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A new approach to the problem of in-plan regularity in seismic design of buildings

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ABSTRACT: The dynamic behaviour of buildings under seismic load is strongly affected by structural regularity. Aim of this paper is to examine the influence of in-plan irregularity on the elastic behaviour of buildings. The main parameters which affect the torsional response are pointed out. Additional eccentricities of horizontal actions are provided in order to obtain the equivalence of static or plane-modal analysis to spatial modal analysis. These values, together with the related procedure, are compared to the EC8 provisions.

1 INTRODUCTION

Regularity is a general concept which concerns many different aspects of the seismic behaviour of a structure. Three main sets of problems, with their respective subsets, may be pointed out:

- *inelastic behaviour*: the distribution of strength and local ductility of structural elements and the presence of non-structural elements, like partition or in-fill walls, influence the global ductility of a structure and the reducing coefficient for design actions (behaviour factor q in EC8)
- *elastic behaviour*: many aspects must be examined to select a proper elastic model:
  - type of analysis (static or modal)
  - model for the evaluation of design actions, both for static and modal analysis (plane or spatial)
  - model for the evaluation of internal actions and stresses in structural elements (plane or spatial)
  - model for the horizontal diaphragms (flexible or rigid)
  - model for non-structural elements (if necessary)
- *action transfer*: discontinuity of structural elements, sharp reduction of sections, re-entrant corners in the floor diaphragm influence the force transfer and require specific checks and careful detailing.

The classical subdivision between vertical and in-plan structural irregularities should be pursued keeping in mind the above mentioned scheme. Such approach is followed by SEAOC Provisions (1990) which try to connect each case of geometrical irregularity to its effects. As an example, the presence of re-entrant corners is related to the local increase of stress and specific provisions are given in order to check both the diaphragms and the connection of diaphragms to the vertical elements. On the contrary, the first draft of EC8 (1988) defined "regular" a building which fulfilled a set of geometric conditions both in the vertical and plan configuration; the behaviour factor and the model of elastic analysis, i.e. the inelastic and the elastic behaviour, were thus connected to the same global definition. A significant improvement is given by its second draft (1993), which distinguishes the implication of regularity on structural model, method of analysis and value of behaviour factor. The approach remains nevertheless over-simplified. In-plan irregularity is assumed to have no influence on the behaviour factor, in spite of the many studies on this matter (Rutenberg et al. 1986, Sedarat and Bertero 1990). The term "plane model" is used indifferently to refer to the evaluation of design actions and to the evaluation of internal actions, ingenerating ambiguity. No reference is given to the problem of action transfer, even when geometrical characteristics connected to it are used to define regularity. Finally, the wider regularity criterion given in annex B substantially overpasses the previously defined criteria for regularity in plan, depriving them of their meaning.

A further research work and a stricter connection of it to the codification appears therefore necessary to help the improvement of Eurocode 8 during its ENV period. In this framework, this paper examines the influence of in-plan irregularity on the elastic behaviour of buildings. The main parameters which affect the torsional response are pointed out. Additional eccentricities of horizontal actions are provided in order to obtain the equivalence of static or plane-modal analysis to spatial-modal analysis. These values, together with the related procedure, are compared to the present EC8 provisions.

2 LATERAL AND TORSIONAL RESPONSE OF STRUCTURES

The most general and correct way to examine the elastic dynamic behaviour of a structure is to perform a spatial modal analysis. It must be noted that, while plane modal analyses commonly use the square-root-of-sum-of-squares (SRSS) rule for modal combination, in spatial analyses more adequate criteria, like the complete quadratic combination (CQC), are often necessary. It is in fact easier to obtain closely spaced natural frequencies, just because the system is three-dimensional.

