

Corso di Laurea in Ingegneria Edile-Architettura

Tecnica delle costruzioni e Laboratorio
A.A. 2023/2024

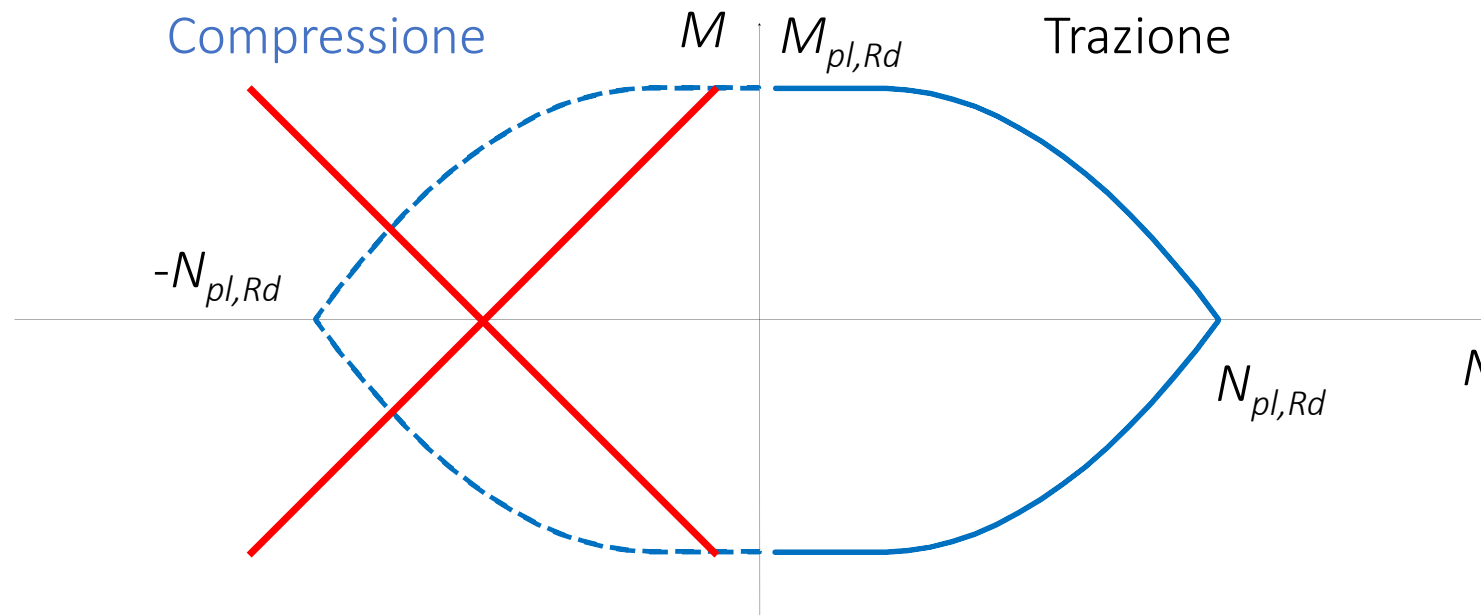
Prof. Edoardo M. Marino

Pressoflessione delle aste

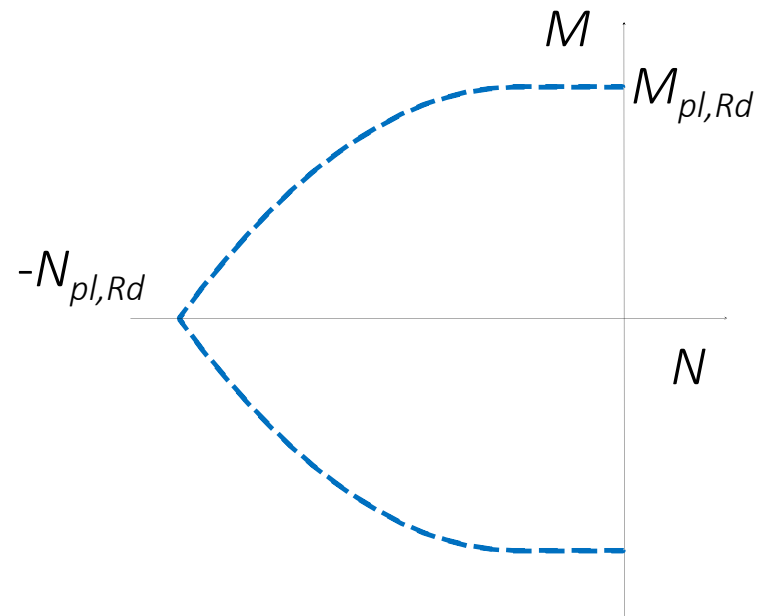
Domini di resistenza - stato limite ultimo

Si possono ottenere semplicemente ribaltando il dominio M-N costruito nel caso di tenso-flessione?

