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Formulation of design eccentricity to reduce ductility demand in asymmetric buildings

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Abstract

A critical analysis of the large number of papers about the seismic behaviour of asymmetric buildings shows some concordant results: the modal analysis correctly predicts their elastic dynamic response to seismic records, while it overestimates deck rotation in the inelastic range. On this basis, the authors propose to design asymmetric structures by twice repeating the modal analysis: the first one with the actual mass distribution, so as to cover the elastic behaviour; the second one by considering the centre of mass displaced towards the centre of rigidity by a design eccentricity, so as to fit the inelastic response. In order to assess a formulation for the design eccentricity that reduces the maximum ductility demand, the paper statistically analyses the inelastic response of an idealised one storey building, symmetric about one direction, to different sets of accelerograms (both natural and artificial) and compares it to that of the corresponding balanced building; the analysis is repeated many times, so as to evaluate the influence of the different geometrical and mechanical parameters governing the inelastic response. The proposed approach and formulations prove to be effective in evaluating the effects of asymmetry, thus providing a design criterion which limits the ductility demand of asymmetric schemes without relevant increments of structural costs.

Keywords: Asymmetric buildings; Inelastic response; Design criteria

Notation

B dimension of the deck along the y direction
C_M mass centre
C_R centre of rigidity
C_{Rb} centre of rigidity of the *transformed basic system*
d_b distance between G and G_b
e_p distance between plastic centre and mass centre
e_s stiffness eccentricity, i.e. distance between C_R and C_M
G geometrical centre of the deck
G_b centre of the *basic system*
K_b total lateral stiffness of the elements of the *basic system* $K_b = \sum_{i=1}^{n_b} k_{ib} = \frac{1}{2} K_y$
k_{ib} stiffness of the ith element of the *basic system*
k'_{ib} stiffness of the ith element of the *transformed basic system*
k_{iy} stiffness of the ith element parallel to y-axis
k_{jx} stiffness of the jth element parallel to x-axis
K_x, K_y total lateral stiffness of the elements parallel to the x and y-axis

$$K_x = \sum_{j=1}^{n_x} k_{jx} \qquad K_y = \sum_{i=1}^{n_y} k_{iy}$$

K_θ total torsional stiffness about C_R $K_\theta = K_{\theta x} + K_{\theta y}$
$K_{\theta b}$ total torsional stiffness of the elements of the *basic system* about G_b $K_{\theta b} = \sum_{i=1}^{n_b} k_{ib}\xi_{ib}^2$
$K'_{\theta b}$ total torsional stiffness of the elements of the *transformed basic system* about their stiffness centre

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$K_{\theta G_y}$ torsional stiffness of the elements parallel to the y-axis about G $K_{\theta G_y} = \sum_{i=1}^{n_y} k_{iy} x_i^2$

$K_{\theta x}$, $K_{\theta y}$ torsional stiffness of the elements parallel to the x and y-axis about C_R

$$K_{\theta x} = \sum_{j=1}^{n_x} k_{jx}(y_j - y_{CR}^2)$$

$$K_{\theta y} = \sum_{i=1}^{n_y} k_{iy}(x_i - x_{CR})^2$$

L dimension of the deck along the x direction

m mass of the deck

n_b number of elements of the *basic system*

n_x, n_y number of resisting elements parallel to the x and y-axis

O_s overstrength ratio, i.e. ratio of total strength of resistant elements along y-direction of an asymmetric system over total strength of the correspondent system designed by multi-modal analysis without design eccentricity

r_k stiffness radius of gyration about C_R

$$r_k = \sqrt{\frac{K_\theta}{K_y}}$$

r_m mass radius of gyration about the mass centre

T_x, T_y uncoupled translational period along the x and y directions

$$T_x = 2\pi\sqrt{\frac{m}{K_x}} \qquad T_y = 2\pi\sqrt{\frac{m}{K_y}}$$

T_θ uncoupled torsional period $T_\theta = 2\pi\sqrt{\frac{mr_m^2}{K_\theta}}$

u_y normalised displacement, i.e. ratio of displacement of asymmetric systems over displacement of the corresponding torsionally balanced systems

x_{CR}, y_{CR} coordinates of C_R

x_i, y_j distance of the resisting elements parallel to the y and x-axis from the y and x-axis respectively

γ_x share of torsional stiffness due to the elements parallel to x-axis $\gamma_x = \frac{K_{\theta x}}{K_\theta}$

ξ'_{CRb} abscissa of the rigidity centre of the *transformed basic system* in the local reference system

ξ_{ib} abscissa of the element of the *basic system* in the local reference system

ω_x, ω_y uncoupled translational frequency along the x and y directions

$$\omega_x = \frac{2\pi}{T_x} = \sqrt{\frac{K_x}{m}}$$

$$\omega_y = \frac{2\pi}{T_y} = \sqrt{\frac{K_y}{m}}$$

ω_θ uncoupled torsional frequency

$$\omega_\theta = \frac{2\pi}{T_\theta} = \sqrt{\frac{K_\theta}{mr_m^2}}$$

Ω_θ uncoupled lateral–torsional frequency ratio

$$\Omega_\theta = \frac{\omega_\theta}{\omega_y} = \frac{r_k}{r_m}$$

1. Inelastic response of asymmetric buildings

Whoever analyses the large number of papers on this subject will probably be struck by the complexity of the problem and the discrepancies among the conclusions of the researchers. In effect, while the elastic seismic behaviour is ruled by few global parameters (eccentricity between mass and stiffness centres, uncoupled lateral–torsional frequency ratio and, in a lesser way, period of vibration, shape of the response spectrum and position of mass centre with respect to the edges of the floor deck), at first sight the inelastic response seems to be influenced by location and strength of each resisting element. A considerable effort is therefore presently devoted to the standardisation of definitions and assumptions and to the identification and evaluation of the effect of every single parameter. One of the goals of this paper is to give proper relevance to the main aspects, putting more emphasis on some concordant results of the research; a few general considerations, which can be found in most papers on this subject, may in fact constitute the basis for a retrospective analysis of past work and for the proposition of a design approach able to limit the negative effects of asymmetry.

At first it must be noted that the conclusions of the researchers seem contradictory mainly when attention is focused on ductility demand, which in different papers is considered to reach the maximum at the stiff or at the flexible side and to be smaller, comparable or much greater than that of the corresponding balanced system. This is obviously related to the different design approaches and to the strength the structural elements are consequently provided with. On the contrary, many authors acknowledge that the inelastic displacements of the elements which constitute a spatial frame are scarcely dependent on their strength, i.e. that different structures, with elements having the same stiffness but designed so as to offer different strength, present approximately the same peak displacements. Goel and Chopra [1] clearly state that “the element deformations of systems designed according to most building codes are not very different" and Tso and Zhu [2] affirm that "the displacement demand is insensitive to the form of torsional provisions adopted". This is confirmed by the numerical analyses described later on; an example is provided by Fig. 1, which points out the peak displacements of a stiffness eccentric system, with a geometrical scheme as shown in Fig. 6, designed many times, with different strength distributions and behaviour factors, subjected to an Italian seismic record (Tolmezzo, Friuli, 1976).

Secondly, it is often recognised that, while the elastic response of asymmetric schemes usually shows a larger rotation, if compared to the prediction of static spatial analysis, the inelastic response is much more translational. According to Goel and Chopra [1], "yielding leads to reduced torsional deformation of medium-period and long-period systems, regardless of their stiffness eccentricity. Thus, if the system is well into the inelastic range, the effects of plan-asymmetry on system response are small". A reason for this is that within inelastic response ranges the eccentricity is not constant, because of the instantaneous variation of the position of the rigidity centre due to the plastification of the elements. A further cause is the torsional contribution of orthogonal elements which remain longer in the elastic range, mainly if the transverse seismic excitation is smaller than the one acting in the normal direction. An example of this is illustrated by Fig. 2, which compares the peak displacements (provided by static analysis, elastic and inelastic dynamic response analysis) of two structural schemes, with different eccentricity between mass and stiffness centre, subjected to El Centro record.

