



## **EVALUATION OF SECOND ORDER EFFECTS ON THE SEISMIC RESPONSE OF VERTICALLY IRREGULAR RC FRAMED STRUCTURES**

**MARIO DE STEFANO**

Department of Constructions, University of Florence  
Piazza Brunelleschi, 6 – 50121 Firenze, ITALY

**EDOARDO MICHELE MARINO**

Department of Civil and Environmental Engineering, University of Catania  
V.le Andrea Doria, 6 – 95125 Catania, ITALY

**STEFANIA VITI**

Department of Constructions, University of Florence  
Piazza Brunelleschi, 6 – 50121 Firenze, ITALY

### **ABSTRACT**

This paper deals with the evaluation of P- $\Delta$  effects on the response of RC framed structures irregular in elevation subjected to severe seismic actions. In order to investigate the sensitivity of such structures to P- $\Delta$  effects, a set of RC plane frames have been designed according to EC8 provisions for high ductility structures and have been subjected to ground motions having increasing PGA. Their seismic response has been evaluated by nonlinear dynamic analysis with and without P- $\Delta$  effects. The adopted numerical model reproduces all the most important mechanical phenomena affecting their nonlinear response including degradation in strength and stiffness, and pinching. The deformation levels obtained from the numerical analyses have been represented in terms of fragility curves which are drawn with reference to the limit states stipulated by the US Federal Emergency Management Agency (FEMA 356, 2000) for RC framed structures. Comparison among fragility curves evaluated with and without P- $\Delta$  effects has pointed out a remarkable influence of such effects in defining structural performance and, then, safety levels related to the assumed limit states.

### **INTRODUCTION**

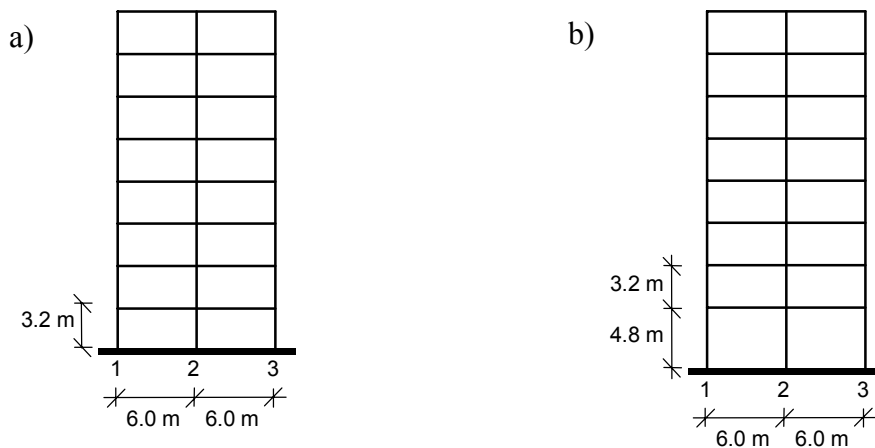
Previous investigations developed by some of the authors [1], [2] showed that the response of RC structures which experience large inelastic deformations during the earthquake may be significantly affected by P- $\Delta$  effects. Such phenomenon is amplified by the hysteretic degradation of the material, which is significant in RC structural members. In particular, the analyses carried out in the aforementioned study on a set of RC plane regular frames, designed according to Eurocode 8 (EC8) [3], and belonging to high ductility class structures, demonstrated that the increase in interstorey drifts due to P- $\Delta$  effects is larger than that expected by

EC8. Such an issue should be even more important for structures which are non-regular in elevation. In fact, such structures tend to develop a story collapse mechanism [4] and, therefore, experience larger plastic deformation with respect to the regular frames.

In consideration of the issues stated above, this paper deals with the evaluation of P- $\Delta$  effects on response of RC framed structures non-regular in elevation subjected to seismic actions. In order to investigate the sensitivity of such structures to P- $\Delta$  effects, a set of RC plane frames non-regular in elevation has been designed according to EC8 provisions for high ductility structures. Then, frames have been subjected to ground motions having increasing PGA. Their seismic response has been evaluated by nonlinear dynamic analysis with and without P- $\Delta$  effects and compared with that of a regular frame. The modelling used for the frames reproduces all the most important mechanical phenomena affecting their nonlinear response including degradation in strength and stiffness, and pinching. Deformation levels obtained from the numerical analyses have been represented in terms of fragility curves which are drawn with reference to the limit states stipulated by FEMA 356 [5] for RC framed structures. Comparison among fragility curves evaluated with and without P- $\Delta$  effects has evidenced a remarkable influence of such effects in defining structural performance and, then, on the safety levels related to the assumed limit states.