Nowadays a number of computer programs allow every structural designer to solve even the most cumbersome problem, making thus possible a wide use of spatial modal analysis. Nevertheless some important aspects must be kept in mind. First of all, the base shear provided by modal analysis (MA) is usually less (about 10% to 30%) than the one given by static analysis (SA). This last one, even if less accurate, is therefore very often safer than MA. The approach commonly suggested by many seismic codes, i.e. to use SA for the regular structures and MA for the irregular ones, gives additional conservatism just to the buildings which present the best seismic behaviour (Fajfar et al. 1988). For this reason Canadian code (1985) prescribes to increase MA results to obtain a value of shear base not less then 90% of the SA one, while SEAOC Provisions ask to increase it up to 100% in the case of irregular buildings. Secondly, the loss of sign of values in modal combination may give rise to uncertainties, e.g. when combining two different internal actions, as in coupled bending moment and axial force checks, or when analysing the trend of bending moment along a member, which influences the stability check of steel columns (Calderoni et al. 1991). Finally, a MA program appears to many engineers like a black-box which transforms data into results in an unpredictable way, to which one must believe with an act of faith. Simpler procedures, like SA, are easier to understand and to control and make thus possible to discover data errors by the analysis of results.

A lot of research has been up to now carried out on the elastic dynamic response of buildings. The basic model used is the single-storey system, i.e. an idealized one-storey structure consisting of a rigid floor supported on inextensible columns. The parameters which rule the lateral and torsional coupling are the radius of gyration of mass and stiffness, r_m and r_k respectively, and the eccentricity e between the centers of mass and stiffness. The three degrees of freedom of the system examined by Kan and Chopra (1977) are the displacements u and v of the centre of mass along the horizontal axes x and y and the rotation θ of the floor about the vertical axis z. Both shear in the direction of ground motion and torque result independent of the transverse (orthogonal to seismic action) lateral stiffness of the system; torque generally decreases as the eccentricity along the direction of ground motion increases. Tso and Dempsey (1980) considered therefore simpler but equally effective to analyse a two-degree-of-freedom model which neglects the transversal motion of the system. A general conclusion is that lateral and torsional motions are strongly coupled when the eccentricity e is large, but also when the centres of mass and stiffness are essentially coincident if r_k is close to r_m. In this last case the SRSS modal combination grossly exaggerate the torsional response. D'Andria and Ramasco (1980) and De Stefano et al. (1987) evaluated for the same model the equivalent eccentricity e^*, i.e. the distance from the centre of stiffness at which the uncoupled shear has to be applied to obtain a deformed shape equivalent to the modal envelope. The use of a single eccentricity to approximate a non linear envelope led to the conclusion that the equivalent static analysis should be used only for torsionally rigid systems ($r_k > r_m$); in the case of torsionally flexible schemes the modal envelope is strongly non linear and the equivalent analysis might be applied only if e is small, using values of e^* smaller than e. In the following sections the use of two eccentricities e_f and e_s is proposed, in order to get with these corrections the same values of the spatial modal analysis both at the flexible and the stiff side of the structure.

3 DYNAMIC RESPONSE OF A SINGLE-STOREY SYSTEM

The equations of motion for a two-degree-of-freedom model (fig. 1) can be written as

$$\begin{aligned} &\ddot{V} + \omega_v^2 V + E\,\omega_v^2\,\theta = -\ddot{V}_g \\ &R_m^2\,\ddot{\theta} + E\,\omega_v^2\,V + (R_k^2 + E^2)\,\omega_v^2\,\theta = 0 \end{aligned} \tag{1}$$

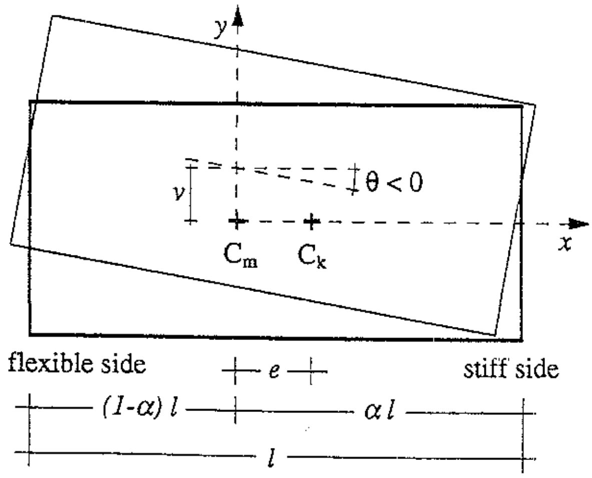

Fig. 1 - two-degree-of-freedom system

where

$$\omega_v = \sqrt{\frac{k}{m}} \qquad V = \frac{v}{l} \qquad E = \frac{e}{l} \qquad R_m = \frac{r_m}{l} \qquad R_k = \frac{r_k}{l}$$