2. Design of asymmetric buildings

The two aforementioned considerations, combined, may be considered a generalisation and a modification of the well known equal displacement assumption proposed in the sixties by Newmark for elastic–perfectly plastic s.d.o.f. systems. We may in this case affirm that the peak displacements of asymmetric schemes well into the inelastic range are independent of the global value of strength and of its distribution among the resisting elements, but they differ from the elastic peak displacements because of the less marked rotation. This assumption is obviously a simplification of the actual behaviour, which is valid in the mean but may be violated in single cases. Nevertheless it proves to be very useful in solving the problem of ductility demand because this may be simply foreseen by comparing the design displacements to the peak values of inelastic analyses. When the strength of each element is assumed proportional to its stiffness, i.e. only translation is considered in design, the

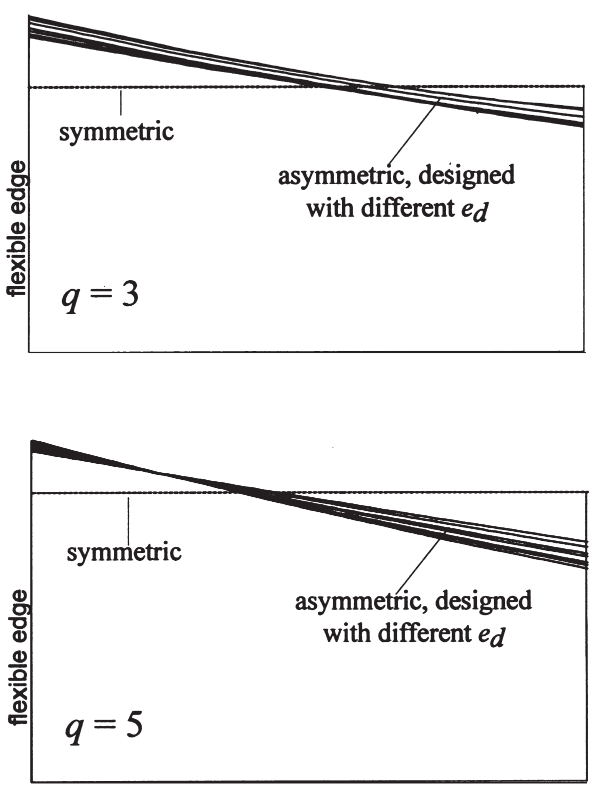

Fig. 1. Inelastic peak displacements of a stiffness eccentric system subjected to Tolmezzo record (design parameters: $\Omega_\theta = 1$, $T_x = T_y = 1$ s, $\gamma_x = 0.2$, $e_s = 0.05\ L$).

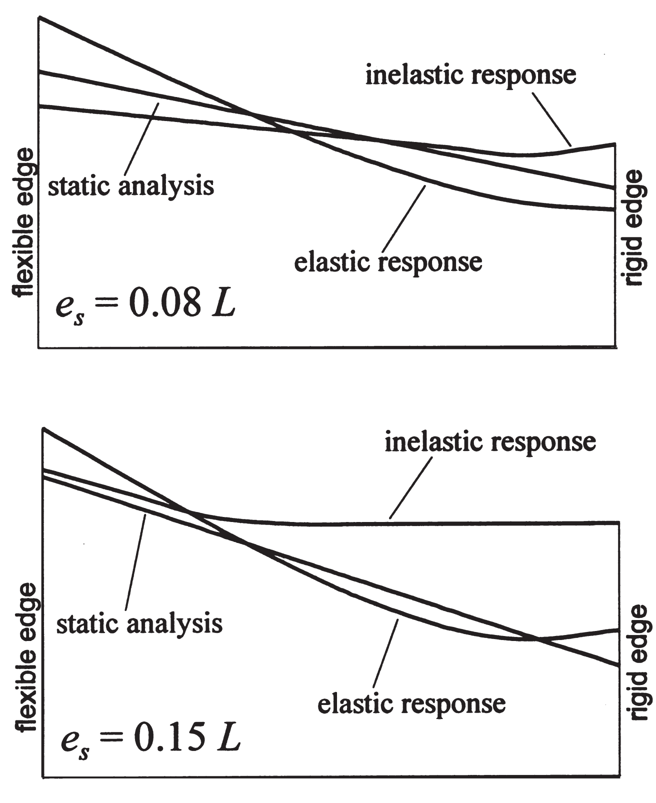

Fig. 2. Peak displacements of a stiffness eccentric system subjected to El Centro record (design parameters: $\Omega_\theta = 1.2$, $T_x = T_y = 1$ s, $\gamma_x = 0.2$, $q = 5$).

maximum ductility is always required at the flexible edge. When static spatial analysis—without additional eccentricities—is used (Fig. 3), the ductility demand is greater at the rigid edge, since the reduction of design displacements due to rotation finds no correspondence in the more translational inelastic behaviour. Finally, the use of multi-modal spatial analysis understimates the actual inelastic displacements (and increases the ductility demand) mainly in the part of the structure next to the stiff edge in torsionally rigid systems and to the flexible edge in torsionally flexible systems. The maximum ductility demands are always smaller than those obtained by using static analysis.

A proper way to face the problem must not forget the two aims of seismic design (no collapse under strong events and damage limitation under seismic actions having a larger probability of occurrence), which in most codes are hidden by the use of a unique value of design

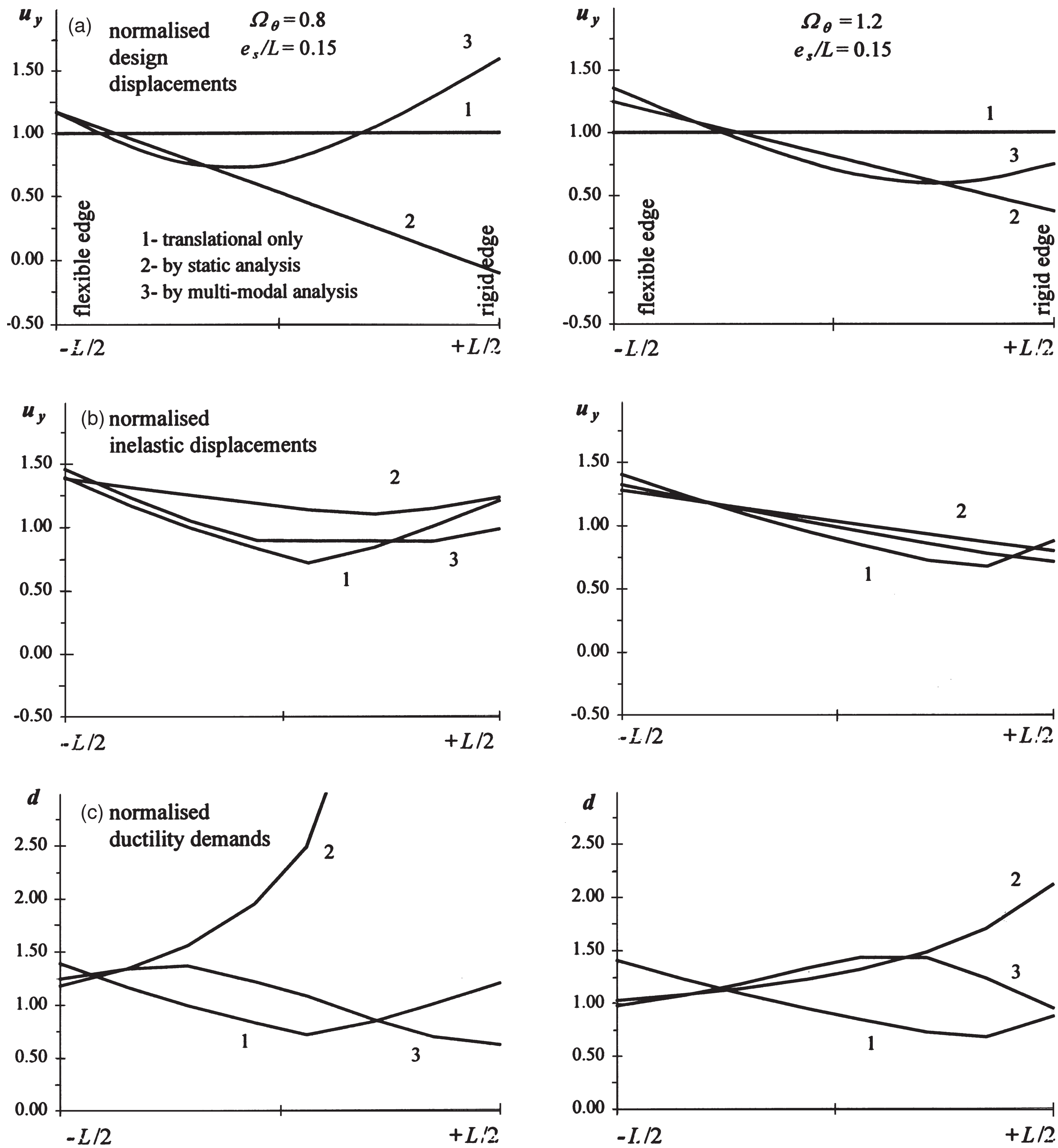

Fig. 3. Response of asymmetric systems designed by either static or multi-modal analyses and subjected to Tolmezzo record: (a) normalised design displacements; (b) normalised inelastic displacements and (c) normalised ductility (design parameters: $T_x = T_y = 1$ s, $\gamma_x = 0.2$, $q = 5$).

actions, provided by the elastic response spectrum divided by a coefficient (behaviour factor q in EC8). Design displacements should therefore cover peak elastic values and peak plastic values (divided by q) at different points of the structure. The use of static spatial analysis as reference method of design, imposed by many codes and followed by nearly all researchers, already requires the use of additional eccentricity to catch the elastic response and thus imposes a double effort to solve both elastic and inelastic aspects. This might constitute one of the reasons for the difficulty in interpreting the results of interesting works. Our basic assumption has therefore been that the strength of the elements should be proportioned by using multi-modal spatial analysis with the actual mass distribution, so as to properly catch the elastic structural response, and with a design eccentricity (i.e. a displacement of the centre of mass towards the centre of rigidity) in order to fit the inelastic response. The present paper shows how such design eccentricity is related to the elastic characteristics and to the mass distribution of the scheme, providing a thorough formulation to reach the proposed goal. The design procedure, based on multi-modal spatial analysis, and the related formulation could be a strong basis for an improvement of the torsional provisions of Eurocode 8 [3]. It is obviously important that the seismic code allows the designer to use static spatial analysis, but the relationship between the results of static and multi-modal analysis must be considered a separate problem, already solved [4,5].

