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Static versus modal analysis: influence on inelastic response of multi-storey asymmetric buildings

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Abstract The paper investigates the influence of design procedures on the seismic response of multi-storey asymmetric buildings. To this end, some structures are designed according to methods based on either static or modal analysis, with or without design eccentricities. The seismic response of these systems is determined by means of inelastic dynamic analyses and the design is thoroughly examined in order to explain the results of the dynamic analyses. Attention is basically focused on the ability of design methods to prevent asymmetric buildings from experiencing ductility demands much larger than those of the corresponding torsionally balanced systems. Numerical analyses underline that while design procedures based on either static or modal analysis are suitable for the design of torsionally rigid structures only those based on modal analysis lead to the satisfactory performance of torsionally flexible buildings. Furthermore, the study highlights the qualities of a design method proposed by the Authors. Its application does not require any explicit calculation of design eccentricities and leads to proper seismic response of both torsionally rigid and flexible asymmetric buildings.

Keywords Asymmetry · Multi-storey buildings · Design methods · Static analysis · Modal analysis · Design eccentricity

1 Introduction

1.1 Seismic response of asymmetric buildings

Asymmetric buildings often perform less well than in-plan regular systems, i.e., they undergo yielding during low intensity earthquakes and large ductility demands during moderate ground motions. This kind of behaviour has repeatedly been put down to the general

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ineffectiveness of seismic code provisions based on the mere application of static analysis. Much research has been carried out to find ways to improve the static method by means of design eccentricities (Goel and Chopra 1990; Chandler and Duan 1992; Duan and Chandler 1992; Tso and Zhu 1992; Zhu and Tso 1992). Two eccentricities are generally proposed to obtain satisfactory seismic performance of asymmetric structures, the first aiming at ensuring elastic behaviour during moderate earthquakes and the second at limiting maximum ductility demands to the demands of in-plan regular buildings in the event of strong ground motions. In this context, modal analysis has often been overlooked despite the unquestionable advantages it provides for the evaluation of the elastic behaviour of structures. Indeed, unlike static analysis, the standard application of modal analysis ensures a reliable estimate of the elastic response and, thus, does not require any design eccentricity. Furthermore, an analytical formula of the design eccentricity intended to limit ductility demands has been proposed by Ghersi and Rossi (2000).

1.2 Models for multi-storey asymmetric buildings

Idealised one-storey models have long been considered as the reference mathematical tool for the analysis of the seismic behaviour of in-plan irregular structures. Investigation of these models has gradually led to a basic comprehension of the dynamic behaviour of asymmetric schemes (Rutenberg et al. 1995; Rutenberg 2002) and to the introduction of relevant torsional provisions in national and international seismic codes. Nevertheless, the question whether the results should be extended to the inelastic behaviour of multi-storey structures has often been raised. Indeed, the use of this simplified model for the study of the influence of asymmetry on the seismic response of multi-storey systems is rigorously valid only for a restricted category of in-plan irregular buildings, called *regularly asymmetric* (Hejal and Chopra 1987), and in the case of an elastic structural behaviour. Many researchers (e.g., Duan and Chandler 1992; Moghadam and Tso 1996; Marino 2000; Stathopoulos and Anagnostopoulos 2003, 2005; De Stefano et al. 2006) have remarked that fundamental aspects of design and response of multi-storey systems (overstrength, influence of higher modes of vibration, change of dynamic properties of structures due to yielding, etc.) are neglected if buildings are schematized by means of one-storey models.

To obtain a more reliable and comprehensive knowledge of the behaviour of asymmetric buildings, most researchers nowadays recognise as compulsory the adoption of multi-storey schemes. Their modelling is strongly conditioned by two requirements which sometimes bring researchers to move in opposite directions. On one hand, there is the need to simplify the structural scheme, in all the geometric and mechanical aspects, in order to reduce the number of parameters involved in the structural response (e.g., strength and overstrength of the single cross-section). On the other hand, there is the desire to avoid applying strict behavioural hypotheses which lead to neglect significant characteristics of the structural response.

Aiming at obtaining a simple evaluation of the inelastic analysis of asymmetric structures, Chandler and Duan (1992) adopted a multi-storey shear-type model, i.e., a scheme with infinitely stiff beams. Unfortunately, as underlined by other researchers (Moghadam and Tso 1996; Stathopoulos and Anagnostopoulos 2003, 2005), it is simple to investigate the behaviour of this model but not very realistic. Indeed, plastic hinges develop only at the ends of columns and give rise to storey mechanisms which are always averted by seismic codes and avoided by practising engineers. With the aim of following more closely the main provisions of modern seismic codes, some other authors (Moghadam and Tso 1996, 2000; Marusic and Fajfar 2005; Stathopoulos and Anagnostopoulos 2005) have examined a multi-storey model with deformable beams. The strength of the cross-sections is determined in compliance with the capacity design criterion so as to favour the development of plastic hinges at the ends of beams and at the base of the first storey columns.

1.3 Aim of the study

This paper investigates the effectiveness of different design methods, based on static or modal analysis, in preventing large ductility demands in asymmetric structures.

Like other researchers, the Authors use multi-storey models with deformable beams and columns; however, in order to lighten the computational burden and ensure the global collapse mechanism required by seismic codes, infinite strength is assigned to all column cross-sections apart from those at the base of the first storey. The model highlights the main aspects of the seismic response of multi-storey framed asymmetric buildings designed in compliance with the capacity design criteria without questioning the reliability of code design rules in fulfilling the requirements of this design philosophy. The assumed behavioural simplicity also makes the post-processing of the numerical analysis results easy.

The study is divided into two parts. In the first, the seismic responses of a set of asymmetric buildings designed by static and modal analysis without any design eccentricity are compared. In the second, the same buildings are re-designed according to three methods characterised by different design eccentricities and structural analyses. The first design procedure is suggested by Chandler and Duan (1992) and is based on static analysis; the second and the third are proposed respectively by Ghersi and Rossi (2000) and Ghersi et al. (1999) and are based on modal analysis.

2 Analysed buildings

The numerical investigation concerns a set of six-storey framed buildings characterised by different structural eccentricity and torsional stiffness. The quality of their seismic response is assessed by comparison with that of the corresponding torsionally balanced systems. Accidental eccentricity is not considered either in the phase of design or in the numerical analyses.

2.1 General properties of the buildings

All the investigated buildings have rectangular decks ($L = 29.5$ m; $B = 12.5$ m) rigid in their own plane. Decks are supported by four seven-bay plane frames arranged along the X-axis and eight three-bay frames arranged along the Y-axis; all the frames are symmetrically disposed with respect to the geometrical centre C_G of the deck (Fig. 1). Masses are considered lumped into the decks and characterised by a radius of gyration about the mass centre C_M equal to 0.3 L. The distribution of the mass is assumed equal at all floors. Therefore, the mass centres are aligned along a vertical axis.

Beams and columns of each frame are characterised by one cross-section each only; the ratio I_b/I_c of the second moment of area of the beams over the second moment of area of the columns is equal to 0.354. The cross-sections of the elements vary from one plane frame to the other, but maintain the same ratio I_b/I_c within the single frame. Therefore, the plane frames have mutually proportional lateral stiffness matrices and give rise to regularly asymmetric systems. In all the examined buildings the cross-sections of the plane frames are symmetric with respect to the geometrical centre and, thus, the stiffness centres C_R coincide with the geometrical centre C_G of the deck.