## ANALYSED STRUCTURES

Four eight story RC plane frames, whose geometrical schemes and cross-sections dimensions are shown in Figure 1 and Table 1, were analysed. The first frame is a regular frame and it is assumed as reference. The others, which present vertical irregularities, were obtained modifying the regular reference frame. These three frames are representative of frames with mass irregularity, strength irregularity and stiffness irregularity, respectively.



**Figure 1:** Geometrical schemes of the analysed frames: (a) regular frame, frame with strength irregularity and frame with mass irregularity, (b) frame with stiffness irregularity.

**Table 1:** Cross-sections dimensions (B x H).

Storey	Beams	Col. 1 and 3	Col. 2
8-th	30 x 45	30 x 30	40 x 40
7-th	30 x 50	30 x 30	40 x 40
6-th	30 x 55	30 x 35	40 x 45
5-th	30 x 60	30 x 40	40 x 45
4-th	30 x 65	30 x 45	40 x 50
3-rd	30 x 65	30 x 50	40 x 50
2-nd	30 x 70	30 x 55	40 x 55
1-st	30 x 70	30 x 55	40 x 55

All the frames were designed according to the Eurocodes provisions for high ductility RC structures. The characteristic value of the compressive cylinder strength  $f_{ck}$  of the concrete and the yield stress  $f_{yk}$  of the reinforcing steel were set at 25 MPa and 450 MPa, respectively. The elastic spectrum stipulated in EC8 for soil type C and high seismicity zone (PGA equal to 0.35g) was adopted. The design seismic force, evaluated by a behaviour factor  $q$  of 5.85, was distributed along the height according to an inverted triangular distribution. Two load conditions were considered for the evaluation of the design internal forces:

- total dead ( $G_k$ ) and live ( $Q_k$ ) loads, increased by the coefficients  $\gamma_g$  and  $\gamma_q$  stipulated for the ultimate limit state approach ( $\gamma_g G_k + \gamma_q Q_k$ );
- total dead load, a rate  $\psi_2$  of live load and seismic action ( $G_k + \psi_2 Q_k + E$ ).

The total dead and live loads were evaluated supposing that the frames are 5 meters spaced. The coefficients  $\gamma_g$  and  $\gamma_q$  were set at 1.4 and 1.5, respectively, while  $\psi_2$  is the combination coefficient for the quasi-permanent value of live loads and was fixed depending on the type occupancy of the floors. The P- $\Delta$  effect was not taken into account in design.

The frame with mass irregularity was obtained modifying the type occupancy of the top floor of the regular reference frame, so that a larger value of the live load was considered for this floor. As a consequence, the top floor mass is larger by about 70% than mass at the other floors. In order to obtain the frame with strength irregularity, the design bending moments of the beams of the regular reference frame were increased by 20% at every floor except the first one. Because columns were designed according to the capacity design criterion, this results in an increase of 20% of design bending moments also for the columns end cross-sections except those of the first order columns and the bottom cross-sections of the second order columns. As a consequence, the first storey is weaker than the others. The frame with stiffness irregularity was obtained modifying the geometrical scheme of the regular reference frame. In particular the inter-storey height of the first storey was increased by 50% (Figure 1b), so that the lateral stiffness of the first storey decreased at about one third that of the regular reference frame.

The stability coefficients  $\theta_i$  and the fundamental periods of the designed frames are summarized in Table 2.

**Table 2:** Stability coefficients and fundamental periods of the analysed structures.

Storey	Regular Reference Frame	Frame with vertical irregularity		
		Mass irregularity	Strength irregularity	Stiffness irregularity
8-th	0.0609	0.0913	0.0609	0.0622
7-th	0.0992	0.1297	0.0992	0.1005
6-th	0.1055	0.1278	0.1055	0.1069
5-th	0.1121	0.1300	0.1121	0.1135
4-th	0.1109	0.1252	0.1109	0.1126
3-rd	0.1150	0.1274	0.1150	0.1175
2-nd	0.1074	0.1173	0.1074	0.1181
1-st	0.0823	0.0889	0.0823	0.1519
$T_1$ (sec)	1.15	1.15	1.15	1.27

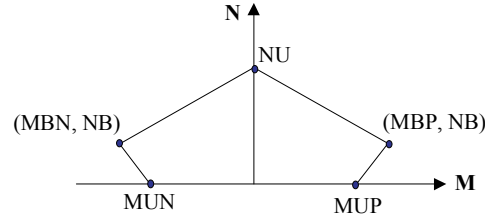
## NUMERICAL ANALYSES

The analysed frames have been subjected to ground motions having increasing PGA. The considered values of PGA range from a minimum of 0.25g to a maximum of 0.65g with a step of 0.10g. Their seismic response has been evaluated by nonlinear dynamic analysis. Despite of its computational effort, nonlinear dynamic analysis is the most effective tool to investigate the inelastic response of structure under seismic excitation. Nevertheless, effectiveness of the analysis and reliability of the results are strongly influenced by the quality of the representation of each phenomenon characterising the inelastic behaviour of the structural members. So the modelling used for the frame reproduce all the most important mechanical phenomena affecting their nonlinear response including degradation in strength and stiffness, and pinching.