and ω_v is the uncoupled lateral frequency, k is the translational stiffness, m is the mass of the floor. The natural frequencies ω_j (j=1,2) of the system are given by

$$\Omega_j = \frac{R_m^2 + R_k^2 + E^2 \mp \sqrt{(R_m^2 + R_k^2 + E^2)^2 - 4 R_m^2 R_k^2}}{2 R_m^2} \qquad (2)$$

where $\Omega_j = \dfrac{\omega_j}{\omega_v}$

The associated mode shapes $\overline{V}_j, \overline{\theta}_j$ and the partecipation factor G_j can be written as

$$\overline{\theta}_j = \frac{\Omega_j^2 - 1}{E} \overline{V}_j \qquad G_j = \frac{\overline{V}_j}{\overline{V}_j^2 + R_m^2 \overline{\theta}_j^2} \qquad (3)$$

For a given spectral acceleration s_a, the modal forces ($\mathcal{F}_j$, $\mathcal{M}_j$) and displacements for the structure are provided, in a non-dimensional form, by

$$F_j = S_a G_j \overline{V}_j \qquad M_j = S_a G_j R_m^2 \overline{\theta}_j \qquad (4)$$

$$V_j = S_a G_j \frac{\overline{V}_j}{\Omega_j^2} \qquad \theta_j = S_a G_j R_m^2 \frac{\overline{\theta}_j}{\Omega_j^2} \qquad (5)$$

beeing

$$F_j = \frac{\mathcal{F}_j}{k l} \qquad M_j = \frac{\mathcal{M}_j}{k l^2} \qquad S_a = \frac{s_a m}{k l}$$

The displacements V_s and V_f of the stiff and flexible side of the scheme are finally obtained, using CQC criterion, by the values for mode j

$$V_{sj} = V_j + \alpha\, \theta_j \qquad V_{fj} = V_j + (\alpha - 1)\, \theta_j \qquad (6)$$

4 CORRECTION OF STATIC OR MODAL PLANE ANALYSIS

For a single storey system, both the static and the plane-modal analysis give a design force $\mathcal{F} = s_a m$, or, in non-dimensional way, $F = S_a$, which must be applied to the centre of mass giving the displacement V_F of C_m and the rotation θ_F

$$V_F = F\left(1 + \frac{E^2}{R_k^2}\right) \qquad \theta_F = -F \frac{E}{R_k^2} \qquad (7)$$

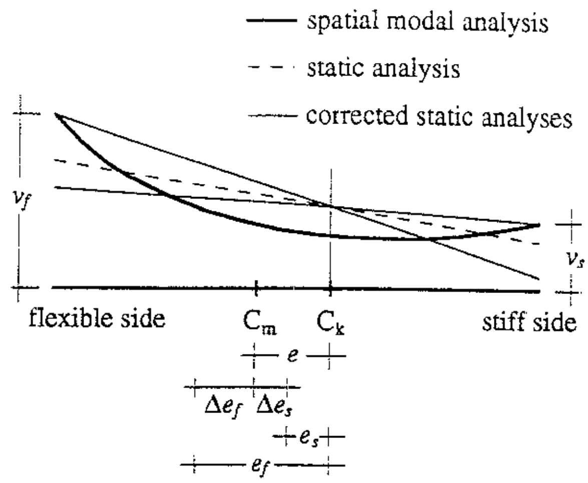

Fig. 2 - static and spatial-modal deformed shapes

The consequent displacements of the end of the building usually differ from the spatial-modal values. To equalize static to spatial-modal displacement at the flexible side, the force F should be applied with an eccentricity $E_f = e_f / l$ from C_k different from E (fig. 2), i.e. with an additional eccentricity $\Delta E_f = M/F$ from C_m which may be evaluated knowing the effect of M

$$V_M = -M \frac{E}{R_k^2} \qquad \theta_M = M \frac{1}{R_k^2} \qquad (8)$$

The additional eccentricity ΔE_s necessary to equalize stiff side values may be evaluated in the same way.