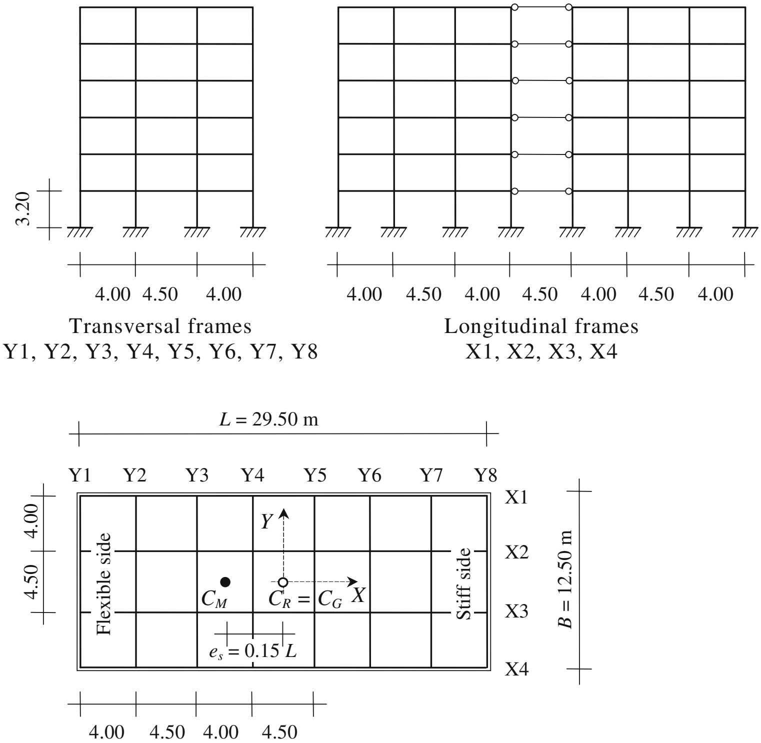

Fig. 1 The structural model of the analysed buildings

The stiffness of the plane frames is defined by means of an automatic procedure (Ghersi and Rossi 2000) so as to obtain given values of the elastic properties of the global scheme. More specifically, all the examined buildings have fundamental lateral periods of vibration in the X and Y directions (T_x and T_y) equal to 1 s and the ratio γ_x of the torsional stiffness due to the elements along the X-axis over the total torsional stiffness equal to 0.2. Furthermore, three different structural schemes are defined with uncoupled lateral-torsional frequency ratios Ω_θ equal to 0.6, 1.0 and 1.4.

Torsionally balanced systems (TB) are generated assuming a uniform distribution of masses, i.e., masses with their centre C_M coincident with C_G (these systems are also symmetric). Torsionally unbalanced systems (TU), instead, are obtained by shifting the position of the mass centre along the X-axis direction; indeed, despite the stiffness symmetry, the new position of the mass centre (not coincident with C_G) causes deck rotations under seismic actions. In accordance with other researchers, the side of the deck which experiences the largest displacements under horizontal static forces is named hereinafter *flexible side*, while the other is named *stiff side*. From each structural scheme two different TU systems are derived, having structural eccentricity e_s equal to 0.05 L (small eccentricity) and 0.15 L (large eccentricity). Only mass eccentric systems are examined because in the past (Goel and Chopra 1990; Ghersi and Rossi 2000; Fajfar et al. 2005) very small differences were highlighted between the seismic behaviour of mass and stiffness eccentric systems.

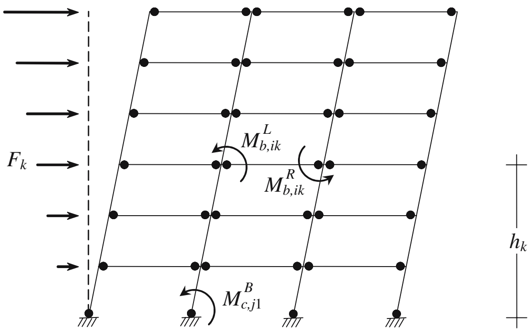

Fig. 2 Global collapse mechanism

2.2 Design of element strength

The strength of the end cross-sections of the resisting elements are evaluated by taking into account the effects of gravity loads and seismic actions. The design bending moment is defined as the maximum value of the bending moments deriving from the following two load combinations:

- $\gamma_g\ G_k + \gamma_q\ Q_k$
- $G_k + \psi_2 Q_k + \gamma_I E$

According to the Italian Application Document for Eurocodes, the coefficients γ_g, γ_q and ψ_2 are fixed to 1.4, 1.5 and 0.2, respectively. Furthermore, the importance factor is assumed equal to unity. The design spectrum is defined equal to the elastic response spectrum proposed by Eurocode 8 (1993) for hard layer soil (soil type A) divided by the behaviour factor q. The elastic response spectrum is characterised by a peak ground acceleration (PGA) equal to 0.35 g and by an equivalent viscous damping factor equal to 0.05. The behaviour factor q is assumed equal to 5.0. This assumption does not undermine the general validity of the study because the response of asymmetric and corresponding TB systems is not strongly influenced by the value of the behaviour factor when this parameter ranges from four to six (Ghersi and Rossi 2000).

For ease of computation, the design flexural strength of beams is considered independent of the sign of the bending moment and constant along the single element. The flexural strength of the single cross-section is assumed equal to the maximum design value of the bending moment of the element to which it belongs. The flexural strength of the base cross-section of the first storey columns is considered perfectly equal to the design value of the bending moment.

2.3 Overstrength

Actual frames can resist seismic shear forces much larger than the design forces because of cross-section overstrength, i.e., of strength values larger than those strictly required by the design analysis. Out of the many causes of overstrength, the use of multiple load combinations in the design phase has a significant role. Indeed, this kind of overstrength always exists in actual multi-storey structures and strongly influences their seismic response (Marino 2000; Stathopoulos and Anagnostopoulos 2005; De Stefano et al. 2006).

Overstrength parameters considered here are defined with reference to a plane frame which develops a global collapse mechanism under a set of horizontal forces F_k (Fig. 2). In this regard, the balance between the energy dissipated by plastic hinges and the work produced by the horizontal forces leads to the equation

$$\sum_{i=1}^{n_c} M_{c,i1}^{B} + \sum_{k=1}^{n_s} \sum_{i=1}^{n_b} \left(M_{b,ik}^{L} + M_{b,ik}^{R} \right) = \sum_{k=1}^{n_s} F_k \, h_k = V_b \, h_V \tag{1}$$

and, therefore, the base shear V_b may be expressed by the following relation

$$V_b = \frac{1}{h_V} \left[\sum_{i=1}^{n_c} M_{c,i1}^{B} + \sum_{k=1}^{n_s} \sum_{i=1}^{n_b} \left(M_{b,ik}^{L} + M_{b,ik}^{R} \right) \right] = \sum_{k=0}^{n_s} V_{b,k} \tag{2}$$

as the sum of the contributions $V_{b,k}$ provided at each level. In particular, the contribution of first storey columns is defined by the expression

$$V_{b,0} = \frac{1}{h_V} \sum_{j=1}^{n_c} M_{c,j1}^{B} \tag{3}$$

while that of the beams at the kth level is

$$V_{b,k} = \frac{1}{h_V} \sum_{i=1}^{n_b} \left(M_{b,ik}^{L} + M_{b,ik}^{R} \right) \tag{4}$$