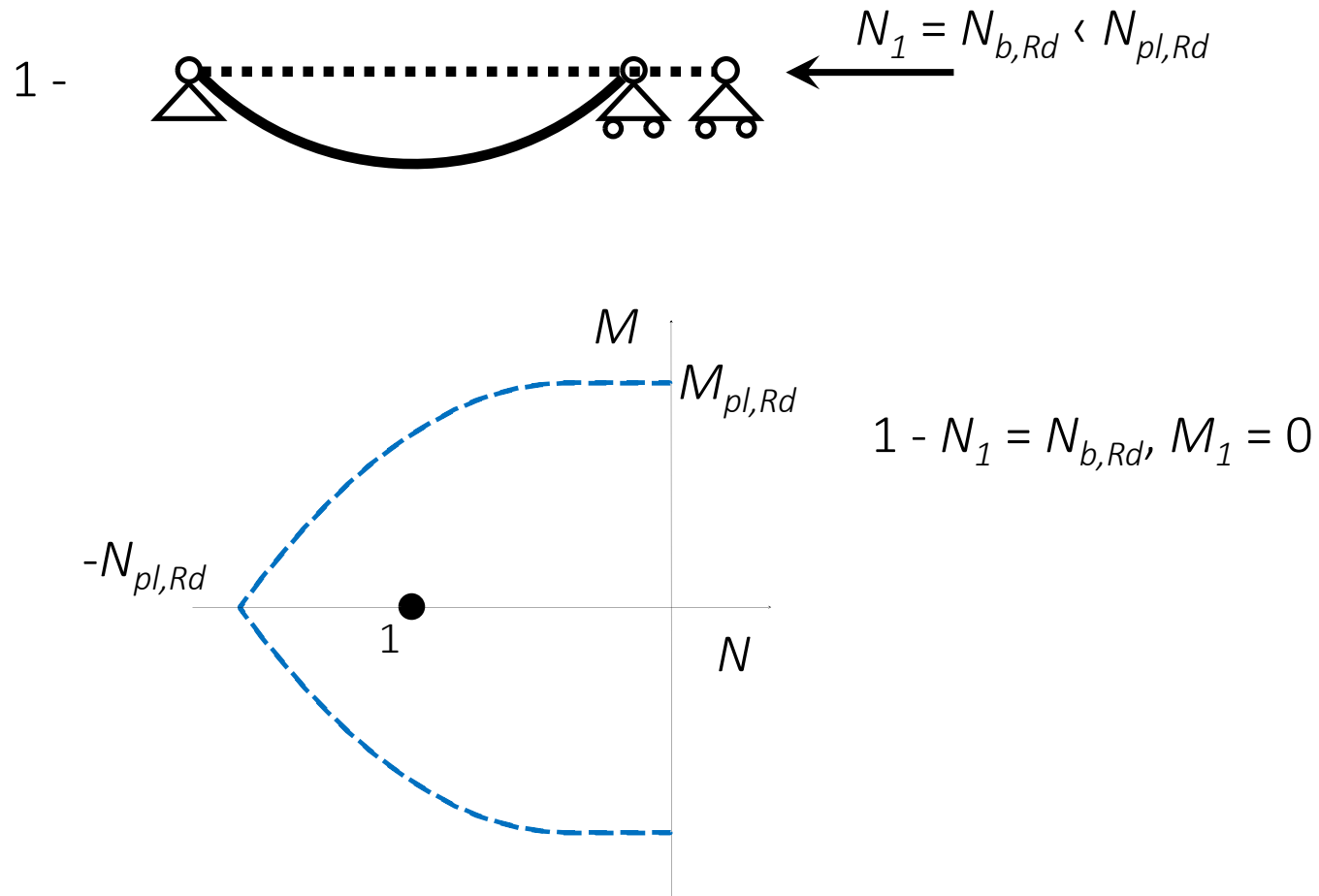
Va bene per la singola sezione, ma per l'asta bisogna tener conto dell'instabilità