A wide numerical analysis has been performed, assuming five values of R_m (from 0.30 to 0.70). For each one of them R_k varies in the range $R_m - 0.20$ to $R_m + 0.20$ and E in the range 0 to 0.20, providing two surfaces in the space R_k, E, ΔE_f (ΔE_s) which may be represented in plane schemes by equally spaced level curves, i.e. sets of points in which ΔE assumes the same value (fig. 3). Only positive values of ΔE are indicated, i.e. only the cases in which the static displacement has to be increased to get the spatial-modal value.

At the flexible side, the static analysis greatly underestimates displacement (up to 40%) when E is small and R_k is close to or slightly greater than R_m. The maximum additional eccentricity ΔE_f is about 0.06 (at R_k / R_m=1.15 and E=0.07) when R_m=0.30 and about 0.08 when R_m=0.70. A smaller correction is necessary when torsional stiffness or proper eccentricity are high; in this second case the effect itself of the additional eccentricity is less relevant, because the per cent difference between static and spatial-modal values is small. Torsionally flexible schemes require very small ΔE_f and the static analysis may be even safer than the spatial-modal one if E is small.

The situation at the stiff side is completely opposite. While torsionally rigid structures require no correction, the value of ΔE_s increases with E and as far as R_k decreases, reaching values many times greater than ΔE_f.