Hence, the global overstrength O_S is defined by the relation

$$O_S = \frac{V_b^u}{V_b^d} = \frac{\sum_{i=1}^{n_c} M_{c,i1}^{B,u} + \sum_{k=1}^{n_s} \sum_{i=1}^{n_b} \left(M_{b,ik}^{L,u} + M_{b,ik}^{R,u} \right)}{V_b^d \, h_V} \tag{5}$$

where V_b^u is the base shear corresponding to the actual ultimate strength of the cross-sections and V_b^d the base shear corresponding to the strength required to sustain the design horizontal forces. Similarly, the storey overstrength $O_{S,k}$ is defined by the relations

$$\text{first storey columns} \quad O_{S,0} = \frac{V_{b,0}^u}{V_{b,0}^d} = \frac{\sum_{j=1}^{n_c} M_{c,j1}^{B,u}}{\sum_{j=1}^{n_c} M_{c,j1}^{B,d}} \tag{6}$$

$$\text{beams at the } k\text{th level} \quad O_{S,k} = \frac{V_{b,k}^u}{V_{b,k}^d} = \frac{\sum_{i=1}^{n_b} \left(M_{b,ik}^{L,u} + M_{b,ik}^{R,u} \right)}{\sum_{i=1}^{n_b} \left(M_{b,ik}^{L,d} + M_{b,ik}^{R,d} \right)} \tag{7}$$

As previously mentioned, the overstrength of the examined buildings is caused by gravity loads. In this study, gravity loads are distributed among frames in proportion to their lateral stiffness so as to produce global overstrength O_S equal to 1.5 in each frame of the TB systems. The height-wise distribution of the storey overstrength $O_{S,k}$ is equal for all the frames. However, as shown in Fig. 3 with reference to torsionally balanced systems, the storey overstrength is not constant along the height of the building because the effect of seismic design forces and design gravity loads varies in elevation. It should also be noted that the storey overstrength is almost identical in buildings designed by either static or modal analysis. More in detail, in the bottom cross-sections of the first storey columns, bending moments due to gravity loads are slightly different from zero and, therefore, overstrength is close to unity. Gravity and seismic design actions instead induce comparable internal actions in the beams of the examined structures and thus produce values of the storey overstrength $O_{S,k}$ which are larger than unity and increasing with the height of the floor above the base.

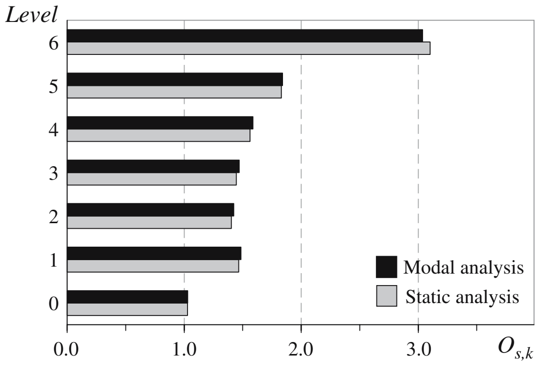

Fig. 3 Storey overstrength distribution of the transversal frames of the torsionally balanced buildings designed by static and modal analysis

2.4 Modelling

A three dimensional model is used which is constituted by plane frames connected by rigid diaphragms and endowed with stiffness and strength in their plane only. The frame members are schematised as one-dimensional deformable elements with concentrated plasticity at their ends; the moment-rotation relationship of the end cross-sections is assumed as elastic and perfectly plastic. The compatibility of the axial deformations of the elements which represent the same column but belong to two different frames is not considered. Furthermore, columns are subjected to independent uni-axial bending about two perpendicular axes and, thus, not to bi-axial bending moments.

Plastic hinges may develop only in beams and at the bottom of the first storey columns, because infinite values of strength are considered in all column cross-sections apart from those at the base. For the sake of simplicity, the plastic bending moment of columns is assumed to be independent of the variation of the axial force.

3 Seismic response of buildings designed by static and modal analysis

In the first part of the study, each torsionally balanced (or asymmetric) building is designed in two ways: by means of either static or modal analysis and without design eccentricity. The seismic response of the resulting structures is evaluated by dynamic nonlinear analyses and compared to that of the corresponding TU systems.

3.1 Design of the buildings and distribution of lateral strength

Owing to the absence of design eccentricities, floor masses are concentrated at nominal positions of mass centres.

In particular, when static analysis is used, the effect of the seismic action is evaluated by means of equivalent seismic forces characterised by an inverted triangular distribution along the height of the building. In this regard, some codes (e.g., Eurocode 8) suggest using reduction factors for the seismic forces of static analysis to obtain similar values of the design base shear forces produced by static and modal analyses. In the present study, instead, no reduction factor is applied to the base shear strength of static analysis because attention is focused on the ratio of response parameters of asymmetric and TB systems.

When modal analysis is adopted, the contributions of the first nine modes of vibration are combined according to the complete quadratic combination rule (CQC rule) by means of the correlation factors proposed by Der Kiureghian (1981).

In asymmetric buildings, deck rotations modify the seismic base shear of frames and, therefore, also the seismic internal actions of structural members with respect to those of the elements of the corresponding TB systems. The normalised values of the seismic design base shear v_b, i.e., the ratios of the seismic design base shear of frames of asymmetric systems to that of the same frames in the corresponding TB systems, are calculated to explain these differences in the seismic behaviour of buildings designed by either static or modal analysis. Such normalised values depend both on the properties of the asymmetric system (e_s and Ω_θ) and on the analysis method used for design. In particular, values smaller than unity indicate that the seismic design base shear forces of the frames under examination are smaller in the asymmetric system than in the corresponding TB system.

As shown in Fig. 4, when static analysis is used, the normalised values of the seismic design base shear v_b is larger than unity in frames on the flexible side and lower than unity in frames on the stiff side. Furthermore, the normalised value of the seismic design base shear varies proportionally to the distance of the frame from the stiffness centre C_R. With reference to the buildings examined in this paper, the largest decrease is achieved in the torsionally-flexible system ($\Omega_\theta = 0.6$) with $e_s = 0.15\ L$, where the normalised values of the seismic design base shear of some frames approach zero. Indeed, owing to the large deck rotations produced by the design analysis the design base shear of this building is negative on the stiff side and positive on the flexible side. As a result, the seismic design base shear has a null point within the deck and the normalised values of the same parameter, which are always positive, present a sharp variation where the base shear is equal to zero. It should be noticed that the presence of accidental eccentricity, not considered in this paper, would have ensured a minimum value of the lateral strength for all the plane frames.

If modal analysis is used, the plan-wise distribution of v_b is quite different. The normalised value of the seismic design base shear decreases below unity on the stiff side of systems only when systems have very high torsional stiffness ($\Omega_\theta = 1.4$). The same parameter is smaller than unity in the interior frames of all the other buildings.

3.2 Numerical analyses and response parameters

Time-history inelastic analyses are carried out for all the considered buildings by means of the DRAIN-building computer program (Prakash et al. 1992), properly modified to include elastic-perfectly plastic beam and column elements. Damping is considered by means of the Rayleigh formulation. Mass and stiffness coefficients are derived so as to have an equivalent viscous damping factor equal to 0.05 for the first and third modes of vibration. Nominal dead loads plus quasi-permanent live loads are assumed as initial gravity loads in the analyses.

Seismic action is applied along the Y-direction and simulated by a set of thirty artificial accelerograms matching the elastic response spectrum proposed by Eurocode 8 with reference to hard layer soil (soil A) and for an equivalent viscous damping factor equal to 5%. The accelerograms are defined by a stationary random process modulated by means of a trapezoidal intensity function characterised by a strong motion phase of 22.5 s (as recommended by EC8 for peak ground accelerations equal to 0.35 g) and by starting and ending connecting parts of 3 and 5 s, respectively. In compliance with EC8 no value of the mean elastic response spectrum of the accelerograms is more than 10% below the corresponding code value.