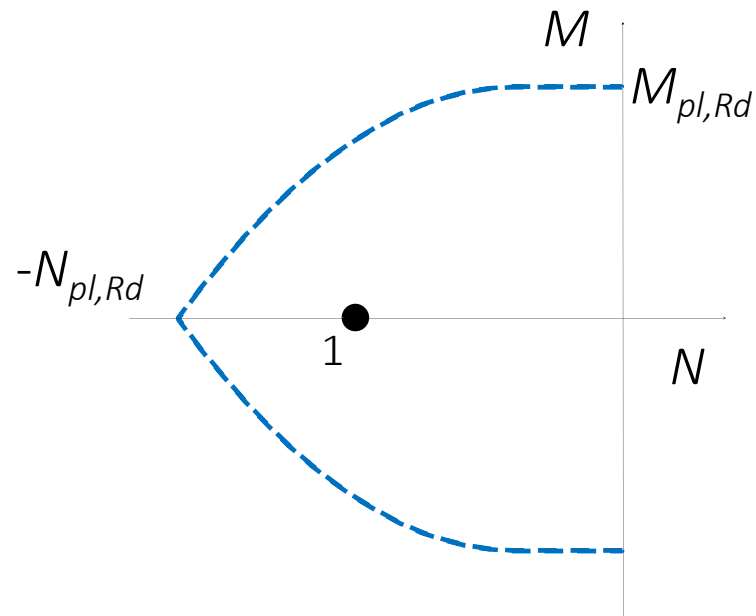
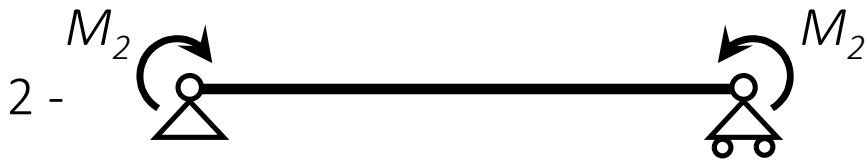
Costruzione del dominio di resistenza



Costruzione del dominio di resistenza

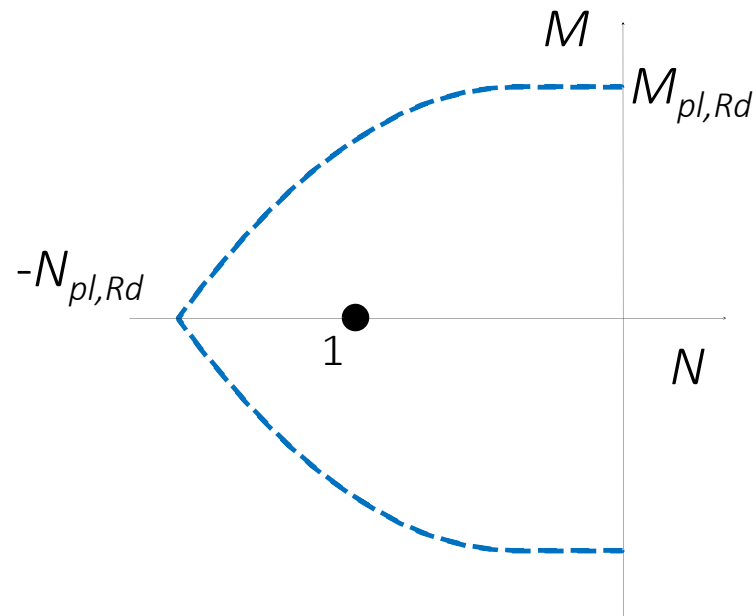


Costruzione del dominio di resistenza



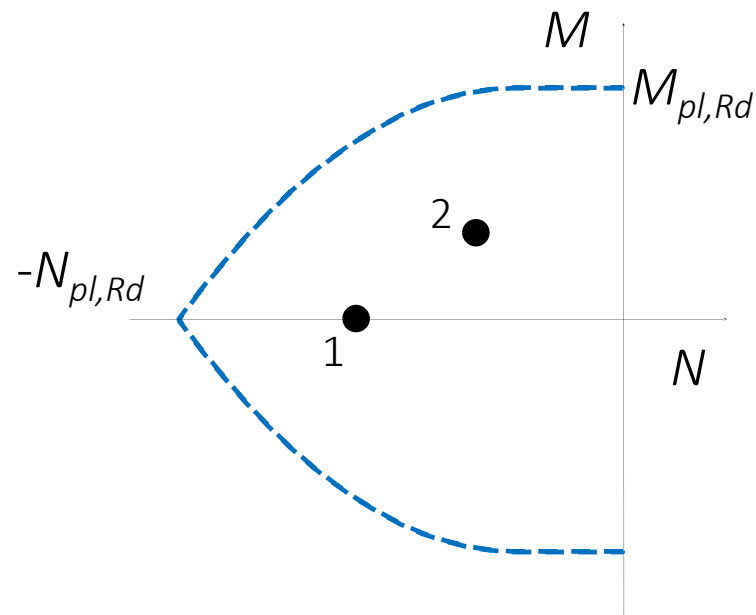
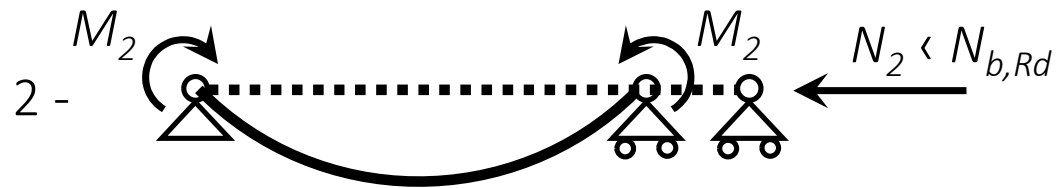
$$1 - N_1 = N_{b,Rd}, M_1 = 0$$