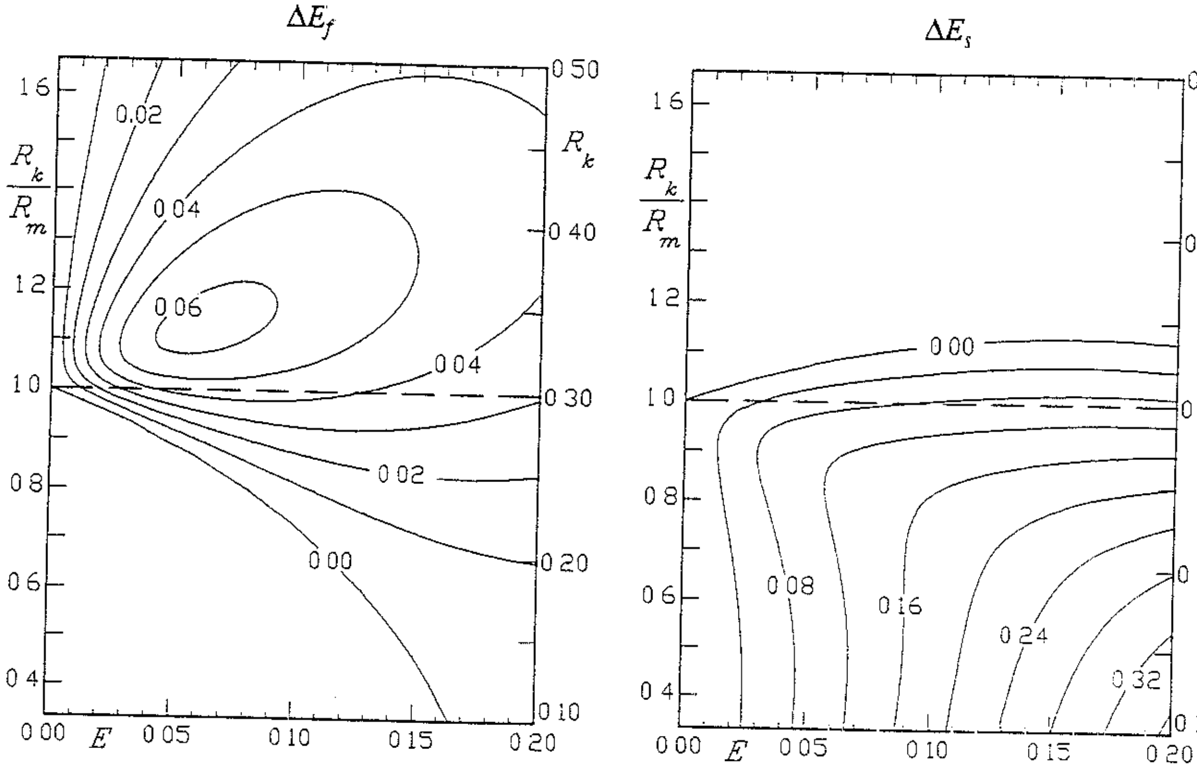

Fig. 3 - Additional eccentricities ΔE_f and ΔE_s as a function of E and R_k/R_m, in the case R_m=0.30

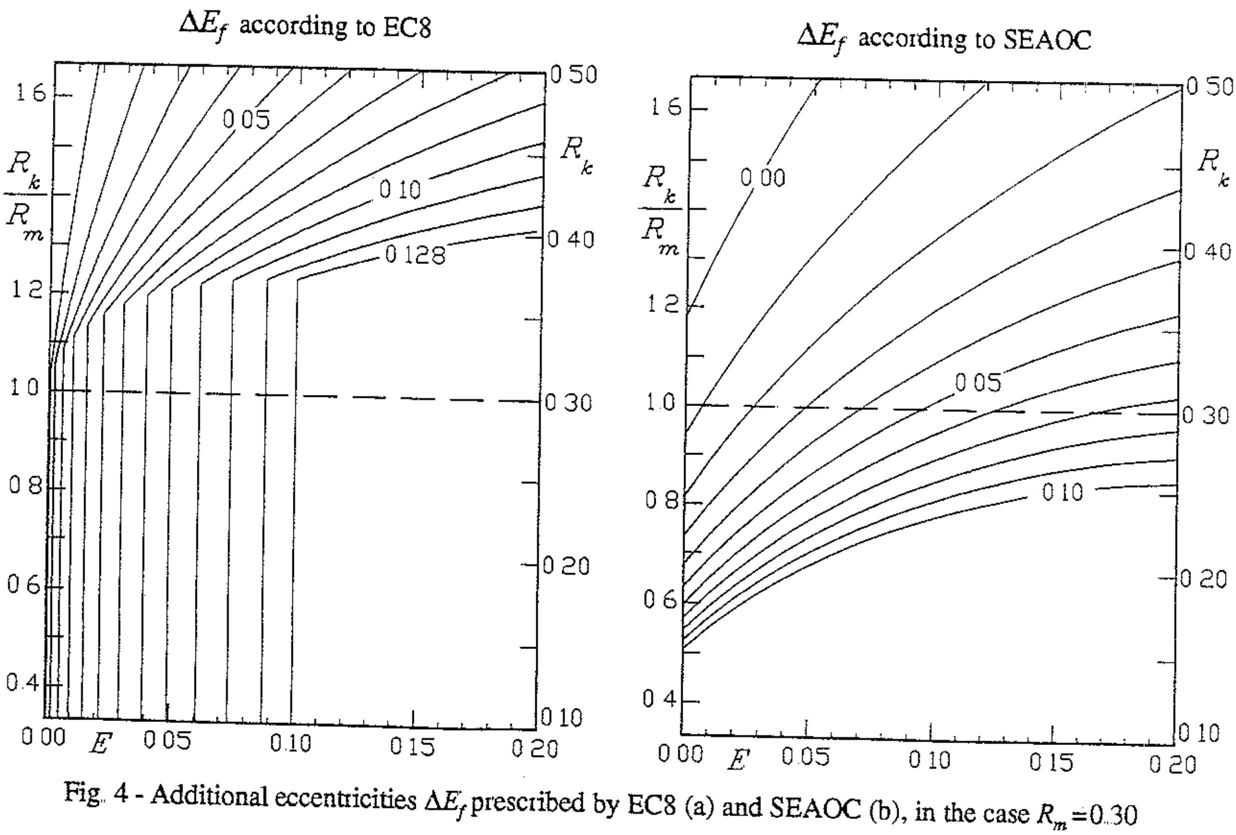

Fig. 4 - Additional eccentricities ΔE_f prescribed by EC8 (a) and SEAOC (b), in the case R_m=0.30

It must be noted that ΔE_s may be even greater than E, i.e. the force shall be shifted to a position opposed to C_m in respect to C_k. In these cases not even the wise suggestion given by previous SEAOC Provisions (to design stiff side under actions not less than the ones given by a purely translational scheme) is safe.