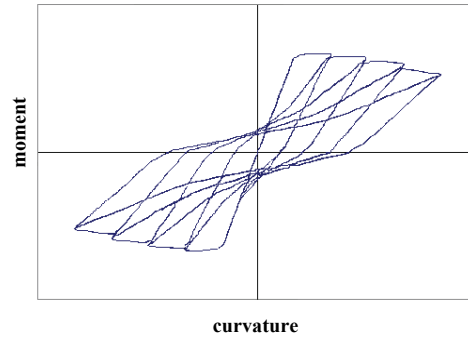
### Modelling

Dynamic analysis has been performed in this study by the IDARC2D program (Valles [6]). The inelastic behaviour of the structural members has been described through bi-linear (for beams) and three-linear (for columns) moment-curvature relationships, taking into account axial loads due to gravity loads. The parabolic-rectangular domain provided by EC2 [7] for the confined concrete and the elastic perfectly plastic relationship for the reinforcing steel were used. Figure 2 shows the obtained interaction domain for bending moments and axial loads.

A linear distribution of the inelastic deformation has been assumed at the critical regions of the elements, and two different evolutive-degrading hysteretic models (Sivalsen [8]) have been assumed for beams and columns. The obtained hysteretic model, shown in Figure 3, takes into account strength and stiffness degradation and pinching.



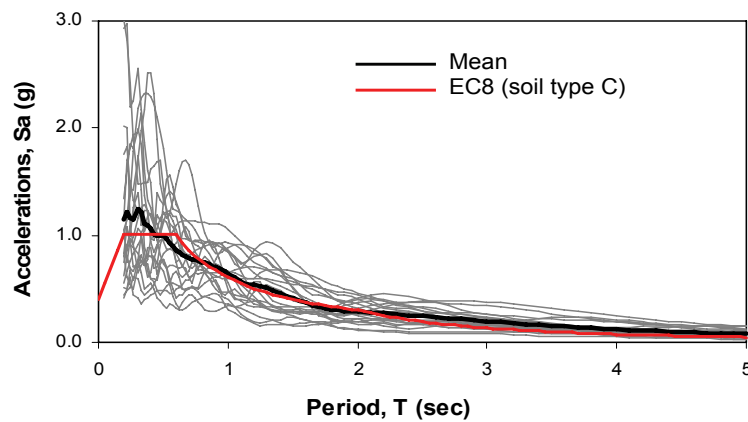
**Figure 2:** Axial load-bending moment domain.



**Figure 3.** Evolutive-degrading hysteretic model.

#### Seismic input

A suite of twenty ground motions [9] adopted in the FEMA/SAC project in the United States was used in this study. The suite of records represents seismic events having a probability of exceedance of 10 percent in 50 years in the Los Angeles area. The main characteristics of the adopted ground motions are listed in Table 3. Figure 4 shows the mean spectrum of the twenty ground motions, together with the EC8 spectrum used in design. Note that for periods larger than 0.75 sec, the mean 5% damped spectrum fits well the elastic spectrum provided by EC8. Since the fundamental periods of the analysed structures varies from 1.15 to 1.27 sec, it can be considered that the selected suite of ground motions represents well the EC8 seismic action for this case study.



**Figure 4:** Comparison between EC8 spectrum and mean spectrum of adopted ground motions.

**Table 3:** Main characteristics of the adopted ground motions.

EQ code	Description	Magnitude	Distance (km)	Scale Factor	Duration (sec)	PGA (g)
La01	fn Imperial Valley, 1940, El Centro	6.9	10.0	1.675	53.48	0.383
La02	fp Imperial Valley, 1940, El Centro	6.9	10.0	1.675	53.48	0.567
La03	fn Imperial Valley, 1979, Array #05	6.5	4.1	0.842	39.39	0.325
La04	fp Imperial Valley, 1979, Array #05	6.5	4.1	0.842	39.39	0.408
La05	fn Imperial Valley, 1979, Array #06	6.5	1.2	0.700	39.39	0.250
La06	fp Imperial Valley, 1979, Array #06	6.5	1.2	0.700	39.39	0.192
La07	fn Landers, 1992, Barstow	7.3	36.0	2.667	80.00	0.350
La08	fp Landers, 1992, Barstow	7.3	36.0	2.667	80.00	0.358
La09	fn Landers, 1992, Yermo	7.3	25.0	1.808	80.00	0.433
La10	fp Landers, 1992, Yermo	7.3	25.0	1.808	80.00	0.300
La11	fn Loma Prieta, 1989, Gilroy	7.0	12.0	1.492	40.00	0.558
La12	fp Loma Prieta, 1989, Gilroy	7.0	12.0	1.492	40.00	0.808
La13	fn Northridge, 1994, Newhall	6.7	6.7	0.858	60.00	0.567
La14	fp Northridge, 1994, Newhall	6.7	6.7	0.858	60.00	0.550
La15	fn Northridge, 1994, Rinaldi RS	6.7	7.5	0.658	15.95	0.442
La16	fp Northridge, 1994, Rinaldi RS	6.7	7.5	0.658	15.95	0.483
La17	fn Northridge, 1994, Sylmar	6.7	6.4	0.825	60.00	0.475
La18	fp Northridge, 1994, Sylmar	6.7	6.4	0.825	60.00	0.683
La19	fn North Palm Springs, 1986	6.0	6.7	2.475	60.00	0.850
La20	fp North Palm Springs, 1986	6.0	6.7	2.475	60.00	0.825