Costruzione del dominio di resistenza



$$1 - N_1 = N_{b,Rd}, M_1 = 0$$

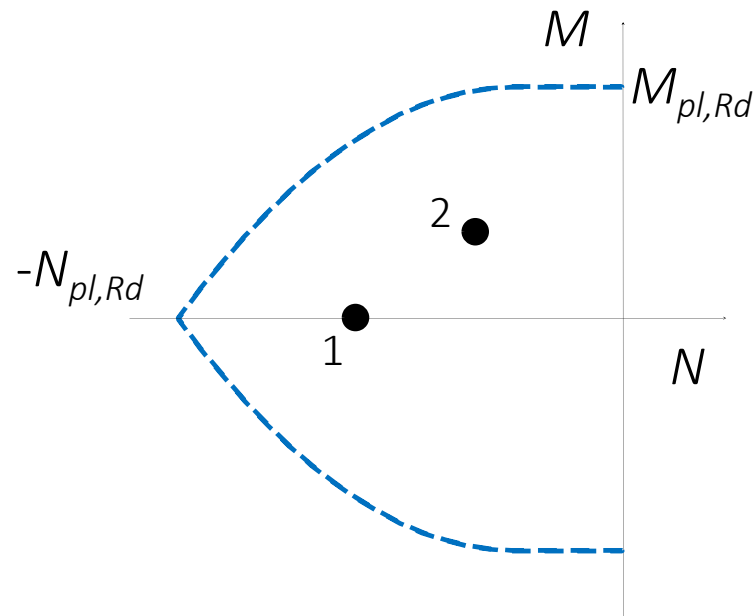
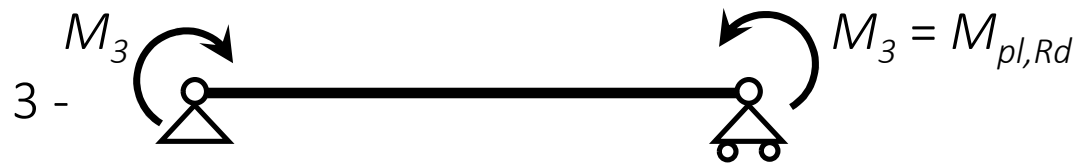
Costruzione del dominio di resistenza