3. Geometry and stiffness of the model

A preliminary step of the research has been the definition of the geometrical and elastic features of the structural model. The scheme is an idealised one-storey building with rectangular rigid deck; it is referred to the reference axes x and y, with origin G coincident to the geometrical centre of the deck, and it is assumed to be symmetric about the x-axis. The position of mass centre and the mass radius of gyration are assigned independently of shape and dimensions of the deck, under the hypothesis that the mass distribution may not be uniform. The main component of the seismic ground motion is considered to act along the y-direction, which is called the "normal direction"; the developed procedure could also take into account the component acting along the x-axis ("transverse direction"), which in the present phase of the work has been neglected. The seismic action is withstood by two sets of resisting elements, parallel to the axes, which are assumed to have a bilinear elastic–perfectly plastic force–displacement relationship and to present no out-of-plane stiffness or strength. The general procedure developed, which allows assignation of the stiffness of each element so as to obtain a required value of the global elastic parameters, hereafter is described separately for the two directions. The criteria used to define the elements' strength are described later on.

3.1. Elements along the normal direction (y-axis)

In order to obtain given values of location of stiffness centre and total translational and torsional stiffness of the elements oriented along this direction, a minimum number of three independent parameters is necessary. In the two-elements models, like the one used by Goel and Chopra [1], the position of the elements must be considered variable and cannot coincide to the edge of the deck. This might have some influence on the inelastic response, because the displacement due to rotation depends on the distance from the rotation centre. Therefore, more common is the use of three-element models [2,6,7], which supply in most cases a satisfactory estimate of the inelastic response. However, the number of resisting elements might sometimes significantly affect the ductility demand [1]; for this reason a more general automatic generation procedure has been developed, able to assign the proper stiffness to any number of elements, from three onwards. For a better correspondence to the actual buildings, in most analysed cases the system constituted eight elements in the main direction, but the effect of assuming a smaller number of elements has been investigated too.

In order to apply the procedure, according to established mathematical rules complying with the same logic, the primary model called the *reference symmetric system* (RSS) is firstly defined (Fig. 4). It is made up by a sub-system of n_b elements, called the *basic system*, duplicated symmetrically with respect to the y-axis. The elements of the *basic system* are themselves symmetrical about their centre G_b, origin of a set of local axes ξ–η, which is located at a distance d_b from the geometric centre G.

The RSS shown in the figure has an even number of elements; an odd number may be obtained by positioning the sub-systems so that the two central elements coincide

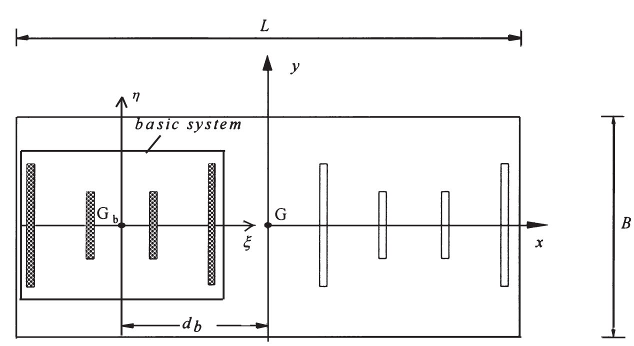

Fig. 4. The *basic system* and the *reference symmetric system* (the length of the elements is drawn in proportion to their stiffness).

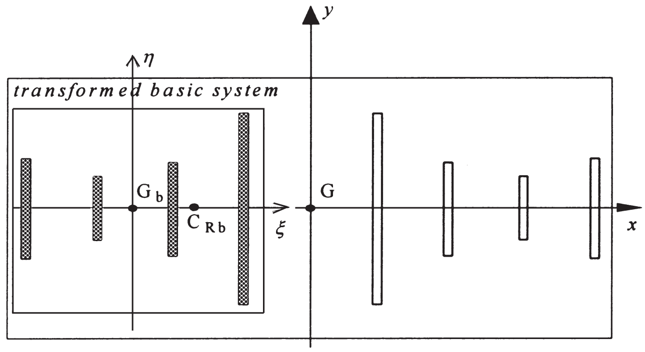

Fig. 5. The *transformed basic system* and the *transformed symmetric system*.

and by substituting these with a unique equivalent element.

A *transformed symmetric system* (TSS), with a given torsional stiffness, is obtained by applying a linear transformation to the stiffness k_{ib} of the elements of the *basic system*, which is modified proportionally to the distance of the elements from G_b and to a parameter β_1 (Fig. 5). The stiffness k'_{ib} of the generic element of the *transformed basic system* is therefore

$$k'_{ib} = k_{ib}(1 + \beta_1 \xi_{ib}) \tag{1}$$

Appendix A demonstrates that such transformation does not change the total translational stiffness and the torsional stiffness about G_b. It is therefore possible to obtain a TSS having a given translational and torsional stiffness simply by selecting a whatsoever *basic system* with $K_b = 0.5\ K_y$ and evaluating the coefficient β_1 my means of Eq. A(7).

An *asymmetric system* may be obtained from TSS by assigning a mass centre not coincident with the geometric centre G. Such a system is usually called a *mass eccentric system* (MES) and its corresponding balanced system is the same TSS. As an alternative, a further linear transformation may be applied to the whole TSS, by modifying the stiffness of each element proportionally to its distance from G and to a parameter β_2 (Fig. 6).

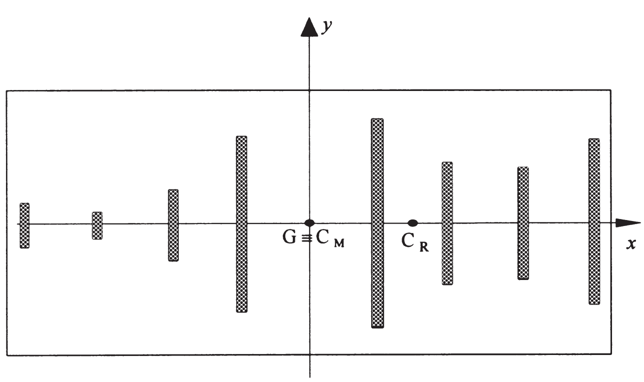

Fig. 6. The *asymmetric system* (SES) obtained by means of the last transformation.

Once again the transformation leaves the translational stiffness and the torsional stiffness about G unchanged, while the abscissa of the centre of rigidity is related to β_2 by an expression analogous to that given in Eq. (A4) in Appendix A, which can be inverted giving

$$\beta_2 = \frac{K_y x_{CR}}{K_{\theta G_y}} \tag{2}$$

If the mass centre is coincident to G the *asymmetric system* so generated is called the *stiffness eccentric system* (SES). The corresponding balanced system is obtained by moving C_M to C_R.

The distinction between MES and SES is considered fundamental by some authors (e.g. Goel and Chopra [1]), while others note that most actual systems are contemporaneously mass and stiffness eccentric (Tso and Zhu [2]). Our opinion is that the ruling parameter is not the type of model (MES or SES) but the position of mass centre with respect to the edges of the deck; this parameter proved to have some importance, although minor, in the elastic analyses [4] and it seems logical that an analogous influence may be found in the inelastic behaviour. Nevertheless the present research has been focused separately on MES and SES models and the results finally obtained show that the effect of such distinction on the design eccentricity, although perceptible, is not relevant.