## RESULTS

The maximum inter-story drift was used as an index to assess the seismic performance of the analysed frames. For each frame (the regular frame and the frames with mass, strength and stiffness irregularities), it was evaluated twice, including and not including P- $\Delta$  effects in the analysis. Then, the increase of the seismic response due to the P- $\Delta$  effects was obtained comparing such two values.

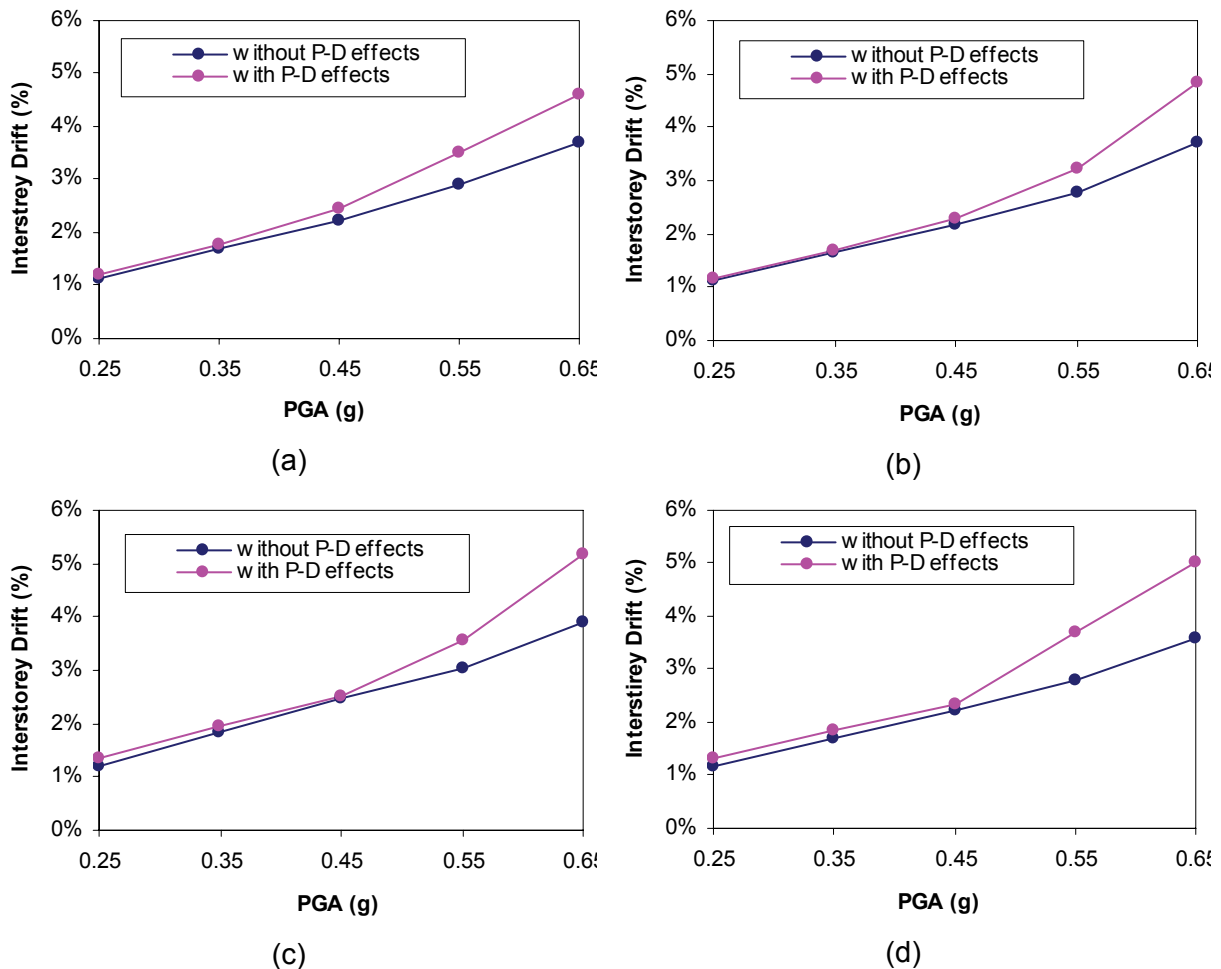
The obtained results were also represented in terms of fragility curves. They were drawn with reference to the performance levels defined in FEMA 356 and named Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP). Here, according to FEMA 356 provisions for RC frames, it is assumed that such performance levels are achieved when the maximum inter-story drift angle is equal to the values listed in Table 4.

