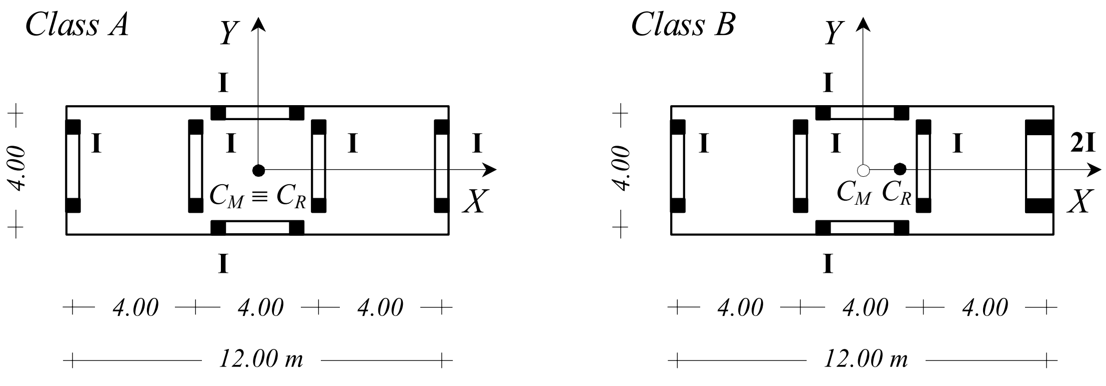


Figure 4. Reference systems for class A and class B irregular buildings

CORRELATION BETWEEN ERROR AND IRREGULARITY

When a simplified method of analysis is used (corrected static analysis or planar modal analysis modified by the normalised response of the single-storey asymmetric system) the results do not coincide to those of the spatial modal analysis. The maximum and minimum difference (non-conservative and conservative errors) may be plotted versus an irregularity parameter Θ, obtaining a couple of points for each analysed building (Fig. 5a) and a large number of points for the whole set of buildings (Fig. 5b). These points are enveloped with two curves corresponding to 95% fractile (Fig. 5c), which may be used form now on to discuss the correlation between error and irregularity, e.g. to determine the value of the irregularity parameter Θ corresponding to a given error (Fig. 5d).

In order to evaluate the different influence of the two parameters Θ_1 and Θ_2 previously defined, subsets of buildings characterised by the same value of Θ_2 have been selected and the corresponding errors have been plotted in function of Θ_1 (e.g. see Fig. 6a). Analogously, error has been plotted versus Θ_2 for subsets of buildings characterised by the same value of Θ_1 (e.g. see Fig. 6b). We observed that, in mean, the increase of error is more manifest in relation with Θ_1, less evident in function of Θ_2. For this reason as unique parameter of irregularity Θ_k has been considered, being:

$$\Theta_k = \Theta_1 + k\,\Theta_2 \qquad \text{with } k=0.5 \tag{7}$$

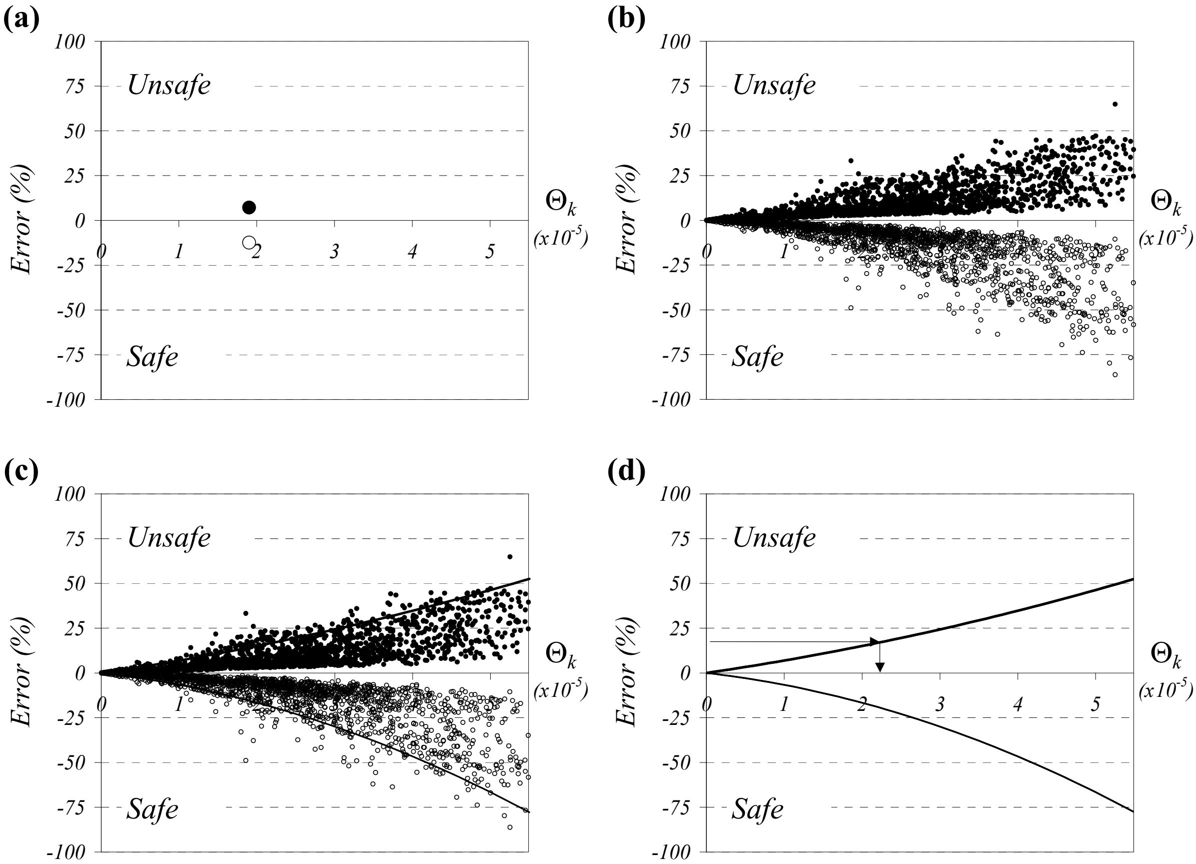


Figure 5. Correlation between error and irregularity parameter

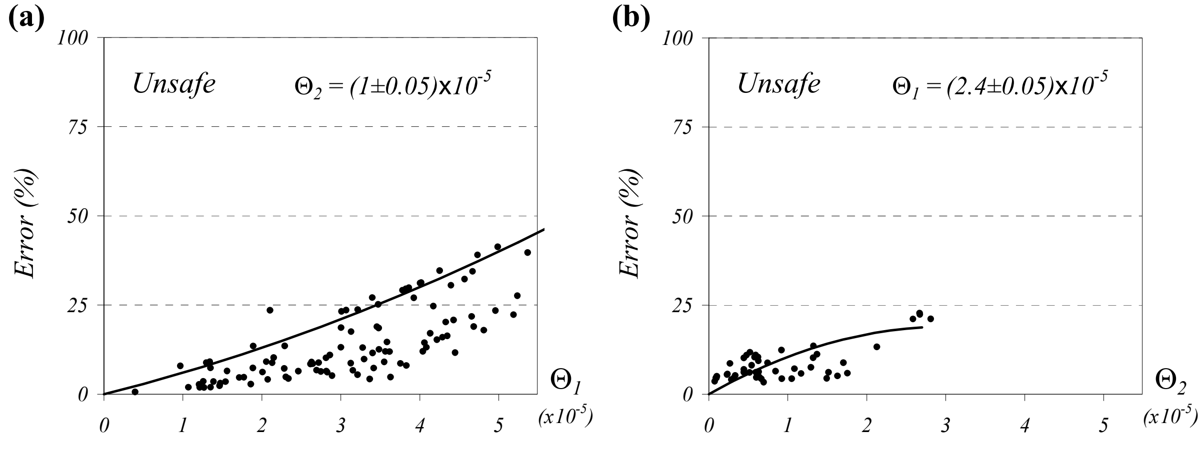


Figure 6. Correlation between error and parameter Θ_1 (a) and Θ_2 (b)