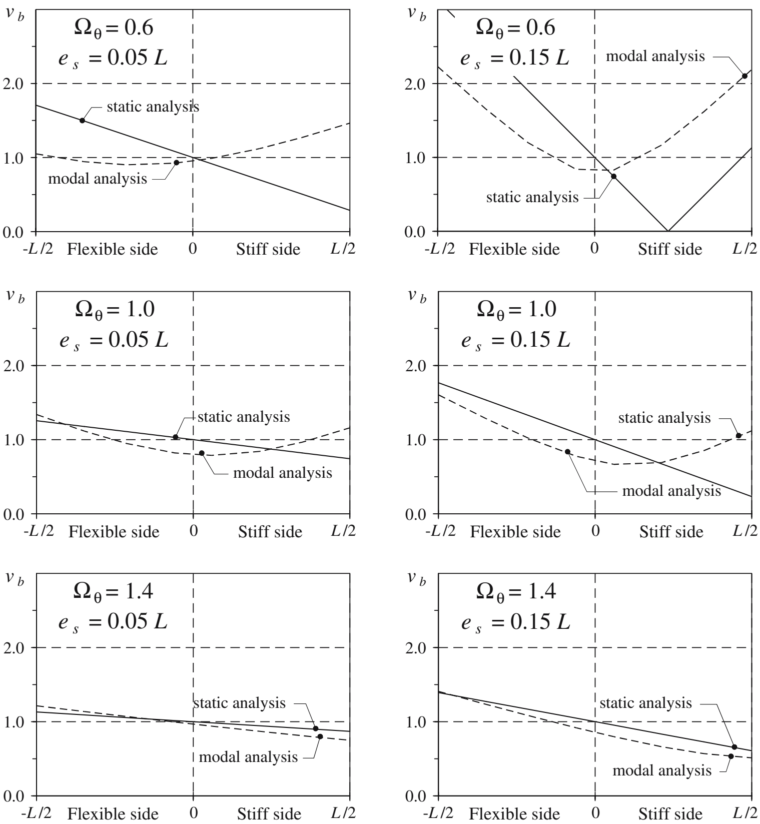

Fig. 4 Normalised design base shear force for the asymmetric buildings designed by static and modal analyses

Furthermore, the mean value of the pseudo-accelerations in the acceleration-sensitive region is not smaller than the value of the code response spectrum.

For each cross-section, the maximum plastic rotation θ_p obtained by the single inelastic analysis is used to evaluate the ductility demand D, i.e., the ratio of the maximum end rotation $\theta_{\max}$ to the yield rotation θ_y

$$D = \frac{\theta_{\max}}{\theta_y} = \frac{\theta_p + \theta_y}{\theta_y} \tag{8}$$

and the normalised ductility demand d, i.e., the ratio of the ductility demands required to asymmetric and corresponding TB systems. A statistical evaluation of the effect of the thirty accelerograms is provided by the mean value $\overline{d}$ of the normalised ductility demands. The behaviour of groups of sections of a single plane frame are finally synthesized by the mean value of the normalised ductility demands of first storey columns $\overline{d}_c$ and beams at a generic storey $\overline{d}_b$.

3.3 Results of numerical analyses

For a better comprehension of the results it is useful to mention that, during earthquakes, random yielding of resisting elements along the direction of the ground motion causes instantaneous variation of the lateral and torsional stiffness of the system and oscillation of the stiffness centre C_R about its nominal position. The lateral stiffness generally decreases more than the torsional stiffness and the uncoupled lateral-torsional frequency ratio increases because resisting elements arranged along the orthogonal direction often remain elastic (Ghersi and Rossi 2000; 2001). Due to this phenomenon, the effect of deck rotation on the seismic response of asymmetric buildings (i.e., the variation of the displacements with respect to those of the corresponding TB systems) is less relevant in the inelastic than in the elastic range (e.g., see Goel and Chopra 1990). As a consequence, large ductility demands are expected in members where (in accordance with the elastic design analysis) strength is reduced with respect to that of the corresponding torsionally balanced system. The risk of extremely high values of the ductility demand is usually avoided by the application of accidental eccentricity.

As is shown in Fig. 5, the mean value of the normalised ductility demand of the first storey columns $\bar{d}_c$ is larger than unity in all the systems with the exception of those which are torsionally rigid and endowed with small eccentricity. The largest values of $\bar{d}_c$ are obtained in frames designed by a seismic shear force significantly smaller than that of the corresponding TB system (see Fig. 4). The result is consistent with those obtained on single-

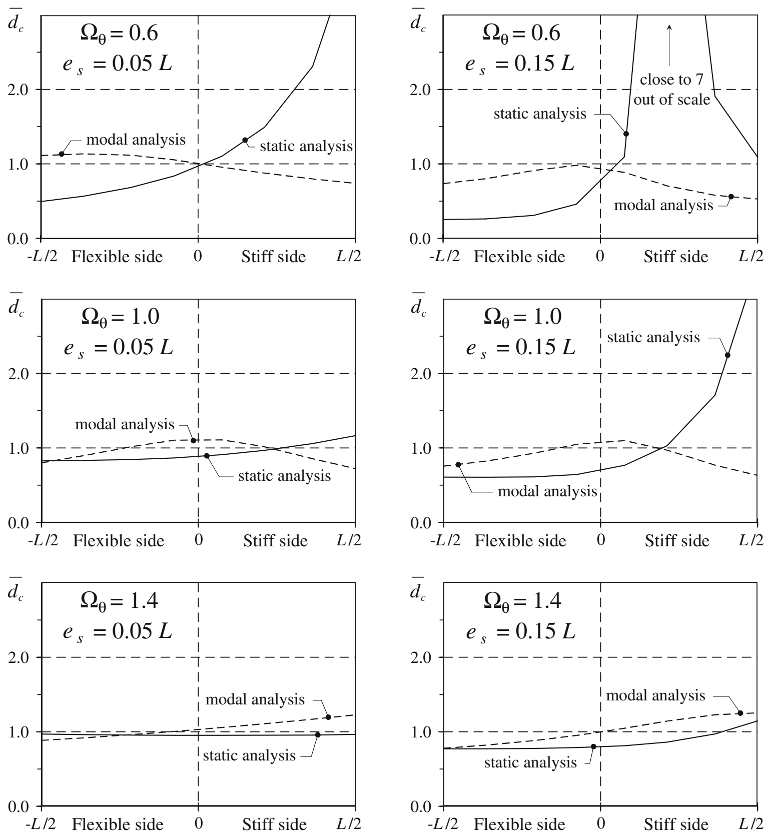

Fig. 5 Mean normalised ductility demand of the columns of asymmetric systems designed by standard static and modal analyses

storey systems because no significant overstrength is present in columns (De Stefano et al. 2006). More interesting observations may arise from the comparison of the seismic behaviour of buildings designed by static and modal analysis. In torsionally stiff asymmetric buildings ($\Omega_\theta = 1.4$), the maximum normalised ductility demand of the first storey columns $\bar{d}_c$ is never significantly larger than unity regardless of the method of analysis adopted in design. In the worst case (the system characterised by $\Omega_\theta = 1.4$, $e_s = 0.15\ L$ and designed by modal analysis), the maximum normalised ductility demand is about 1.25. In the case of buildings with smaller torsional stiffness, the maximum normalised ductility demand of the first storey columns is very large if static analysis is applied (when $\Omega_\theta = 0.6$ and $e_s = 0.15\ L$, $\bar{d}_c$ is close to seven) but only slightly larger than unity if modal analysis is used. This is because, as is evident from Fig. 4, modal analysis does not allow the significant reduction of the design base shear produced by static analysis on the stiff side of these buildings.