3.2. Elements along the transverse direction (x-axis)

The presence of elements oriented along the *transverse* direction contributes to reduce the rotation in the inelastic range, in particular when the transversal component of the seismic ground motion is small or it is totally neglected in the analysis. However, the analyses carried out aim at evaluating two limiting behaviours, with increased and reduced rotation, given respectively by the elastic and the inelastic response. The absence or the early plastification of the elements along the transverse direction, although actually possible, can limit the reduction of inelastic rotation, which is not safe for our purpose. For this reason the model utilised has elements in the *x*-direction able to provide a translational stiffness (equal to the one in the *y*-direction) and a torsional stiffness (in most cases 1/5 of the total torsional stiffness, although other values have been assumed too, in order to evaluate the influence of this parameter). A number of three elements, located symmetrically along the *x*-direction has been fixed; their stiffness, necessary to comply with the above requirements, is

$$k_{1x} = k_{3x} = \frac{K_{\theta x}}{2y_{1x}^2} \qquad k_{2x} = K_x - 2k_{1x} \tag{3}$$

Table 1
Reference code by ENEA–ENEL, origin, component and date of the thirty selected accelerograms

Ref. code	Record		Date	Ref. code	Record		Date	Ref. code	Record		Date
32	Codroipo	ew	06–05–76	168	Forgaria	ew	15–09–76	621	Bagnoli I.	ew	23–11–80
32	Codroipo	ns	06–05–76	168	Forgaria	ns	15–09–76	621	Bagnoli I.	ns	23–11–80
38	Tolmezzo	ns	06–05–76	169	San Rocco	ew	15–09–76	627	Merc. S. Sev.	ew	23–11–80
143	Buia	ew	11–09–76	169	San Rocco	ns	15–09–76	627	Merc. S. Sev.	ns	23–11–80
143	Buia	ns	11–09–76	177	Buia	ew	15–09–76	636	Calitri	ew	23–11–80
152	Forgaria	ew	15–09–76	301	Patti	ew	15–04–78	636	Calitri	ns	23–11–80
152	Forgaria	ns	15–09–76	301	Patti	ns	15–04–78	643	Rionero	ew	23–11–80
153	San Rocco	ew	15–09–76	302	Naso	ew	15–04–78	643	Rionero	ns	23–11–80
156	Buia	ew	15–09–76	302	Naso	ns	15–04–78	644	Bisaccia	ew	23–11–80
156	Buia	ns	15–09–76	350	Norcia	ew	19–09–79	644	Bisaccia	ns	23–11–80

in which k_{1x} and k_{3x} are referred to the outer resisting elements and k_{2x} to the central one.

4. Seismic ground motion

It is well known that proper selection of the input ground motion has great importance in every response analysis. When one or a few seismic records are used, large differences in response are to be expected. To overcome this problem, a probabilistic approach has been used, i.e. each structural scheme has been subjected to a set of accelerograms and statistical information has been extracted by the set of the results. Thirty historical Italian accelerograms having different characteristics (duration, peak ground acceleration and frequency content), recorded by the national accelerometric network installed in Italy by ENEA–ENEL, have been selected in order to constitute a representative set of national accelerograms (Table 1). In order to homogenise them, the records have been scaled so that each one presents the same mean elastic response (evaluated in the range 0.5–3 s) and the mean elastic response spectrum of the whole set has a given value (0.35 $\boldsymbol{g}$) in correspondence of the period of 1 s. The mean elastic response spectrum so obtained (Fig. 7) sufficiently recalls the elastic spectrum imposed by EC8 for firm soil in areas characterised by expected peak ground acceleration of 0.35 $\boldsymbol{g}$.

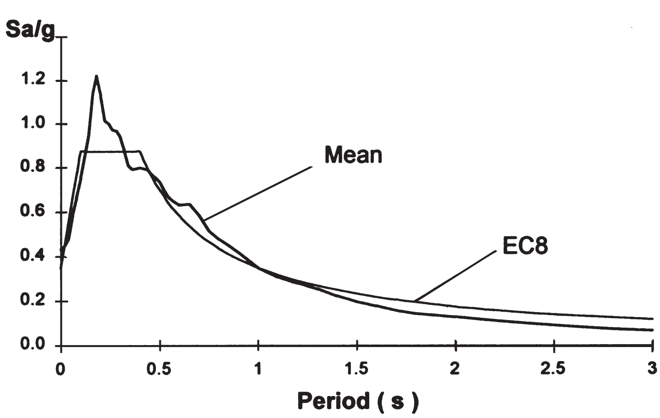

Fig. 7. Mean elastic response spectrum of the set of accelerograms.

In order to confirm that the results obtained are not dependent on the selected set of seismic records, a second set of thirty artificial accelerograms has been generated, so as to match the EC8 elastic response spectrum shown in Fig. 7. Their amplitude, frequency content and duration are consistent with the prescriptions given by Eurocode 8 about the use of artificial accelerograms in time-history analyses.

5. Strength of the elements and design eccentricity

The design procedure we are proposing consists of assigning the strength of the elements by repeating two times the multi-modal spatial analysis: in the first one the mass centre is located in its actual position; in the second one the mass centre is displaced by a quantity e_d (which we name *design eccentricity*) towards the centre of rigidity. The strength of each element is assumed as the largest of the two values so determined.

In order to identify the best value of e_d each system, having given geometrical and inertial characteristics, has been designed many times with design eccentricity ranging from 0 to 1.5 e_s (e_s being the eccentricity between mass and stiffness centre), using as a design spectrum the mean elastic response spectrum of the selected ground motions divided by a fixed value of the behaviour factor q. The resisting schemes so obtained have been subjected to the set of selected accelerograms. In parallel, the corresponding *balanced system*, in which the mass centre has been displaced to coincide with the stiffness centre in order to obtain a purely translational behaviour, has been designed and subjected to the ground motions. Among the output data, attention has been focused on the largest peak ductility demand among all elements: the value required by each seismic event has been normalised by the corresponding value of the *balanced system* and a global estimate is provided by the mean value $d_{0.50}$ and by the 95% fractile $d_{0.95}$ of

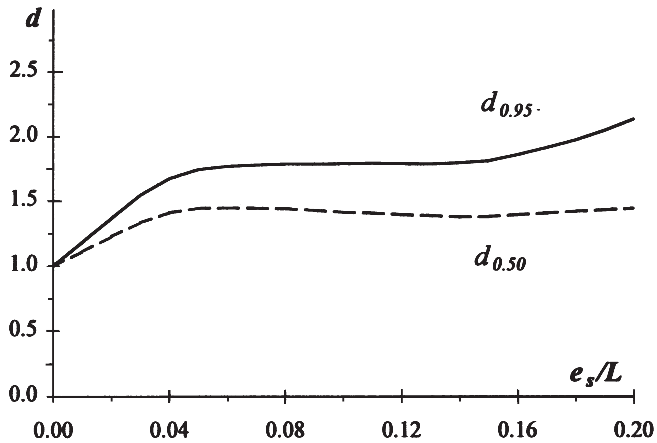

Fig. 8. Normalised ductility demand versus stiffness eccentricity (design parameters: MES, $\Omega_\theta = 1.2$, $\gamma_x = 0.2$, $T_y = 1$ s, $q = 5$).

the normalised ductility demand of the thirty accelerograms.

The numerical analyses show that when only the actual stiffness eccentricity is used in the phase of design, i.e. when $e_d = 0$, the maximum ductility is mostly demanded by the element at the stiff edge in torsionally rigid systems and at the flexible edge or at the centre of the structure in torsionally flexible systems; both parameters, $d_{0.50}$ and $d_{0.95}$, increase in a non linear way with the stiffness eccentricity of the model (e.g. see Fig. 8), reaching values which can be very high depending on the characteristics of the scheme. The use of the above named design eccentricity generally reduces this effect as much as e_d is larger (e.g. see Fig. 9). From the relation of $d_{0.50}$ and $d_{0.95}$ versus e_d it is possible to define the value of e_d necessary to limit the ductility demand to a given value. In the performed analyses the limit $d_{0.95} = 1.3$ has been primarily imposed, but other values have been used too in order to analyse their influence on e_d.

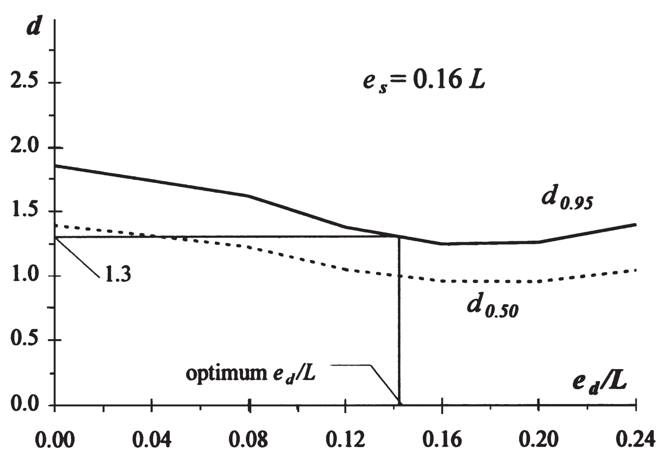

Fig. 9. Normalised ductility demand versus design eccentricity (design parameters: MES, $\Omega_\theta = 1.2$, $\gamma_x = 0.2$, $T_y = 1$ s, $q = 5$).

6. Formulation of design eccentricity

The procedure described above allows the evaluation of the optimum value of e_d for a scheme with assigned elastic and inertial characteristics. In order to find a general formulation, able to provide safe values of the design eccentricity in all actual situations, we investigated the influence of the position of mass centre (i.e. of the type of model, MES or SES) and that of the parameters e_s (stiffness eccentricity), Ω_θ (uncoupled lateral–torsional frequency ratio), T_y (uncoupled translational period) and q (behaviour factor). In all the numerical analyses we assumed dimensions of the rigid deck $L = 29.50$ m and $B = 12.50$ m, total mass corresponding to 1 t/m^2, mass radius of gyration $= 0.312\, L$. In most cases the *basic system* is defined by $n_b = 4$, $d_b = 8.25$ m, $\xi_{3b} = 2.00$ m, $\xi_{4b} = 6.50$ m, $k_{3y} = 0.075\, K_y$, $k_{4y} = 0.175\, K_y$ and the share of torsional stiffness due to the transverse elements is $\gamma_x = 0.2$, although these data have been changed in a few cases in order to check the effect of the number of resisting elements and of the contribution given by the transverse elements.