ANALYSIS OF THE RESULTS

The errors committed when evaluating the absolute displacements of class A buildings by means of corrected spatial static analysis (Fig. 7a) and planar modal analysis modified by the normalised response of the single-storey asymmetric system (Fig. 7b) show significant differences between the two approaches. The field of application of corrected static analysis is larger: e.g., if we accept an error up to 10% static analysis may be used for buildings having $\Theta_k \leq 7.0\times10^{-5}$ while corrected planar modal analysis is valid only up to $\Theta_k = 1.9\times10^{-5}$. The figure confirms also the already mentioned fact that regularly asymmetric structures (Θ_k=0) are conservatively analysed by static analysis (results up to 6% safe), while corrected planar modal analysis gives exact results (error=0) for these schemes. Once again we have to remember that the comparison has been carried on using the same base shear in the two approaches.

The errors committed when evaluating relative displacements (Fig. 8), which are proportional to internal actions because all the buildings are composed by shear-type frames, are larger but confirm the different range of application of the two approaches.

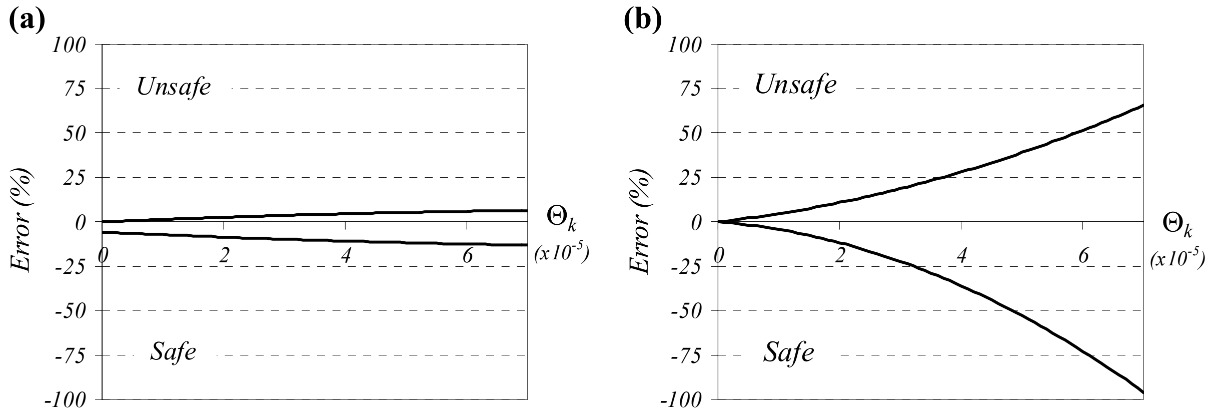


Figure 7. Error committed in the evaluation of absolute displacements: class A buildings analysed by means of corrected spatial static analysis (a) and corrected planar modal analysis (b)