For all the buildings, the mean value of the normalised ductility demand of the beams $\bar{d}_b$, shown in Fig. 6 only for buildings with large eccentricity ($e_s = 0.15\ L$), has similar values at all the floors (apart from the top one, as discussed later on). Furthermore, the normalised ductility demand is larger than unity in the beams located in the part of the building where the strength is reduced with respect to that of the corresponding TB building.

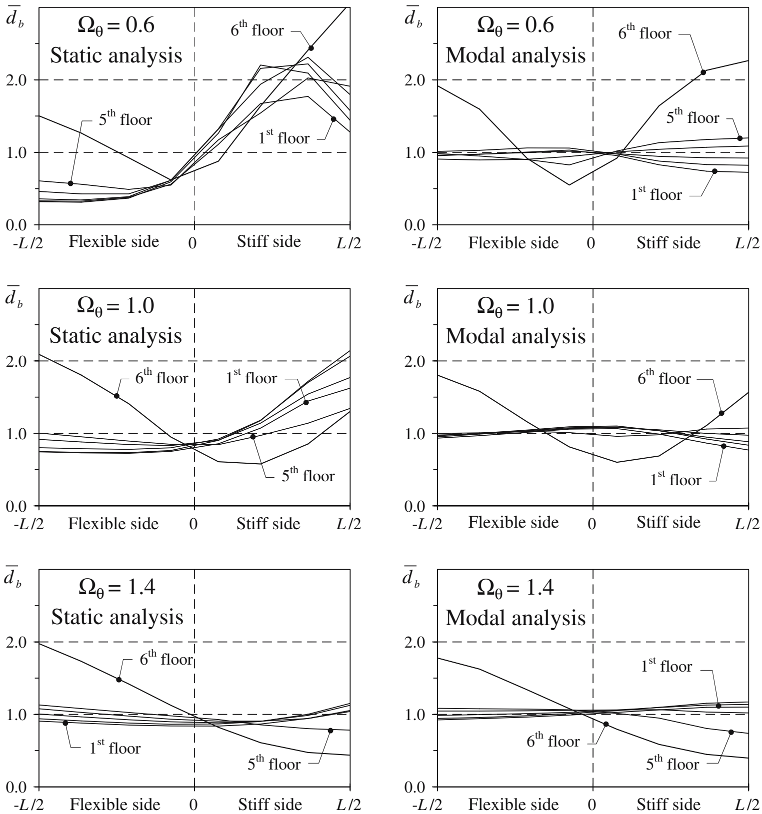

Fig. 6 Mean normalised ductility demand of the beams of asymmetric systems designed by standard static and modal analyses ($e_s = 0.15\ L$)

The comparison of Figs. 5 and 6 shows that the trend of $\bar{d}_b$ is qualitatively similar to that of $\bar{d}_c$ but that the maximum value of $\bar{d}_b$ is always smaller than that of $\bar{d}_c$. This result can be explained by the fact that beams, unlike columns, are always provided with remarkable overstrength $O_{s,k}$ (Fig. 3) because bending moments due to gravity loads constitute a relevant part of the total flexural strength of beams. In the design of asymmetric structures, only bending moments due to seismic forces are modified with respect to those of the corresponding TB systems. Therefore, the decrease of the design flexural strength of the beams in terms of percentage is smaller than that of the columns and the latter is almost equal to that of the base shear force (Fig. 4). Consequently, the normalised ductility demand of beams is smaller than that of columns.

The large overstrength of the beams of the top floor (Fig. 3) explains the different behaviour of the beams of this floor with respect to that of the others. The design strength at the top floor is due to the non-seismic load combination and is, therefore, not influenced by seismic design actions and provisions. As a consequence, the beam strength is the same in TB as in TU systems designed by either static or modal analysis. Conversely, in TU systems the maximum displacements and, thus, the ductility demands are amplified by the deck rotation. Nevertheless, it should be noted that the absolute ductility demand at the top floor is almost always smaller than that of the other floors (e.g., see Fig. 7).

In conclusion, the results indicate that the design procedures based on both static and modal analysis must be improved to avoid normalised ductility demands larger than unity. However, this need is less pressing for modal analysis as it never leads to the poor seismic performance observed in torsionally flexible asymmetric buildings ($\Omega_\theta = 0.6$–1.0) designed by static analysis.

4 Seismic response of buildings designed by non-standard design methods

In the past, a design approach (later on called *non-standard*) consisting in a double structural analysis with proper design eccentricities has been proposed and subsequently included in some seismic codes to improve the design of asymmetric structures by either static or modal analysis. In order to investigate the effect produced by the application of design eccentricities, the asymmetric buildings described in Sect. 2 are designed according to three non-standard design methods, selected as representative of approaches based on static and modal analysis. In particular, the first design method is based on static analysis and is proposed by Chandler and Duan (1992); the other two are based on modal analysis and are suggested by the Authors on various occasions. In order to investigate their reliability, the seismic response of these

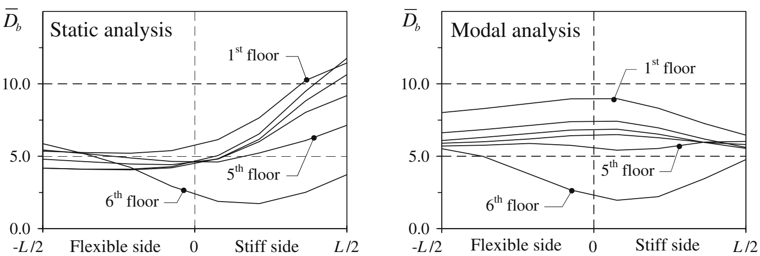

Fig. 7 Ductility demand of the beams of asymmetric systems designed by standard static and modal analyses ($e_s = 0.15\ L$ and $\Omega_\theta = 1.0$)

buildings is evaluated by dynamic non-linear analysis and the performance of corresponding TU and TB systems is compared.

4.1 Static and modal non-standard analyses

As previously mentioned, if static analysis is applied two design eccentricities are necessary. The first, called *primary design eccentricity*, is intended to guarantee an accurate estimate of the seismic response of the elements on the flexible side of the structure while the second, called *secondary design eccentricity*, is considered in order to reduce the favourable effect of the elastic deck rotation on the stiff side of the structure; as reported by several researchers (e.g., see Ghersi and Rossi 2000; Fajfar et al. 2005) this effect may disappear almost completely in the inelastic range of behaviour. The above-mentioned design eccentricities are larger and smaller than the structural eccentricity e_s, respectively.

The elastic seismic response of asymmetric buildings depends on many parameters (e_s, Ω_θ etc.). In some cases it is close to that produced by the application of equivalent seismic forces at the mass centres (e.g., in torsionally rigid systems, particularly for small values of structural eccentricity) while in other cases it is totally different (e.g., in torsionally flexible systems with large structural eccentricity). Owing to the dependence of the elastic response on both e_s and Ω_θ, it is difficult to define simple analytical equations of the above-mentioned design eccentricities, i.e., of eccentricities able to lead to reliable estimate of the elastic response of both torsionally flexible and stiff systems, (Calderoni et al. 2002; Anastassiadis et al. 1998). For this reason, when dealing with design procedures based on static analysis, formulations of design eccentricities found in literature are often restricted to torsionally rigid systems ($\Omega_\theta > 1$).