5 COMPARISON WITH EC8 AND SEAOC PROVISIONS

While in the first draft of EC8 (1988) the conditions for regularity were so strict as to make necessary a spatial-modal analysis for the majority of buildings, in its second draft (1993) static or plane-modal analysis are allowed if both the centre mass and the centre of stiffness at each floor are approximately located along vertical lines, condition usually met by multi-storeys buildings. In this case the torsional effect may be taken into account by analysing a single static load condition, which considers the accidental eccentricity of storey mass e_1=0.05 l plus an additional eccentricity e_2, function of the basic parameters e, r_m and r_k. The additional eccentricity increases the displacement of the flexible side, thus corresponding to the above Δe_f. Its non-dimensional values are plotted in fig. 4a for the case R_m=0.30. No further correction is prescribed for the values at the stiff side.

SEAOC (1990) prescribes to consider in any case the accidental eccentricity, which is again 0.05 l. A building is defined torsionally irregular if the maximum storey drift δ_{max}, computed including accidental torsion, is more than 1.2 times δ_{avg}, average of the storey drifts at the two ends of the structure. In this case, the accidental torsion must be increased by an amplification factor A_x

$$A_x = \left[\frac{\delta_{max}}{1.2\,\delta_{avg}}\right]^2 \tag{9}$$

which corresponds to consider the additional eccentricity 0.05 l (A_x−1), plotted in non-dimensional way in fig. 4b. Once again only the effect of irregularity at the flexible side of the structure is in this way considered.

The differences between the actual values of additional eccentricity Δe_f and the ones proposed by these codes are evident. EC8 gives a close approximation only when $R_k > R_m + 2E$, but it overestimates Δe_f (more than two times) in the case of torsionally stiff structures with high proper eccentricity and prescribes high values of it in the case of torsionally flexible schemes, when no additional eccentricity is necessary. Similar differences may be found in SEAOC values, which moreover are smaller than the ones given by EC8 and thus often unsafe for torsionally stiff structures. It must be furthermore underlined that no particular prominence is given by codes to the ratio R_k/R_m, which on the contrary appears to be a basic point of torsional behaviour, and that the lack of provisions for the stiff side may lead to rough approximations which may have repercussions also on the inelastic behaviour.

6 PROPOSED DESIGN PROCEDURE

Results as safe as those given by spatial-modal analysis, together with a thorough comprehension of the lateral-torsional behaviour, may be obtained in a simple way by means of the following procedure:

- the static analysis of a spatial model of the structure, subjected to two load conditions (design forces F and moment M=$F\,E_1$ corresponding to accidental eccentricity, respectively), is performed
- R_m is evaluated at every storey by means of geometrical considerations or simplified assumptions
- R_k and E are evaluated at every storey by the results of the two static analyses
- if E and R_k/R_m are approximately the same at every level, the additional eccentricities ΔE_f and ΔE_s are evaluated by the graphic results of the present paper or by the equations here presented, using an average value of the above parameters; otherwise a spatial-modal analysis is necessary
- the results of the second load condition are proportionally increased to include the effect of the additional eccentricities and combined to those of the first load condition.

The evaluation of R_k and E by the results of the static analyses is extremely easy. By the equations (7) and (8) we obtain, for a one storey system

$$R_k = E_1\sqrt{\frac{V_F}{\theta_M} - \left(\frac{\theta_F}{\theta_M}\right)^2} \qquad E = -E_1\,\frac{\theta_F}{\theta_M} \tag{10}$$

The above equations, even if not rigorous, gives substantially exact values also for multi-storey schemes.