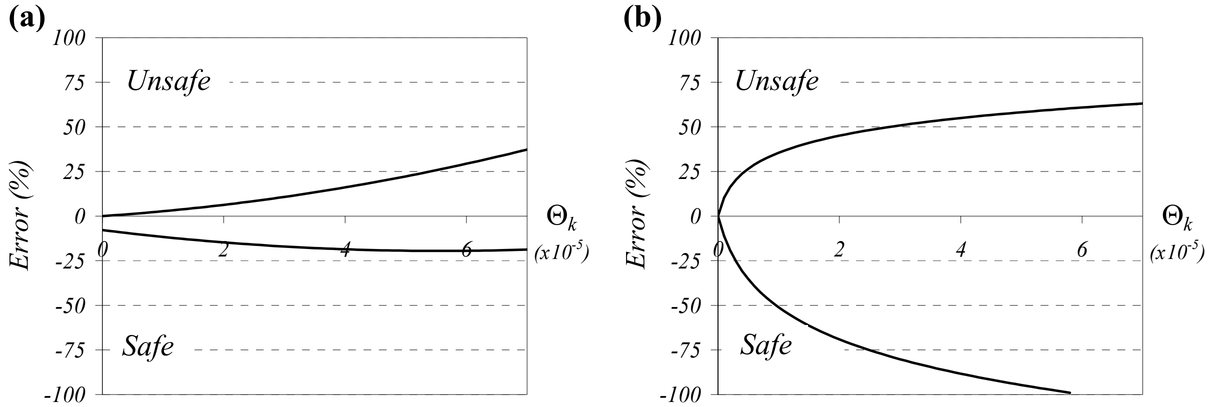


Figure 8. Error committed in the evaluation of relative displacements: class A buildings analysed by means of corrected spatial static analysis (a) and corrected planar modal analysis (b)

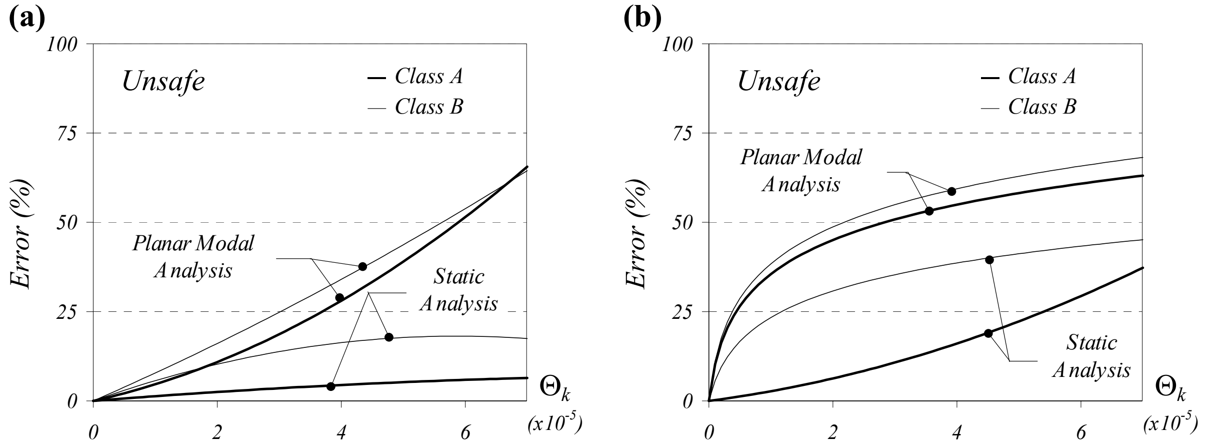


Figure 9. Comparison between errors committed for buildings of class A and B in the evaluation of absolute displacements (a) and relative displacements (b)

Finally, the comparison between the results obtained for class A and class B buildings (Fig. 9) confirms once again the differences between the two methods of analysis, but at the same time shows that the error committed using corrected static analysis is more sensitive to the geometry of the scheme. Further investigation should thus be carried on about the influence of the base model; in particular, an important aspect to analyse seems to be the effect of the structural eccentricity of the base model.

CONCLUSIONS

The paper demonstrates that it is possible to find a correlation between a parameter of irregularity and the maximum error connected to the application of approximate methods of analysis in the evaluation of dynamic response of non regularly asymmetric structure. This result has a quantitative validity based on the analysis of a large set of buildings with random variation of stiffness. Furthermore, the proposed parameter is quite simple to be evaluated also in the ordinary professional practice. Basing on these results, it will be possible to seismic codes to define clear limits to the applicability of simplified approaches, related to the maximum acceptable error and to the level of irregularity of

the structure. The performed analyses show that the field of application of the corrected static analysis is much larger than that of the planar modal analysis of a balanced system, corrected with the normalised displacements of the corresponding single-storey system. The field of application of corrected static analysis is slightly different in function of the typology of the buildings; for this reason a deeper analysis on the influence of structural eccentricity has to be done to generalise the results of this study.

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