If modal analysis is used, the actual elastic response of both torsionally flexible and rigid structures may be predicted with excellent approximation without use of design eccentricity. As in the case of the static approach, a second analysis is however required to take into account that in the inelastic range of behaviour the favourable effect of the deck rotation is generally much smaller than in the elastic field. Hence, in this second analysis the mass centre is shifted from its nominal position towards the stiffness centre of a quantity e_d, named *design eccentricity*.

4.1.1 Design method of Chandler and Duan

The design method proposed by Chandler and Duan (1992) is based on static analysis and requires the evaluation of two design eccentricities. The primary design eccentricity e_{d_1} is evaluated as

$$e_{d_1} = A_1 e_s \tag{9}$$

where

$$A_1 = 2.6 - 3.6\,(e_s/L) \geq 1.4 \tag{10}$$

while the secondary design eccentricity e_{d_2} is defined by the relation

$$e_{d_2} = 0.5 e_s \tag{11}$$

Furthermore, Chandler and Duan recommend that in the presence of large structural eccentricity a behaviour factor q' smaller than q should be used for the evaluation of the strength of the outermost stiff-edge resisting elements in order to avoid large ductility demands coming to bear on these elements. The reduced behaviour factor q' is evaluated as follows

$$
\begin{array}{ll}
q' = q & 0 < e_s/L < 0.1 \\
q' = q - (e_s/L - 0.1)\ /0.1 & 0.1 < e_s/L < 0.2 \\
q' = 0.8\, q & 0.2 < e_s/L
\end{array}
\tag{12}
$$

The behaviour factor is linearly decreased in this study from the stiffness centre (where the value q was applied) to the stiff edge (where the value q' was applied) so as to increase the strength of all the vertical resisting elements on the stiff side. This design provision was not specified by Chandler and Duan, because they adopted a structural model with three resisting elements only. Chandler and Duan also suggested using a concentrated force at the top of the building for asymmetric systems having structural eccentricity larger than 0.2 L. This provision is not considered in the present study because the structural eccentricity is lower than 0.2 L in all the investigated structures.

It should also be noted that the described design method was recommended by Chandler and Duan for torsionally stiff structures only and verified specifically in asymmetric buildings having uncoupled lateral-torsional frequency ratio Ω_θ equal to 1.0 (Duan and Chandler 1992). Nevertheless, for the sake of completeness the procedure is applied here also to torsionally flexible structures. Therefore, the ineffectiveness of the method in such schemes must not be ascribed to the procedure itself. In fact, as shown in the previous section, static analysis is not reliable enough to predict the seismic response of systems having very low torsional stiffness and the design method proposed by Chandler and Duan does not suggest a solution to this problem.

4.1.2 Design method of Ghersi and Rossi

The design procedure proposed by Ghersi and Rossi (2000) is based on modal analysis. Like the method suggested by Chandler and Duan, this method requires that the seismic analysis should be performed twice. Nevertheless, it involves the evaluation of only one design eccentricity e_d. The mass centres remains in their nominal position in the first design analysis and move towards the stiffness centres in the second design analysis. The design eccentricity e_d is defined by the equations

$$
e_d = \max \begin{cases} k(e_s - e_r) \\ 0.6\, e_s \end{cases}
\tag{13}
$$

where:

$$
k = \max \begin{cases} 3.3 - 2.5\, \Omega_\theta + 0.04\, q \\ 1 \end{cases}
\tag{14}
$$

$$
e_r = \max \begin{cases} 0.1(0.5\, \Omega_\theta - 0.4) L \\ 0.01\, L \end{cases}
$$

The formulas reported above, resulting from the study of the seismic response of mono-eccentric one-storey models subjected to mono-directional ground motions, were calibrated so as to limit the mean and characteristic values of the maximum normalised displacement ductility demands of both torsionally flexible and stiff systems to 1.0 and 1.3, respectively (Ghersi and Rossi 2000). The method has subsequently been verified with reference to mono-eccentric one-storey models subjected to bi-directional ground motions (Ghersi and Rossi 2001) and to bi-eccentric models (Ghersi and Rossi 2006).

4.1.3 The proposed design method

A different design method based on modal analysis has more recently been proposed by Ghersi et al. (1999). This procedure considers a double application of modal analysis to fulfil the requirements of a dual-level design, but without explicit calculation of any design eccentricity. Indeed, in the first analysis mass centres are considered in their nominal positions and all the degrees of freedom of the decks are taken into account (3D analysis). In the second analysis, instead, deck rotations are restrained (2D analysis) according to a design strategy suggested by some major seismic codes, e.g., the Uniform Building Code (ICBO 1997). The method takes advantage from the ability of modal analysis to predict in asymmetric systems the amplification of the seismic response with respect to that of the corresponding TB system; at the same time it does not allow for any reduction of the seismic response due to deck rotation.

This design method is much easier to handle than those presented before because of the lack of design eccentricities. Indeed, it should be noticed that calculation of any design eccentricity preliminarily requires evaluation of the parameters e_s and Ω_θ, which can be rigorously defined only with reference to regularly asymmetric buildings and evaluated in actual buildings only by means of complex and generally approximate procedures (Calderoni et al. 2002, Makarios and Anastassiadis 1998a and b; Marino and Rossi 2004).

4.2 Distribution of lateral strength

The normalised values of the seismic design shear force of the frames of the structures designed by the described non-standard methods are represented in Fig. 8. As is evident from the comparison of Figs. 4 and 8, the normalised values of the seismic design shear force of the vertical resisting elements on the flexible side of the buildings designed according to the Chandler and Duan method are larger than those of the corresponding buildings designed by the standard static analysis. Furthermore, the decrease of the normalised values of the seismic design shear force below unity is smaller for the resisting elements located on the stiff side (in the case of the building with $e_s = 0.15\ L$ and $\Omega_\theta = 0.6$, v_b is even larger than unity). These differences are due to primary and secondary design eccentricities, which are larger and smaller than e_s, respectively. The reduced value of the behaviour factor adopted for the frames of the stiff side in buildings having $e_s = 0.15\ L$ also contributes to limit the decrease of the normalised values of the seismic design shear force below unity. Both non-standard methods based on modal analysis similarly modify the plan-distribution of v_b obtained by the simple use of modal analysis. In particular, where standard modal analysis (Fig. 4) predicts normalised values of the seismic design base shear force smaller than unity, the non-standard methods based on modal analysis lead to values of the same parameter equal (or very close, in the case of the design method proposed by Ghersi and Rossi) to unity. Either no difference or a very small one is observed, however, where modal analysis predicts normalised values of the seismic design base shear force larger than unity. Small differences between the two non-standard design methods based on modal analysis are evident only with reference to torsionally-flexible systems; in these systems the method proposed by Ghersi and Rossi is slightly more conservative in the resisting elements located on the flexible side.