For every assigned value of the above parameters, the automatic generation procedure defines the stiffness of each element. The total stiffness of the resisting elements oriented along the y-direction and the torsional stiffness of all elements about C_R are given by

$$K_y = m\left(\frac{2\pi}{T_y}\right)^2 \qquad K_\theta = \Omega_\theta^2 r_m^2 K_y \tag{4}$$

while the share of the torsional stiffness due to the elements along the y-axis evaluated about C_R and G are respectively

$$K_{\theta y} = K_\theta(1 - \gamma_x) \qquad K_{\theta G_y} = K_{\theta y} + K_y x_{CR}^2 \tag{5}$$

These values allow the evaluation of the parameters β_1 and β_2.

As a first step of the study we examined the influence of e_s on e_d. In all cases examined the relation between these two parameters is about linear (e.g. see Fig. 10) and can be approximated by a straight line having equation

$$e_d = k(e_s - e_r) \tag{6}$$

in which k is the slope of the line and e_r is its intersection with the x-axis.

The second step consisted therefore in the search of a relationship among the parameters k, e_r and the elastic characteristics of the scheme. Starting from a basic case ($\Omega_\theta = 1$, $T_y = 1$ s, $q = 5$) each parameter has been separately varied, in the following range: $\Omega_\theta = 0.6$–1.6; $T_y = 0.4$ s–2 s; $q = 1.5$–5. Fig. 11 shows the relation

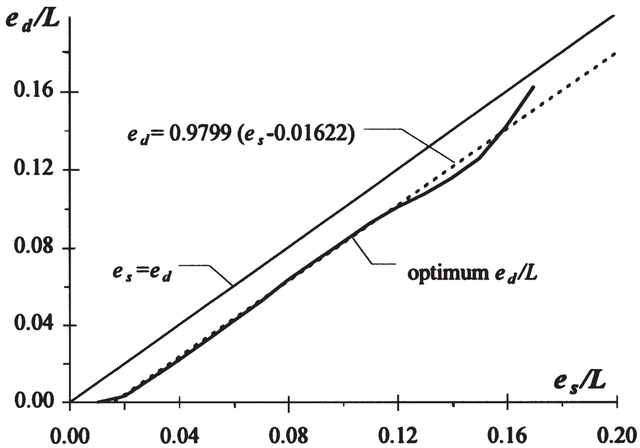

Fig. 10. Design eccentricity (necessary to obtain $d_{0.95} = 1.3$) versus stiffness eccentricity (design parameters: MES, $\Omega_\theta = 1.2$, $\gamma_x = 0.2$, $T_y = 1$ s, $q = 5$).

of e_r and k versus Ω_θ for the value of $d_{0.95} = 1.3$. The parameter e_r grows with Ω_θ in an approximately linear way, with a slope which depends on the required value of $d_{0.95}$ but it is independent of the type of system (SES or MES). An abnormal behaviour is highlighted only by systems with an uncoupled lateral–torsional frequency ratio close to unity. The parameter k is slightly decreasing as Ω_θ increases. Figs. 12 and 13 show the relation of e_r and k versus q and T_y respectively. The parameter e_r is practically independent of q, while k increases slightly as q increases, but without perceivable differences between SES and MES. The effect of T_y, practically negligible, does not always display a clear tendency. The results obtained by varying γ_x from 0.001 to 0.4 and by changing the number of resisting elements, not reported in any figure, show that these parameters have almost no influence. Finally, also the set of accelerograms (natural or artificial) shows just a minor effect, thus confirming the general validity of the obtained results.

From the results described above we propose to express the parameters k and e_r by means of the following equations

$$k = \max\begin{cases} 3.3 - 2.5\Omega_\theta + 0.04q \\ 1 \end{cases} \tag{7}$$

$$e_r = \max\begin{cases} 0.1(0.5\Omega_\theta - 0.4)L \\ 0.01L \end{cases} \tag{8}$$

The values provided by Eq. (7) and (8) are shown in Figs. 11–13, confirming the effectiveness of the formulations.

Fig. 14 compares the values of design eccentricity given by Eq. (6) to those evaluated by means of the numerical analyses. In nearly all cases the proposed values are greater than those numerically calculated, showing the safety of the aforementioned formulation. In some cases this safety is higher, but still acceptable, and only in very few situations the numerical results are slightly larger than those given by the formula. This is confirmed by Figs. 15 and 16, which compare the values

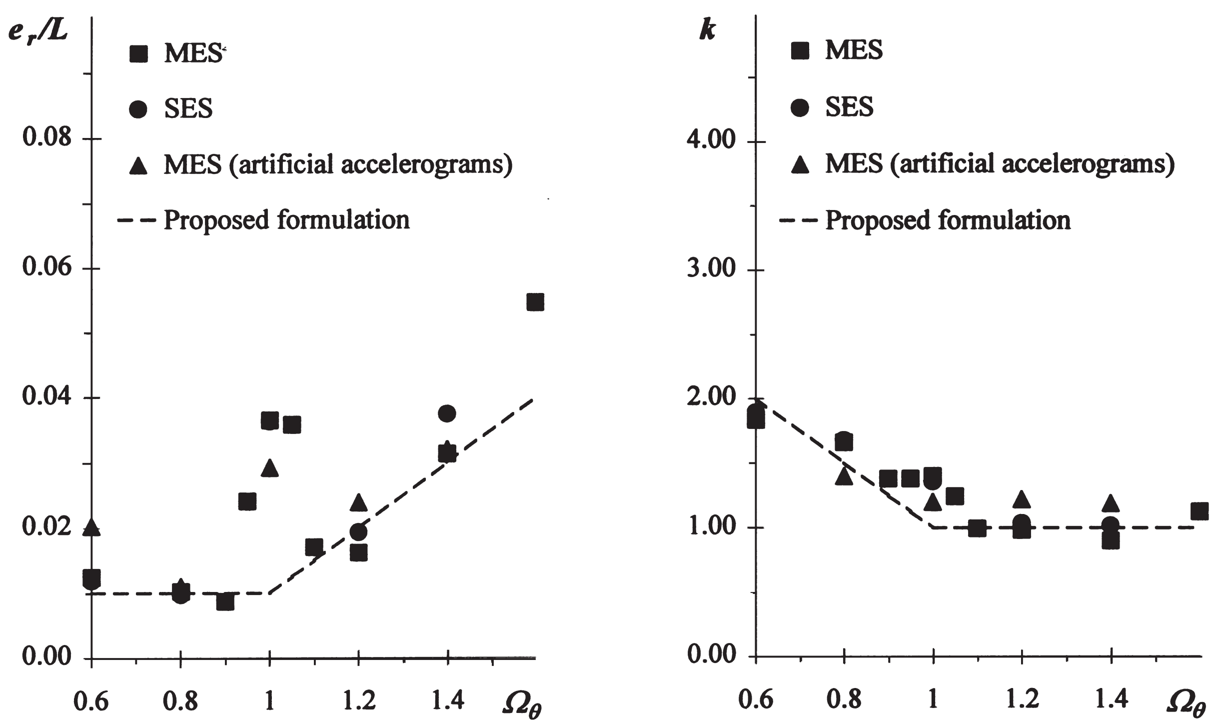

Fig. 11. Values of k and e_r versus Ω_θ (constant parameters: $\gamma_x = 0.2$, $T_y = 1$ s, $q = 5$).