The additional eccentricities ΔE may be obtained also by the curves of fig. 5, in which the ratio ρ of the minimum over the maximum end displacement given by the first load condition is used as co-ordinate instead of the eccentricity E.

The proposed design procedure has been tested by applying it to five r.c. buildings with different degree of in-plan irregularity. Both these results and those given by the static analysis with EC8 eccentricities have been compared to the ones provided by the spatial-modal analysis, performed using SAP 90. The ratio of the base shear given by plane-static and plane-modal analysis has also been evaluated and accounted in the comparison. The results of the proposed procedure are always close to the exact ones, with a maximum difference which is less than ±6%. On the contrary, EC8 in some cases gives differences of about +30% or −12%.

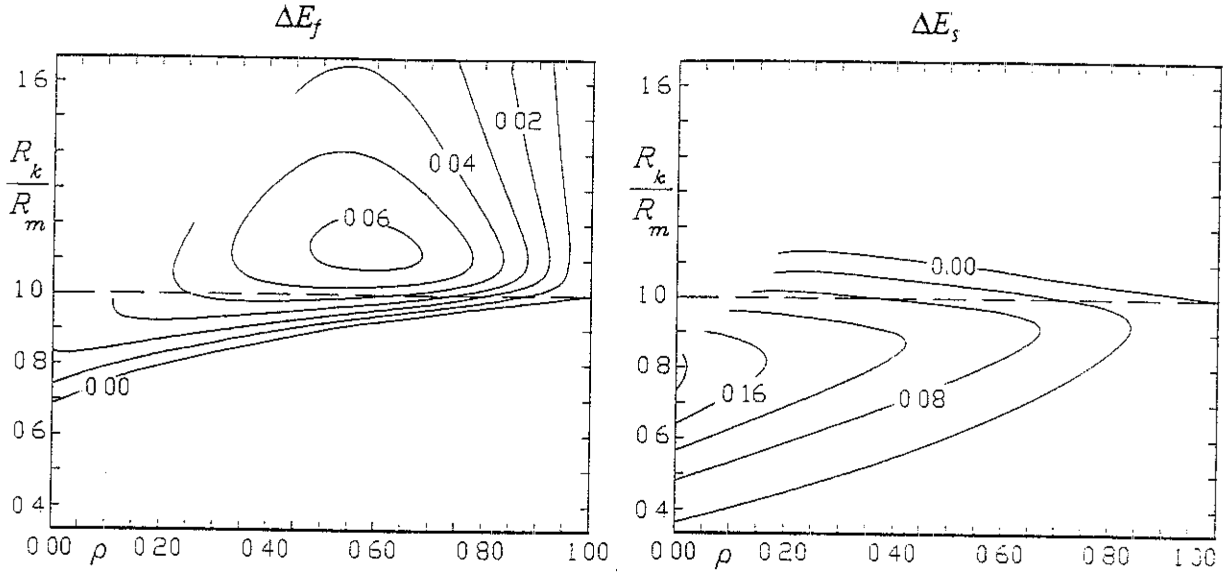

Fig. 5 - Additional eccentricities ΔE_f and ΔE_s as a function of ρ and R_k/R_m, in the case R_m=0.30

7 CONCLUSIONS

The theoretical study points out the limits of the EC8 approach to structural regularity, which is not well referred to the three main aspects of the problem (inelastic behaviour, elastic behaviour and action transfer). In particular, with reference to the effect of in-plan irregularity on the dynamic elastic response of buildings, the additional eccentricity prescribed by this code appears to be in contrast with the results here referred. EC8 in fact leads to over-estimate torsional effects in the case of torsionally rigid structures and completely neglects corrections for the stiff side in the case of torsionally flexible schemes.

The design procedure here proposed easily and clearly individuates the basic parameters which govern the dynamic elastic spatial problem and bases on them the evaluation of additional eccentricities. The in-plan regularity is thus connected to the actual behaviour of the structure, i.e. to its response to given static actions, without limiting the judgement to morphological considerations only. The analysis of several multi-storey buildings confirmed the weakness of the EC8 provisions and the effectiveness of the proposed procedure.

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