The comparison of the normalised values of the design shear force required by the non-standard methods under examination proves that the design method of Chandler and Duan is always more conservative for frames of the flexible side. The difference is small for torsionally-stiff buildings, but also extremely large for torsionally-flexible buildings. This result

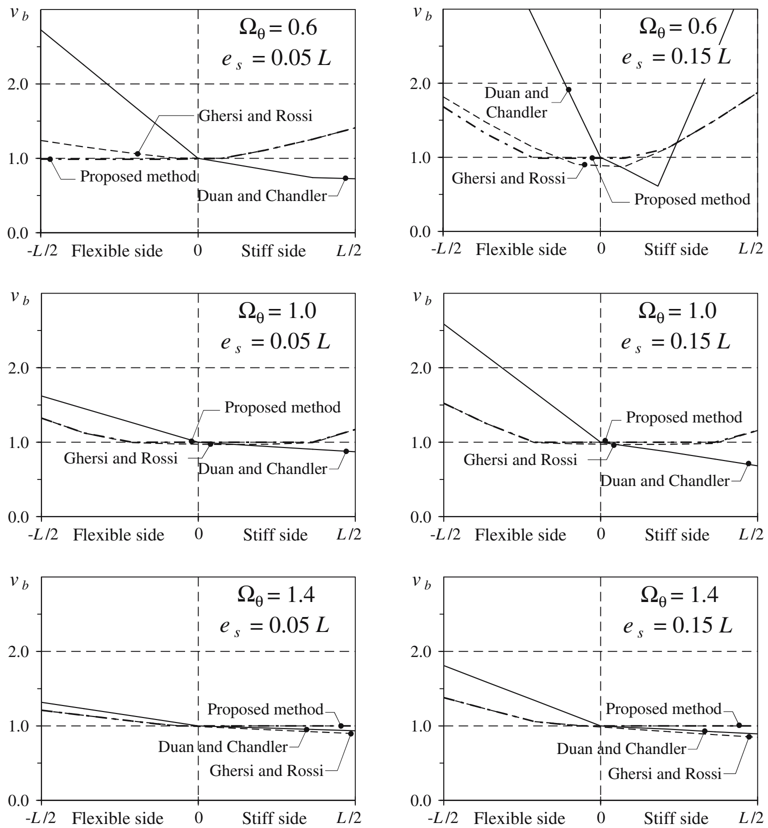

Fig. 8 Normalised design base shear force for the asymmetric systems designed by methods of Duan and Chandler, Ghersi and Rossi and according to the proposed design procedure

confirms that torsionally flexible schemes do not fall into the range of application of the design method proposed by Chandler and Duan. With the exception of torsionally-flexible systems, all the examined non-standard design methods require similar design base shear forces for the frames of the stiff side. Large differences, either positive or negative, may be observed on the stiff side of torsionally-flexible systems.

4.3 Results of numerical analysis

The comparison of the design methods in terms of normalised ductility demands of first storey columns $\overline{d}_c$ is shown in Fig. 9. As non-standard methods prevent excessive reductions of the seismic design shear force below the value of the corresponding TB system, $\overline{d}_c$ is now smaller than unity in all the buildings apart from the torsionally flexible building which is characterised by small eccentricity ($\Omega_\theta = 0.6$, $e_s = 0.05\ L$) and designed according to Chandler and Duan. Furthermore, the results of the non-linear dynamic analyses of the buildings designed by the non-standard methods based on modal analysis are nearly identical. This result was expected because the above-mentioned design methods basically lead to the same plan-wise distribution of the element strength, as shown in Sect. 4.2 (Fig. 8).

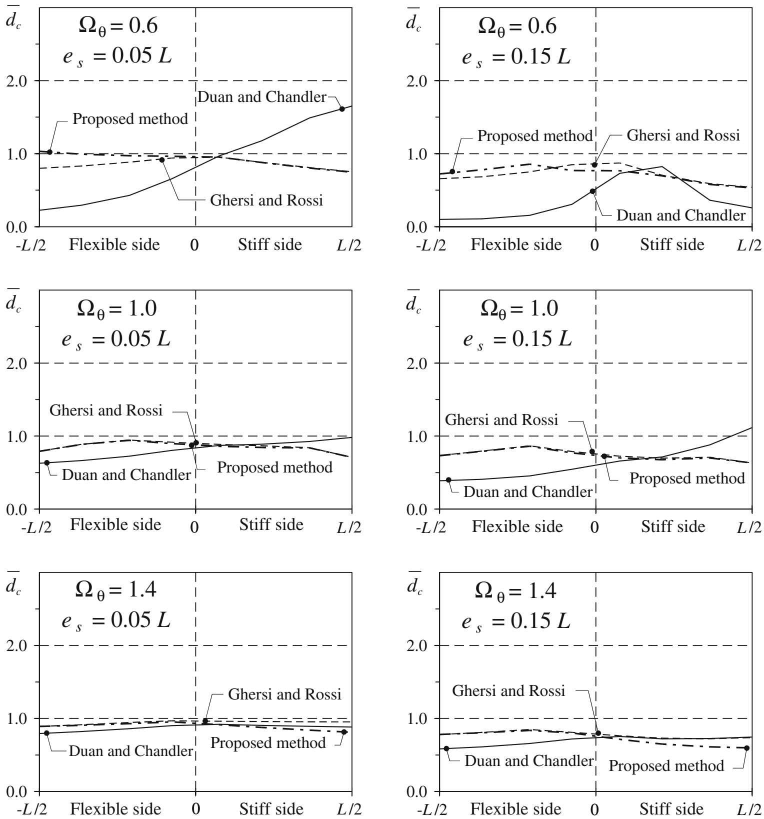

Fig. 9 Mean normalised ductility demand of the columns of asymmetric systems designed by means of non-standard procedures

However, remarkable differences may sometimes be observed between the ductility demands of structures designed by non-standard methods based on either static or modal analyses (Fig. 9). For torsionally-stiff buildings ($\Omega_\theta = 1.4$), all the non-standard methods ensure the reduction of the ductility demand of the first storey columns below that of the corresponding TB systems and do not lead designers to oversize such elements ($\overline{d}_c$ is always close to one). As regards structures with $\Omega_\theta = 1.0$, this conclusion is still valid if small structural eccentricities are considered ($e_s = 0.05\ L$). In the presence of moderate or large structural eccentricity the design method proposed by Chandler and Duan is significantly more conservative for the flexible side of the building but not for the stiff side. In the case of $e_s = 0.15\ L$, for instance, the ductility demand produced in the first storey columns of the flexible side by the examined design methods based on static and modal analysis is close to 0.4 and 0.8, respectively. Finally, for torsionally-flexible buildings ($\Omega_\theta = 0.6$), the results demonstrate that, while methods based on modal analysis still produce satisfactory seismic performance, those based on static analysis need to be adjusted by design eccentricity formulations more complex than those proposed by Chandler and Duan. In fact, torsionally-flexible systems designed by the latter method are sometimes undersized (for $\Omega_\theta = 0.6$ and $e_s = 0.05\ L$, $\overline{d}_c$ is equal to about 1.60 on the stiff side) and at other times largely oversized (for $\Omega_\theta = 0.6$ and $e_s = 0.15\ L$, $\overline{d}_c$ attains values close to zero).