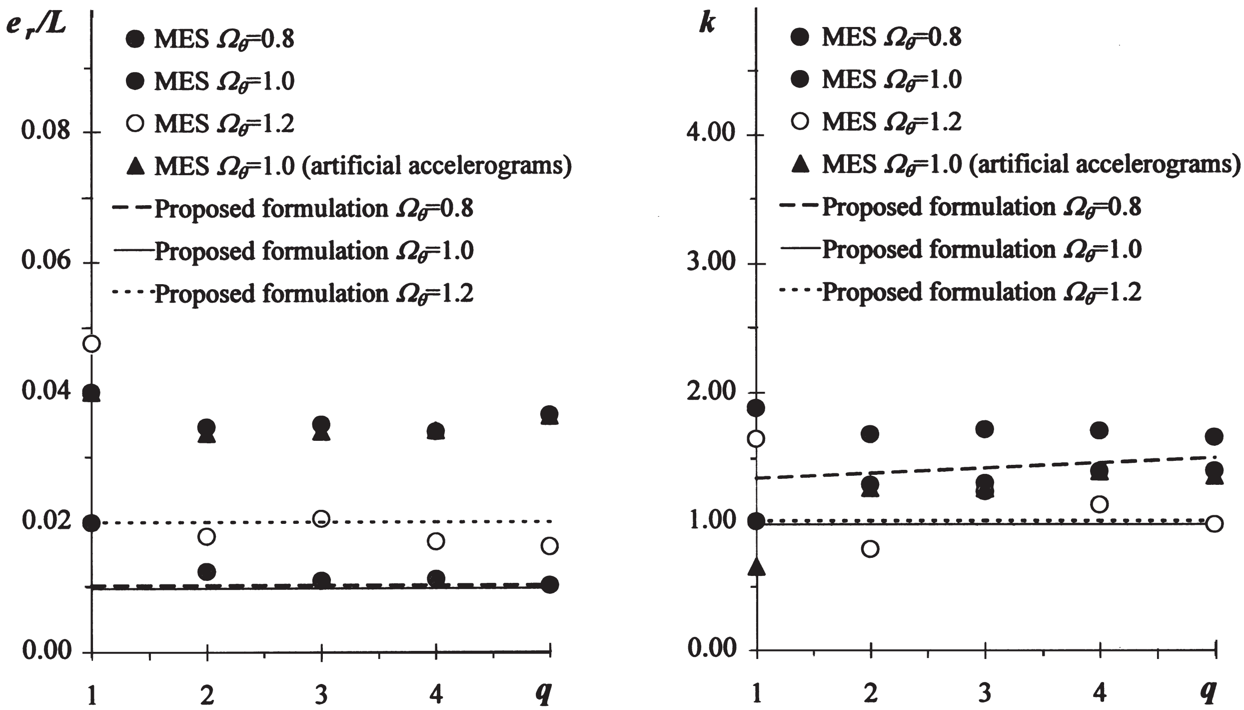

Fig. 12. Values of k and e_r versus q (constant parameters: $\gamma_x = 0.2$, $T_y = 1$ s).

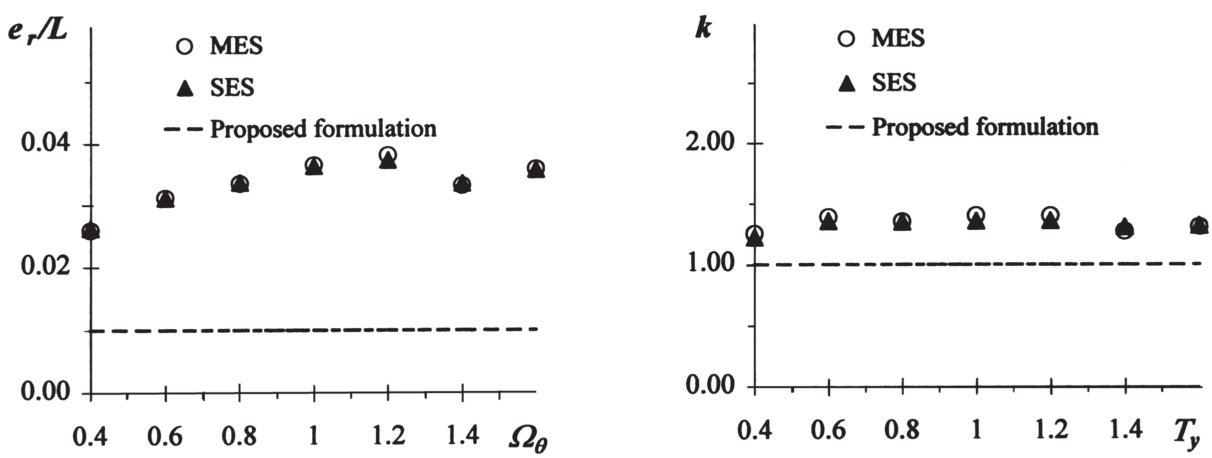

Fig. 13. Values of k and e_r versus T_y (constant parameters: $\Omega_\theta = 1$, $\gamma_x = 0.2$, $q = 5$).

of ductility demand for schemes designed by taking into account only the actual stiffness eccentricity or by following the proposed approach, using the values of design eccentricity provided by the above formulations. It is apparent that the goal of limiting the ductility demand has been achieved, with a partial exception for torsionally flexible schemes with large stiffness eccentricity. It must be noted furthermore that, having imposed the value 1.3 to the 95% fractile of the normalised ductility demand, the mean value of the normalised ductility ($d_{0.50}$) is often close to 1, showing that the ductility demand of asymmetric structures designed according to the proposed rules and parameters is coincident, in the mean, to that of the corresponding balanced schemes. Only torsionally rigid systems having small stiffness eccentricity experience mean normalised ductility demand up to 1.20; in order to overcome this problem a minimum value of the design eccentricity to be used in combination with Eq. (6) could be

$$e_d = 0.6e_s \tag{9}$$

Some consideration may be also given to the location of the centre of strength. If the torsional contribution of the transverse elements is neglected, it coincides with the mass centre. Both the contribution of transverse elements and the use of design eccentricity move it toward the stiffness centre. In particular, the proposed

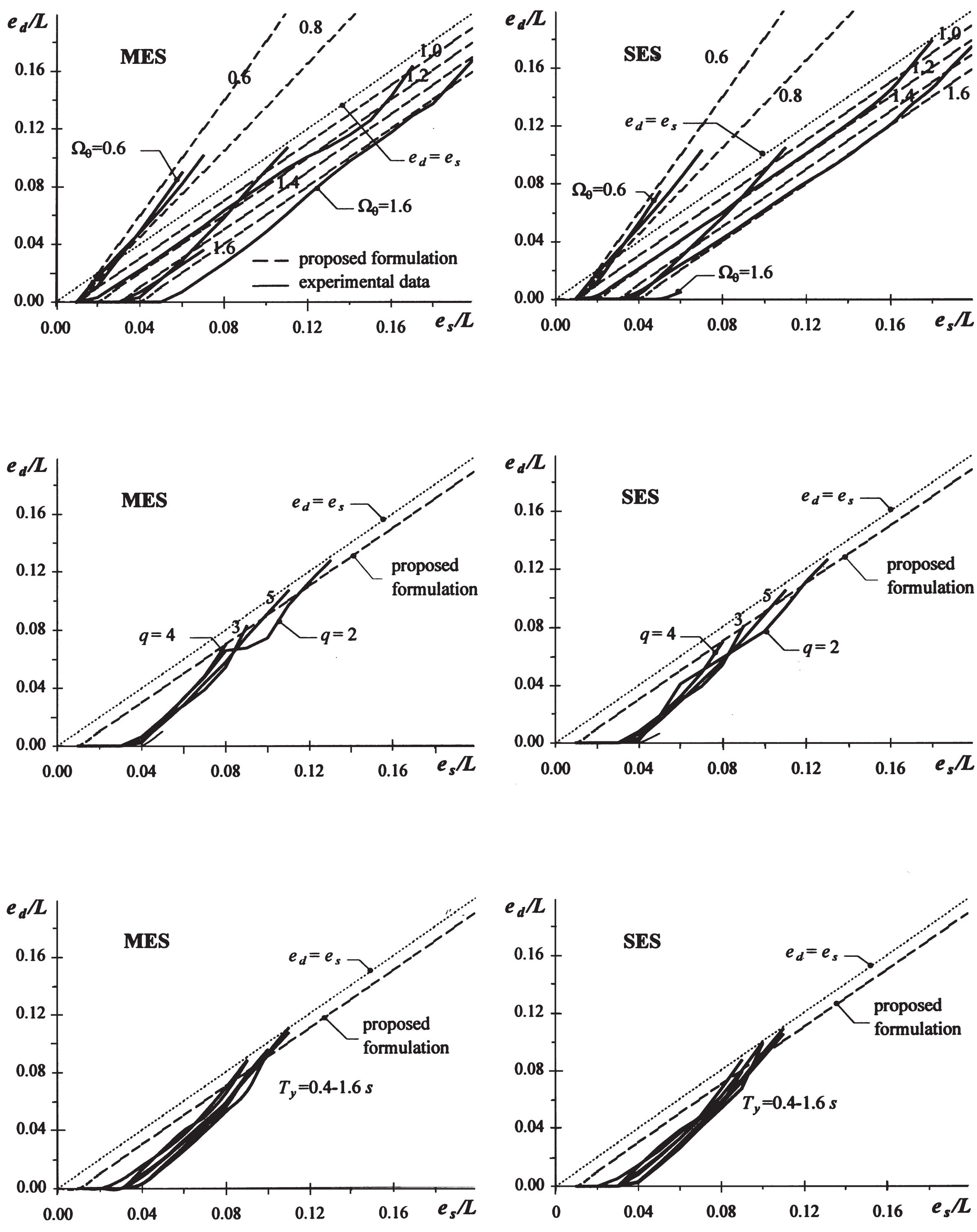

Fig. 14. Calculated and proposed values of design eccentricity (necessary to obtain $d_{0.95} = 1.3$) plotted versus stiffness eccentricity.

values of e_d shift the strength centre to a position not far from the mid-way between the mass and stiffness centres, a position which many researchers have suggested to be optimal for a good response of the structure (e.g. see Chandler et al. [7] and De Stefano et al. [6]).