**Table 4:** Limit values provided by FEMA for RC framed structures.

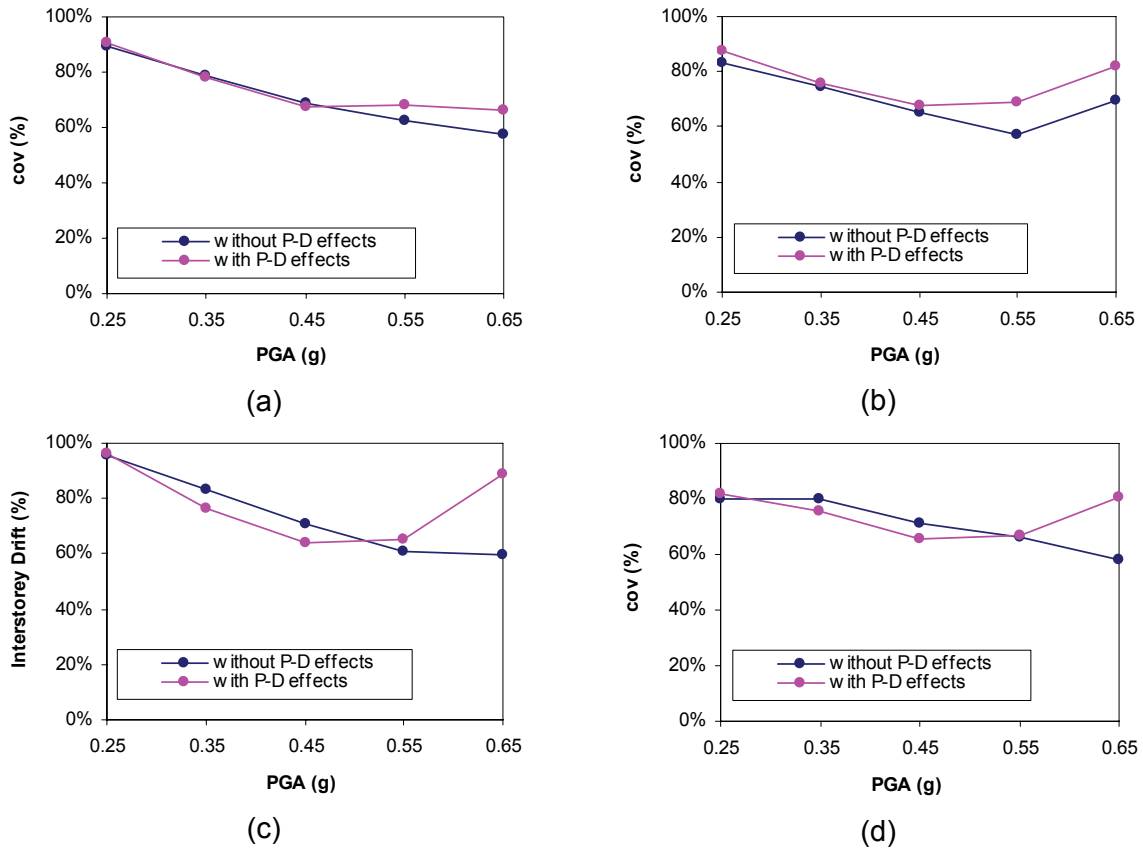
Performances levels	Inter-story drift
Immediate Occupancy	1 %
Life Safety	2 %
Collapse Prevention	4 %

### Inter-story Drifts

For each ground motion the maximum inter-story drift was determined. Then, the mean over the twenty values obtained for the twenty ground motions was calculated. This procedure was repeated for all the PGA values considered in the dynamic analyses (from 0.25g to 0.65g). The obtained results are summarized in Figure 5 for the four analysed frames. Here, the inter-story drift sustained by each frames and evaluated including and not including P- $\Delta$  effects in the analysis are plotted against the PGA. In Figure 6 the coefficient of variation (cov) found for each of the response domains is shown.