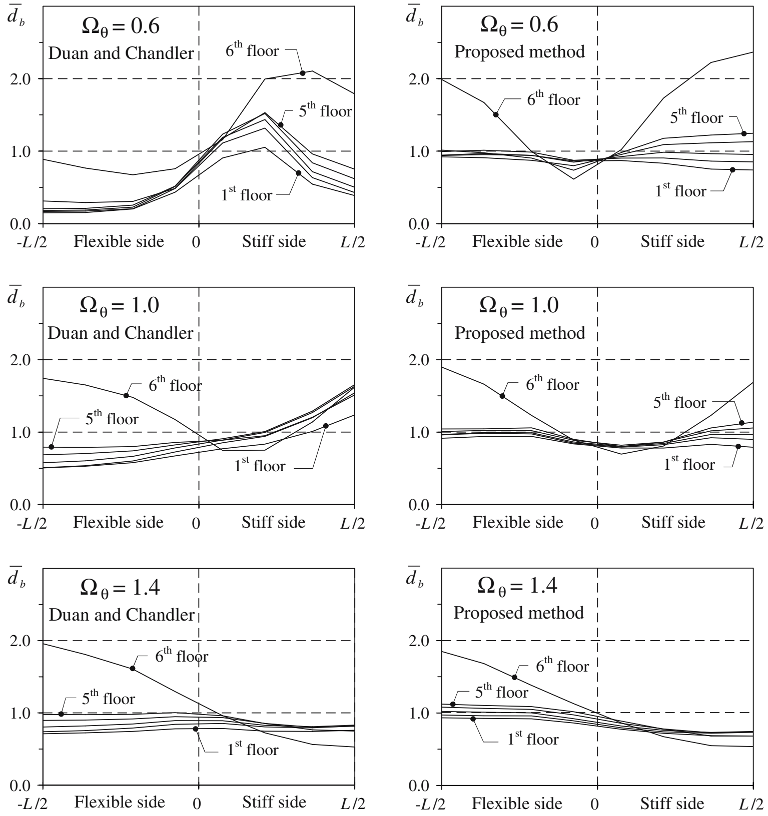

Fig. 10 Mean normalised ductility demand of the beams of asymmetric systems designed by non-standard methods ($e_s = 0.15\ L$)

The plan-wise distribution of the normalised ductility demand of the beams $\bar{d}_b$ is shown in Fig. 10. The results refer only to the buildings with large eccentricity and designed according to either the method suggested by Chandler and Duan or that proposed in this paper; results obtained by the method of Ghersi and Rossi are not shown because they are very similar to those of the method proposed by Ghersi et al. (1999), as we have already observed in columns. The analysis confirms that overstrength remarkably influences the ductility demand of beams (De Stefano et al. 2006). In the lower floors, where overstrength is relatively small (Fig. 3), the plan-wise distribution of $\bar{d}_b$ is similar to that of $\bar{d}_c$. Therefore, the observations previously developed to explain the effectiveness of non-standard design methods in limiting the ductility demand of columns can be repeated here with reference to the beams of the lower floors. For beams of the upper floors, instead, the larger overstrength strongly contributes to reduce the effectiveness of all the considered design methods. This is particularly evident in the beams of the top floor. The bending moment due to gravity loads, as in the case of the standard application of static and modal analyses, largely prevails over that produced by seismic actions. Regardless of the design method, top floor beams of asymmetric buildings are provided with the same strength as those of the corresponding TB systems. As a consequence, $\bar{d}_b$ reaches values much larger than unity (close to 2) and has a plan distribution which is similar for all the considered design methods.

5 Conclusion

In this paper the effectiveness of static and modal analysis for the design of asymmetric structures has been investigated. The research analyses and compares the seismic response of a set of multi-storey asymmetric buildings designed by standard static and modal analyses and by non-standard design methods based on static and modal analyses (Chandler and Duan's method, Ghersi and Rossi's method and a method proposed by the Authors). The following findings were obtained.

- Both static and modal analyses of asymmetric structures need to be adjusted by appropriate design eccentricities if ductility demands of members larger than those of the corresponding torsionally balanced systems are to be avoided. Nevertheless, standard modal analysis never leads to the poor seismic performance observed in buildings with moderate or small torsional stiffness ($\Omega_\theta = 1.0$ and 0.6) and designed by static analysis.
- The design method of Chandler and Duan (based on static analysis) improves the seismic response with respect to that of buildings designed by the standard application of static analysis. For Ω_θ larger than 1.0 the design method of Chandler and Duan leads to ductility demands of members similar to those of the corresponding torsionally balanced systems. However, this method is not appropriate for the design of torsionally-flexible buildings, because it is inadequate and overconservative for small and large values of the structural eccentricity, respectively.
- Modal analysis, if adjusted by proper design eccentricities, represents a valid design tool for asymmetric structures. Indeed, all the buildings designed by the non-standard design methods based on modal analysis have good seismic performance; i.e., ductility demands close to those of the corresponding torsionally balanced systems. A satisfactory seismic response is obtained also in the case of torsionally-flexible buildings.
- The design method proposed by the Authors leads to a plan-wise distribution of strength very similar to that provided by the method of Ghersi and Rossi. Nevertheless, it does not require explicit evaluation of any design eccentricity. This is an easy method to handle because the evaluation of design eccentricities requires the knowledge of the elastic parameters e_s and Ω_θ, which can be evaluated in actual buildings only by means of complex and approximate procedures.

Appendix – Notation

B	Dimension of the deck, measured along Y-axis
C_G	Geometrical centre of the deck
C_M	Mass centre
C_R	Stiffness centre; design forces applied at C_R at every floor level induce only translation of the deck
D	Ductility demand of a cross-section
d	Normalised ductility demand, i.e. ratio of the ductility demand of a cross-section of the TU system to that of the same cross-section of the corresponding TB system
$\overline{d}$	Mean value of the normalised ductility demand, computed with reference to the thirty considered accelerograms

$\overline{d}_b$	Mean normalised ductility demand of the beams of a plane frame, at a generic storey
$\overline{d}_c$	Mean normalised ductility demand of the first-order columns of a plane frame
E	Design value of the seismic action
e_d	Design eccentricity, according to Ghersi and Rossi
e_{d_1}, e_{d_2}	Primary and secondary design eccentricity, according to Chandler and Duan
e_s	Structural eccentricity, i.e., distance between mass centre and stiffness centre
F_k	Horizontal force applied to the kth floor
G_k	Characteristic value of dead loads
h_k	Height (above the base of the frame) of the kth floor.
h_V	Height (above the base of the frame) of the centre of horizontal forces F_k
I_b	Second moment of area of a beam cross-section
I_c	Second moment of area of a column cross-section
L	Dimension of the deck, measured along X-axis
$M^L_{b,ik}$, $M^R_{b,ik}$	Flexural strength of the left and right end of the ith beam at the kth storey
$M^B_{c,j1}$	Flexural strength of the bottom end of the jth column at the first storey
n_b	Number of bays of the frame
n_c	Number of columns of the frame
n_s	Number of storeys of the frame
O_S	Global overstrength
$O_{S,0}$	Overstrength of first storey columns
$O_{S,k}$	Overstrength of beams at the kth level
q	Behaviour factor
q'	Reduced behaviour factor, according to Chandler and Duan
Q_k	Characteristic value of live loads
TB	Acronym for torsional balanced system, i.e., a scheme with $C_M = C_R$
TU	Acronym for torsional unbalanced system, i.e., a scheme with $C_M \neq C_R$
T_x, T_y	Fundamental lateral periods of vibration in X and Y direction
V_b	Base shear
v_b	Normalised design seismic base shear, i.e., ratio of the seismic design base shear in a frame of an asymmetric system over that of the same frame in the corresponding TB system

$V_{b,0}$	Contribution to the base shear, due to the first storey column strength
$V_{b,k}$	Contribution to the base shear, due to the beam strength at the kth level
X, Y	Reference axes for the plan of the building
γ_g, γ_q	Partial safety factors of loads, for the ultimate limit state approach
γ_I	Importance factor
γ_x	Ratio of the torsional stiffness due to the elements along the X-axis to the total torsional stiffness
θ_{max}	Maximum end rotation
θ_p	Maximum plastic end rotation
θ_y	Yield rotation
ψ_2	Combination coefficient for the quasi-permanent value of live loads
Ω_θ	Uncoupled lateral-torsional frequency ratio, i.e., ratio of the torsional to lateral frequencies of the torsionally balanced system (Hejal and Chopra 1987)

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