The mean increase of strength due to the proposed approach is small, comparable to that obtained by complying with the prescriptions of seismic codes. The global overstrength, i.e. the ratio of the total strength of the resisting elements oriented along the y direction over the strength of those of the corresponding system designed by multi-modal analysis without design eccentricity, is plotted in Fig. 17. The influence of the uncoupled lateral–torsional frequency ratio is really clear: torsionally

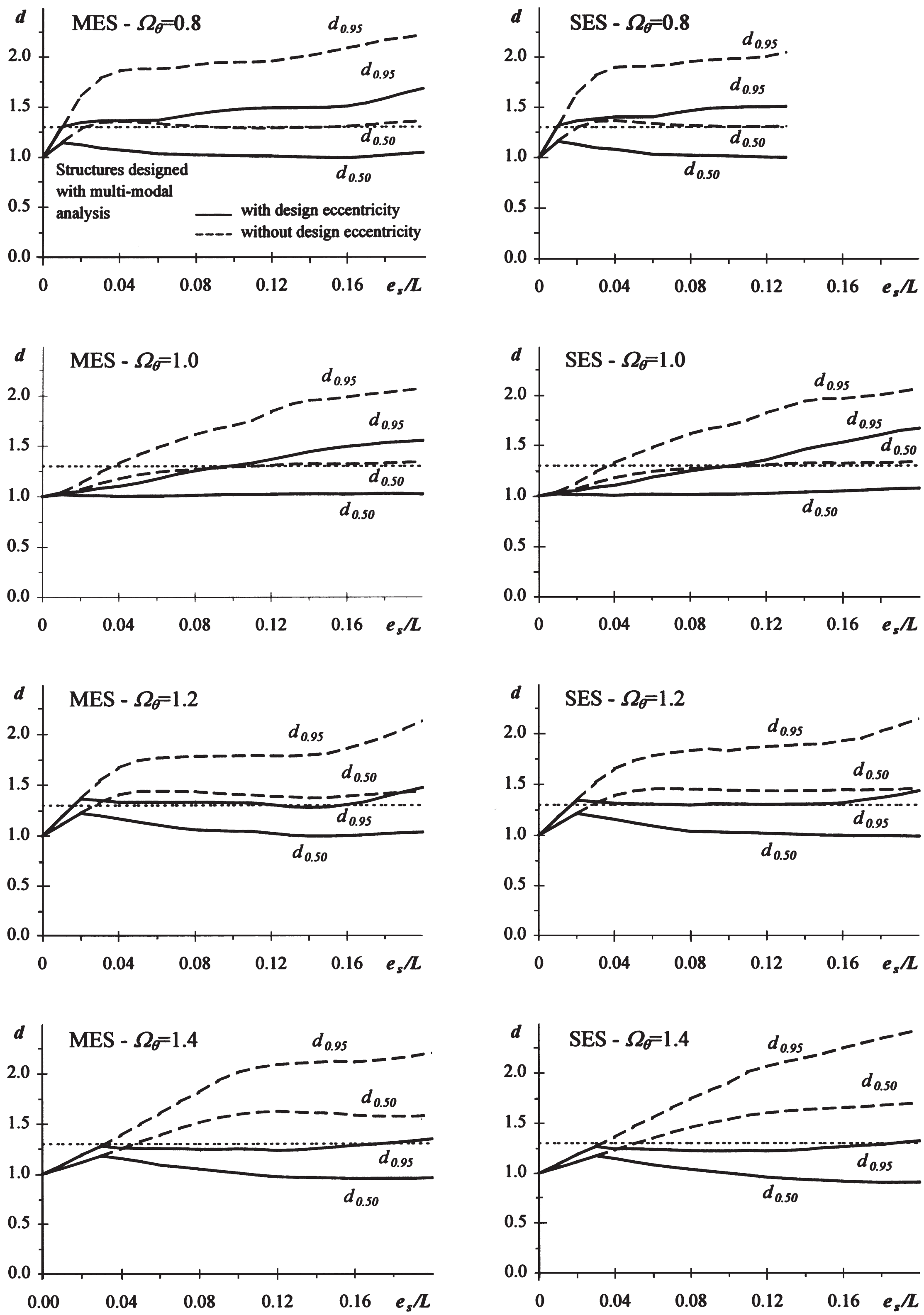

Fig. 15. Ductility demand of schemes designed with and without the proposed approach (constant parameters: $\gamma_x = 0.2$, $T_y = 1$ s, $q = 5$).

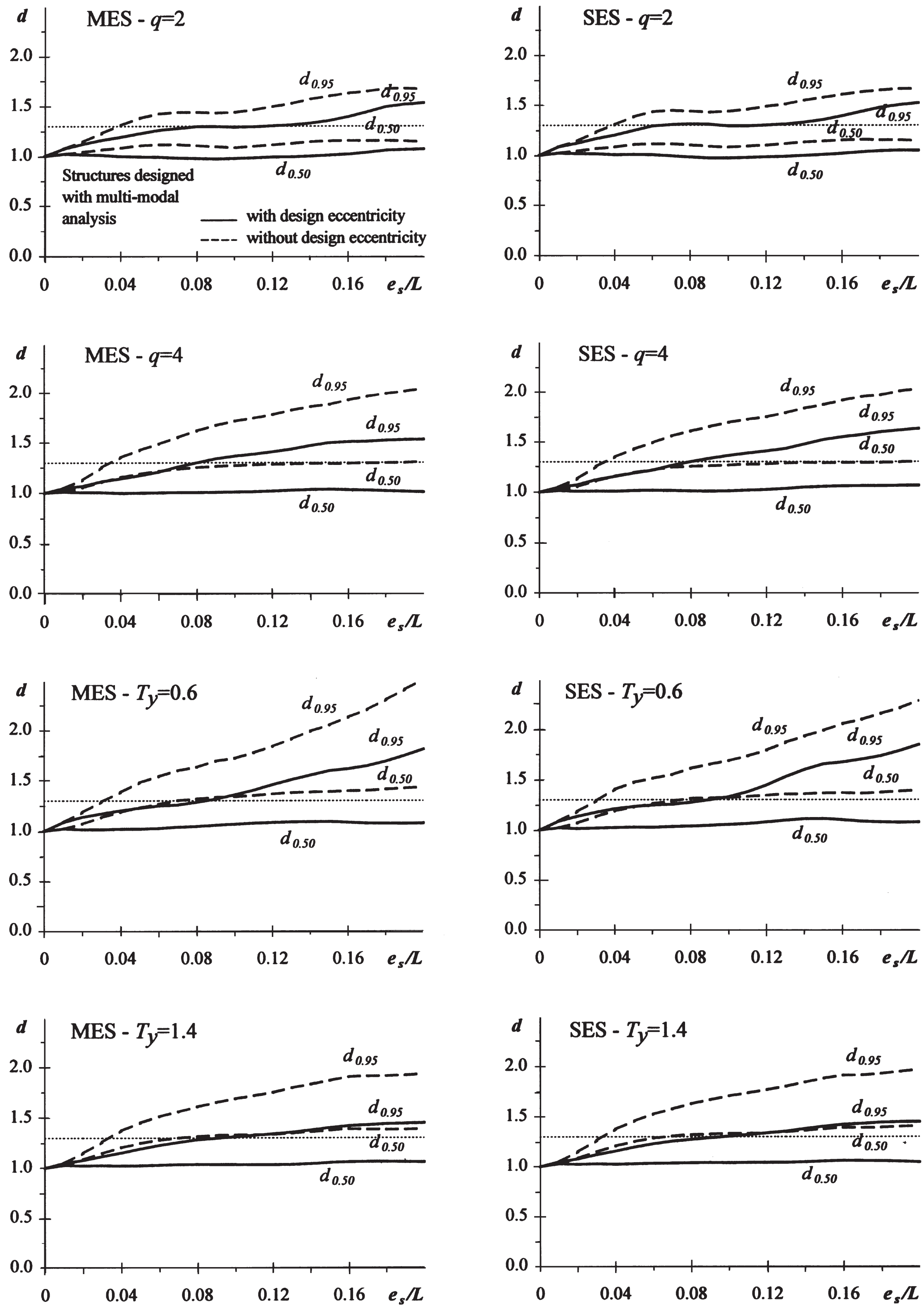

Fig. 16. Ductility demand of schemes designed with and without the proposed approach (constant parameters: $\Omega_\theta = 1$, $\gamma_x = 0.2$).