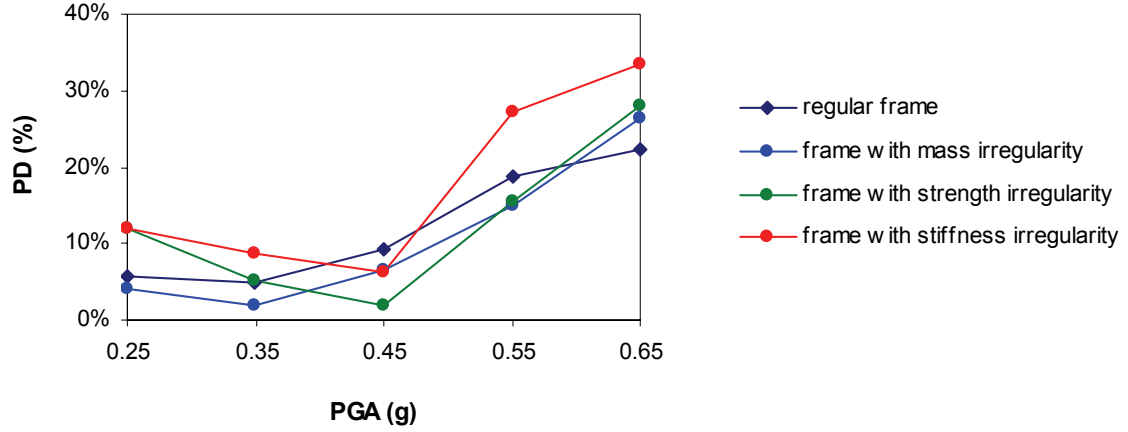
**Figure 5:** Mean values of the inter-storey drift: (a) regular frame, (b) frame with mass irregularity, (c) frame with strength irregularity and (d) frame with stiffness irregularity.



**Figure 6:** Cov of the inter-storey drift domains: (a) regular frame, (b) frame with mass irregularity, (c) frame with strength irregularity and (d) frame with stiffness irregularity.

The obtained results show that the P- $\Delta$  effects always increases the seismic response of the frames. The increase is negligible for PGA values smaller than 0.45g, while it becomes important for larger PGA values. Such phenomenon is observed for all the considered frames. The percent increase of the inter-story drift due to P- $\Delta$  effects is plotted in Figure 7 as a function of PGA. Figure 7 shows that the increase of the inter-story drift due to P- $\Delta$  effects achieves always remarkable values (close to 10% or even larger) for PGA larger than 0.45g. It also shows that in the regular frame P- $\Delta$  effects are smaller than those found in the irregular ones. For PGA = 0.65 g, while the increase in maximum interstory drift is about 22% for the regular frame, it is around 27% for the frames with strength and mass irregularities, and it achieves the value of 33.5% for the frame with stiffness irregularity.





**Figure 7:** Percent difference computed for inter-story drift.

### Fragility Domains

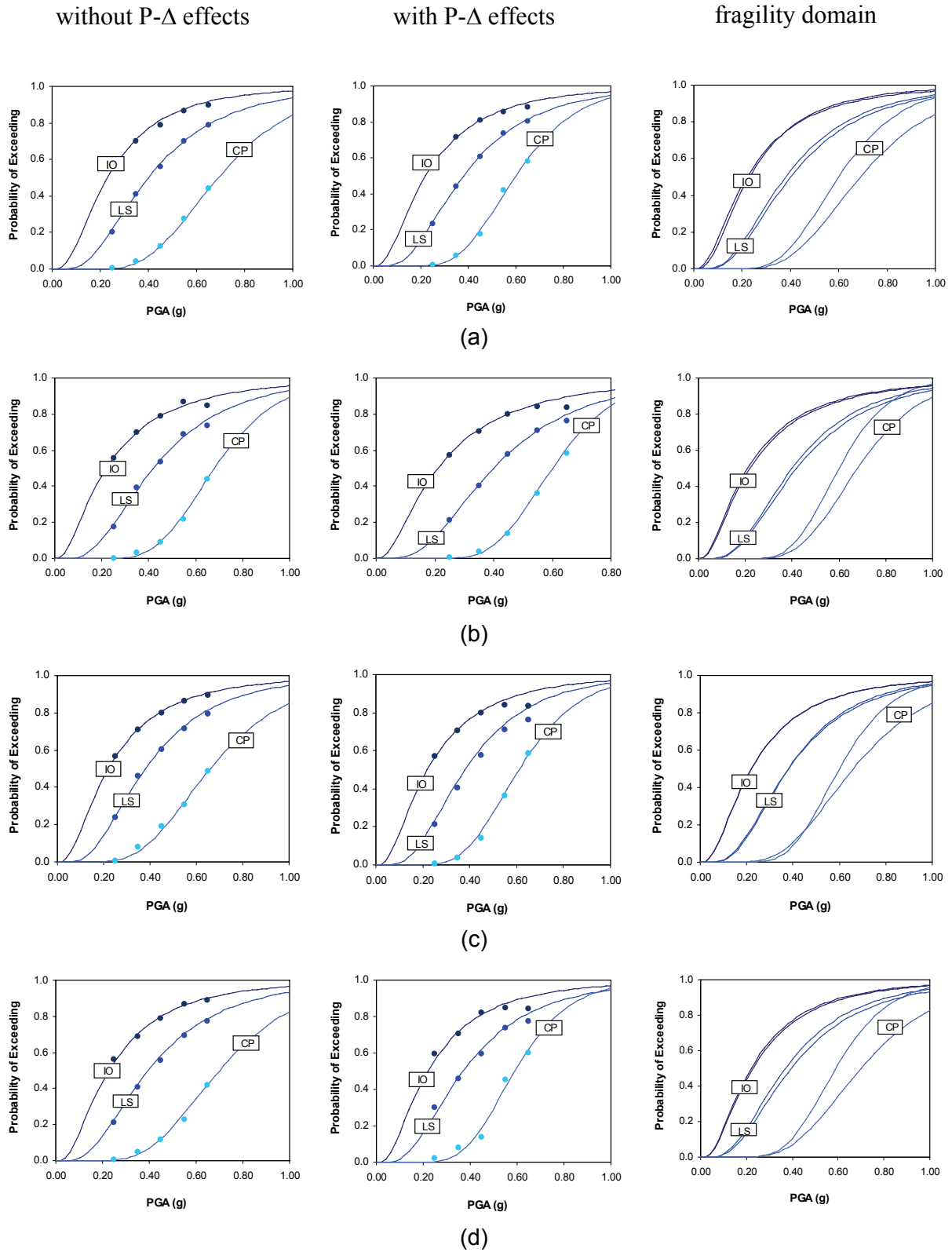
Fragility curves of the considered frames with and without P-Δ effects have been determined. A Gaussian statistical distribution over the domain of the response parameter was assumed. The mean value and the standard deviation evaluated from the sample of the twenty results for every PGA values was used. Each distribution has been compared with the assumed limit values, so obtaining the corresponding probability of exceedance them. For each performance level, therefore, a probability of exceedance, and therefore a point of the fragility curve, has been calculated for every PGA. Each point of the fragility curve represents the probability  $P$  of the response parameter ( $r$ ) of the frame to exceed the limit value (l.v.) corresponding to the assumed performance level under a given intensity of the ground motion (PGA), according to the following expression:

$$\text{Fragility} = P [r > \text{l.v.} \mid \text{PGA}] \quad (3)$$

The most adopted function to represent the fragility curves is the two-parameters lognormal distribution (Barron Corvera [10]), that can be determined when at least three points are known. In order to fit the curve the points have to belong to the intermediate part of the curve, that is for values not too close to 0 and 1. In this case, the available points are quite suitable to fit curves representative of Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP) performance levels. The obtained families of fragility curves for the two studied models (with and without second order effects) are shown in Figure 8.

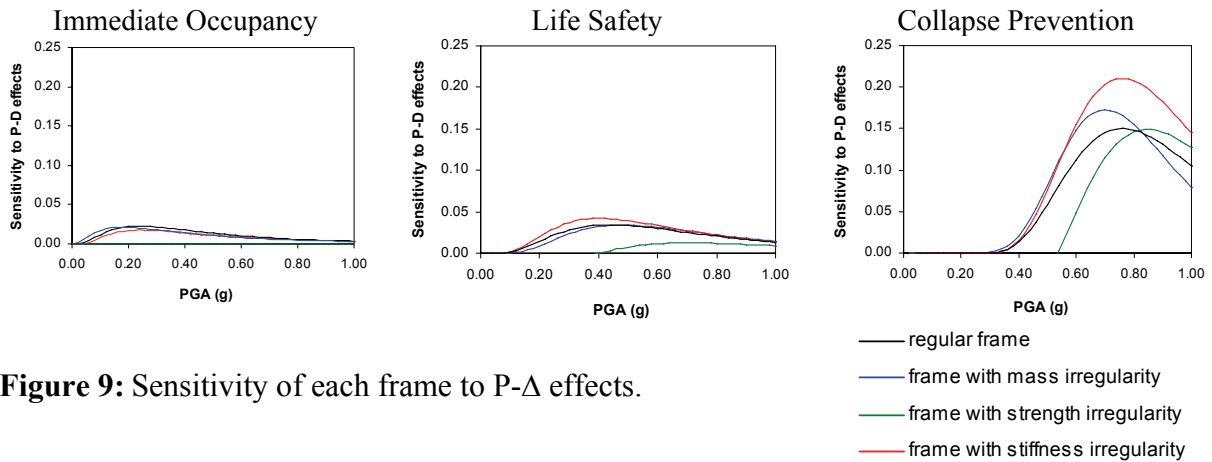
Figure 8 also shows that the comparison between the two families of fragility curves for each performance level. The domains bounded by the two curves referred to the same performance level represent the fragility domain due to second order effects for such performance level. As it can be seen, fragility domains of the IO performance level have always a negligible extension.

The fragility domains of the LS performance level have remarkable extension only for the frame with stiffness irregularity. Finally, the CP domains are much larger. In conclusion, the analysis of the fragility domains confirms that the P-Δ effects are relevant especially for large PGA values. Also, the frame with stiffness irregularity is the most sensitive to P-Δ effects among the analysed frames.

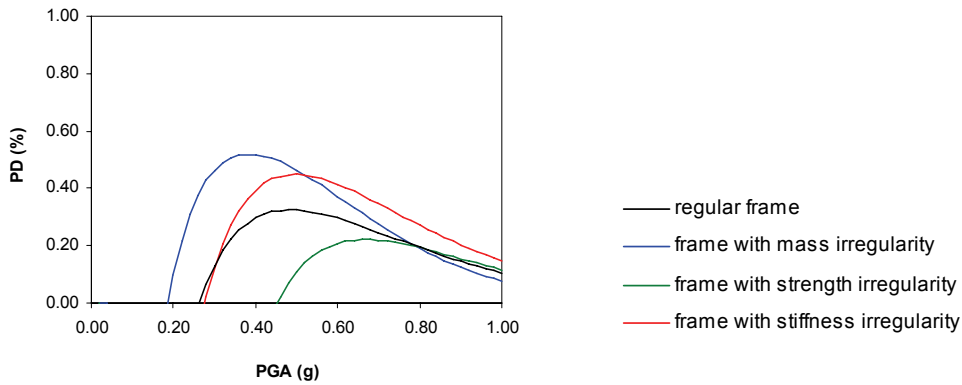


**Figure 8:** Fragility curves with and without P-Δ effects: (a) regular frame, (b) frame with mass irregularity, (c) frame with strength irregularity and (d) frame with stiffness irregularity.

Figure 9 shows the variation between the fragility curves evaluated with or without the second order effects as functions of the PGA for the four frames. As it can be seen, the sensitivity to the second order effects, for each limit state, strongly depends on PGA value. For the limit state IO the peak variation is attained for PGA around 0.20g, where second order effects are negligible, for the limit state LS variation is more significant for values of PGA around 0.40g, while for the limit state CP it becomes significant for PGA over 0.40g. Since the second order effects become more significant for high values of PGA, the limit state more affected by such effects is the CP limit state. Figure 10 shows, in percentage terms, the increase in probability of exceedance the CP limit state. As it can be seen, the increase ranges between 25% and 55% for the four considered frames. In particular, it can be observed that the frames having mass and stiffness irregularities are the most sensitive to P- $\Delta$  effects.



**Figure 9:** Sensitivity of each frame to P- $\Delta$  effects.



**Figure 10:** Increase of the probability of exceedance the CP limit state.

## CONCLUSIONS

In this paper the increase in second order effects due to vertical irregularities has been investigated. An eight storey, two span framed RC structure has been assumed as case study (regular frame), and three different vertical irregularities (stiffness, mass and strength irregularity)

have been introduced in the sample structure. The four investigated structures have been designed according to the EC8 provisions.

The obtained results show that the second order effects increase significantly the seismic response of all the four examined frames. The increase in interstorey drift, for the maximum considered PGA (0.65g), ranges between 22% and 34%. Comparing the second order effects in the regular frame with those in the irregular ones, it can be observed that all the considered irregular cases are more sensitive to the second order effects for large PGA values.

The maximum interstorey drifts obtained for each frame have been compared with the limit values provided by FEMA 356 for the three limit states: Immediate Occupancy, Life Safety and Collapse Prevention. For each value of the PGA, the response domain of the structure, assumed to be normal and therefore characterized by the mean value and the standard deviation calculated over the twenty ground motions, has been compared with the three limit values. The obtained probability of exceeding the assumed limit states has been expressed in terms of fragility curves, in the space PGA-probability of exceedance.

The comparison between fragility curves of the sample frame with and without P- $\Delta$  effects evidenced the role played by such effects on the safety level for each limit state. From the comparison it emerges the correlation between the role played by the second order effects for the probability of exceeding each limit state and the value of PGA. In fact, each limit state evidenced a sensitivity to P- $\Delta$  effects over a different range of PGA: for the limit state IO increase in probability of exceedance is maximum for PGA around 0.20g, for the limit state LS it is more significant for values of PGA around 0.40g, while for the limit state CP it becomes significant for PGA greater than 0.40g. Since the second order effects become more remarkable for high values of PGA, the increase of the probability of exceeding the CP limit state has been further investigated, by measuring the increase in the probability of exceeding the CP limit state in percentage terms. An increase ranging between 25% and 55% has been found for the four considered frames. In particular, it can be observed that the frames having mass and stiffness irregularities have the maximum sensitivity to P- $\Delta$  effects.

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