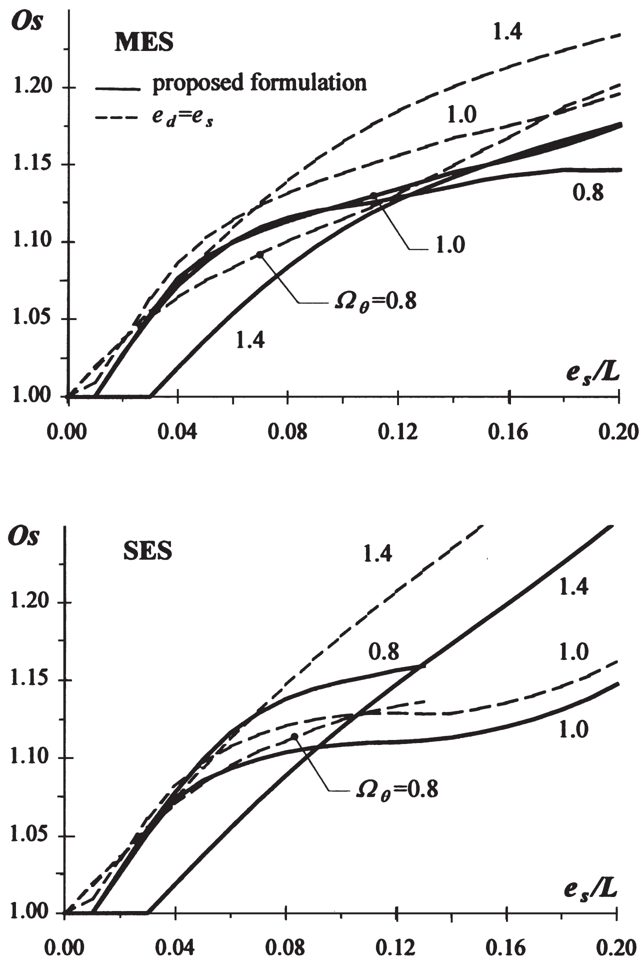

Fig. 17. Overstrength of systems designed according to the proposed procedure, compared to the one obtained by using $e_d = e_s$ (constant parameters: $\gamma_x = 0.2$, $T_y = 1$ s, $q = 5$).

stiff schemes need just a small overstrength even in the case of relevant eccentricities, while torsionally flexible structures require a large overstrength also for small eccentricities. The same figure shows a dotted line, as a term of comparison, which is the overstrength obtained by satisfying a clause given by some codes (like UBC), which require that the strength of the elements should not be reduced when a spatial analysis is performed; this is equivalent to considering a design eccentricity equal to the stiffness eccentricity ($e_d = e_s$). A comparison between the two sets of curves shows that the last provision ($e_d = e_s$) requires an overstrength that is acceptable in the case $\Omega_\theta = 1$, but it is insufficient for torsionally flexible structures and excessive for torsionally stiff schemes. The introduction of this prescription in seismic codes is probably a good solution to the problem of limiting ductility demand in asymmetric buildings, because of its simplicity, but it should be connected to explicit limitations to avoid torsional flexibility; at the same time the use of a more precise approach, like the one proposed in this paper, should be allowed.

There have been few researchers in the past who have proposed formulations of design eccentricity: Mittal and Jain [8] have studied the influence of the location of the plastic centre on the response of structures characterised by different inertial and mechanical properties and designed by means of static analysis. They have imposed the optimum value of the eccentricity of the plastic centre from the mass centre as that for which the demand of normalised ductility at the stiff edge equals that at the flexible side. Fig. 18 shows a comparison of the eccentricity of the plastic centre from the mass centre obtained by the present approach and that proposed by Mittal and Jain. Diagrams underline a good agreement for large structural eccentricity, while the values obtained by the authors are always lower for small structural eccentricity.

7. Conclusions

The proposed approach (i.e. to perform twice the multi-modal analysis, the first one with the mass centre

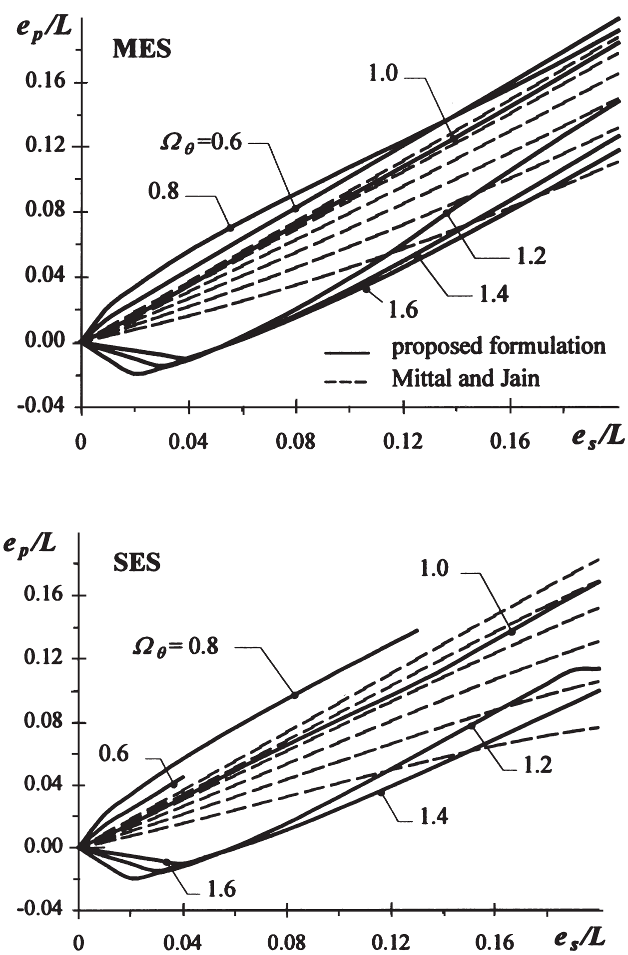

Fig. 18. Comparison of the eccentricity of plastic centre from mass centre, according to the proposed approach (continuous line) and the formulation by Mittal and Jain [8] (dashed line).

in its nominal position and the second one with the centre displaced by a quantity, named design eccentricity, towards the stiffness centre) appears to be a powerful tool for overcoming the problems connected to asymmetry, by limiting the ductility demand without a relevant increment of structural costs. The proposed formulation of design eccentricity has been tested for a wide set of values of geometrical and inertial parameters, proving a large effectiveness. The given formula may therefore constitute a good basis for an improvement of the European seismic code.

Appendix A

Properties of the transformation from "reference symmetric system" to "transformed symmetric system"

In a *transformed symmetric system* is

$$\sum_{i=1}^{n_b} k'_{ib} = \sum_{i=1}^{n_b} (k_{ib} + \beta_1 k_{ib}\xi_{ib}) = \sum_{i=1}^{n_b} k_{ib} + \beta_1 \sum_{i=1}^{n_b} k_{ib}\xi_{ib} = K_b \tag{A1}$$

$$\sum_{i=1}^{n_b} k'_{ib}\xi^2_{ib} = \sum_{i=1}^{n_b} (k_{ib} + \beta_1 k_{ib}\xi_{ib})\xi^2_{ib} = \sum_{i=1}^{n_b} k_{ib}\xi^2_{ib} + \beta_1 \sum_{i=1}^{n_b} k_{ib}\xi^3_{ib} = K_{\theta b} \tag{A2}$$

in which

$$\sum_{i=1}^{n_b} k_{ib}\xi_{ib} = \sum_{i=1}^{n_b} k_{ib}\xi^3_{ib} = 0$$

because of the symmetry of the *basic system*.The first moment of the new distribution about G_b is

$$\sum_{i=1}^{n_b} k'_{ib}\xi_{ib} = \sum_{i=1}^{n_b} (k_{ib} + \beta_1 k_{ib}\xi_{ib})\xi_{ib} = \sum_{i=1}^{n_b} k_{ib}\xi_{ib} + \beta_1 \sum_{i=1}^{n_b} k_{ib}\xi^2_{ib} = \beta_1 K_{\theta b} \tag{A3}$$

and the abscissa of the centre of rigidity of the *transformed basic system* is therefore

$$\xi'_{\mathrm{CR}b} = \frac{\sum_{i=1}^{n_b} k'_{ib}\xi_{ib}}{\sum_{i=1}^{n_b} k'_{ib}} = \frac{\beta_1 K_{\theta b}}{K_b} \tag{A4}$$

The torsional stiffness of the *transformed basic system* about its centre of rigidity ($K'_{\theta b}$) and that of the TSS about G ($K_{\theta G_y}$) can finally be expressed as

$$K'_{\theta b} = K_{\theta b} - K_b \xi'^2_{\mathrm{CR}b} \tag{A5}$$

$$K_{\theta G_y} = 2[K'_{\theta b} + K_b(d_b - \xi'_{\mathrm{CR}b})^2] = 2[K_{\theta b} + K_b d_b^2 - 2K_b d_b \xi'_{\mathrm{CR}b}] = 2K_{\theta b} + 2K_b d_b^2 - 4\beta_1 K_{\theta b} d_b \tag{A6}$$

It is therefore possible to obtain a TSS having a given translational and torsional stiffness simply by selecting a *basic system* with $K_b = 0.5\ K_y$ and evaluating the coefficient β_1 as

$$\beta_1 = \frac{2K_{\theta b} + 2K_b d_b^2 - K_{\theta G_y}}{4K_{\theta b} d_b} \tag{A7}$$

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