$$1 - N_1 = N_{b,Rd}, M_1 = 0$$

$$2 - N_2 < N_{b,Rd}, M_2 < M_{pl,Rd}$$

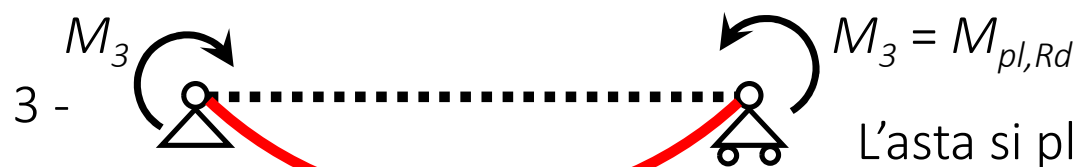
Costruzione del dominio di resistenza



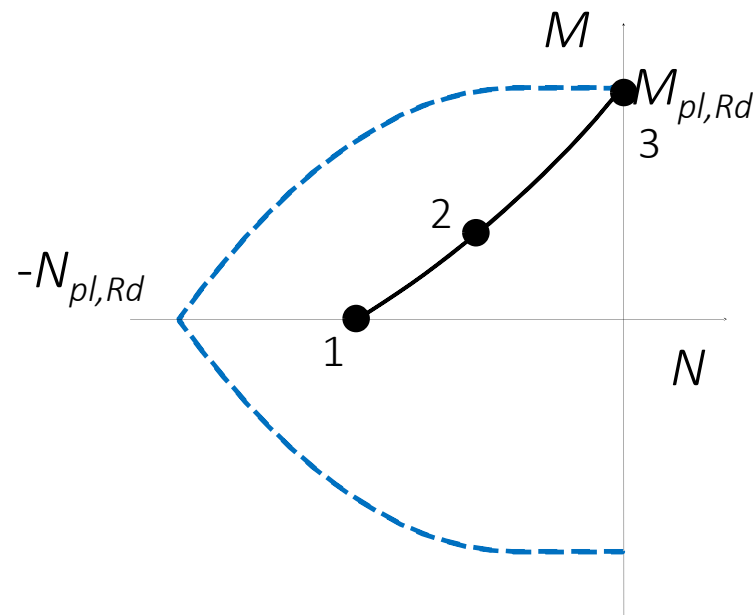
$$1 - N_1 = N_{b,Rd}, M_1 = 0$$

$$2 - N_2 < N_{b,Rd}, M_2 < M_{pl,Rd}$$

Costruzione del dominio di resistenza



L'asta si plasticizza e collassa in assenza di sforzo normale



$$1 - N_1 = N_{b,Rd}, M_1 = 0$$

$$2 - N_2 < N_{b,Rd}, M_2 < M_{pl,Rd}$$

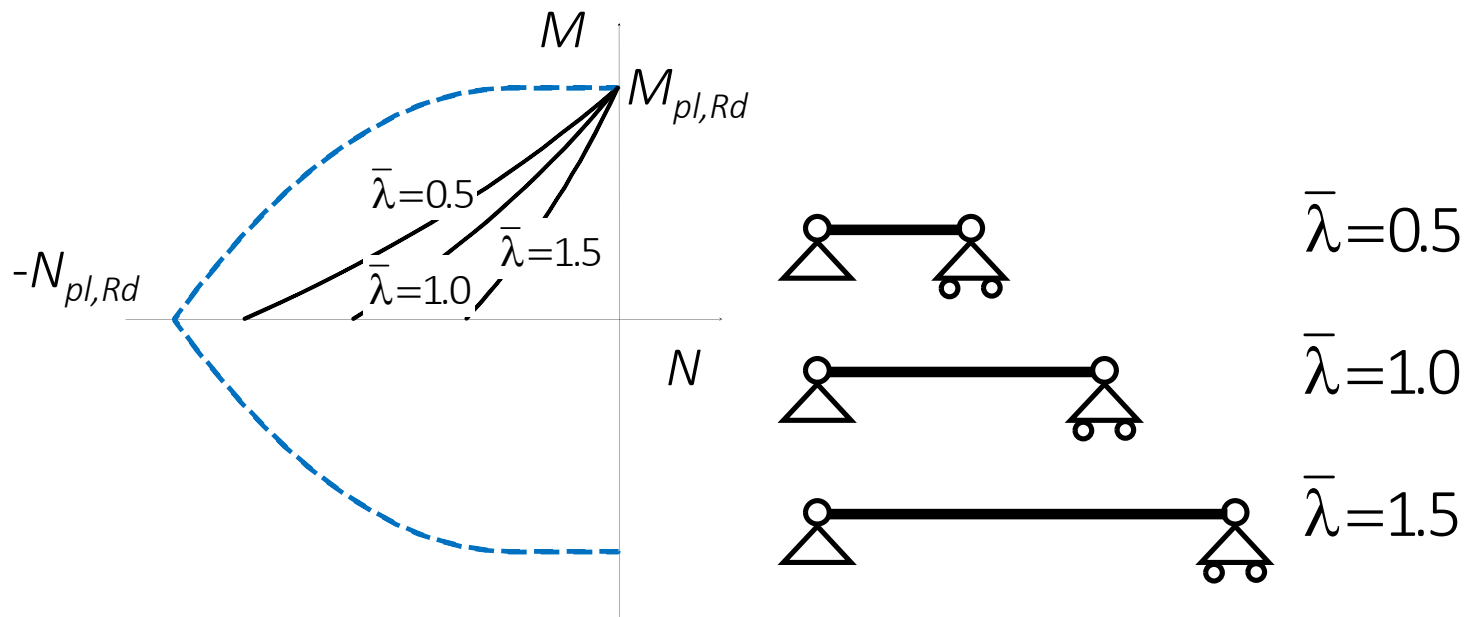
$$3 - N_3 = 0, M_3 = M_{pl,Rd}$$

Collegando i punti si ottiene il dominio

Influenza della snellezza

Il dominio dipende dalla snellezza dell'asta:

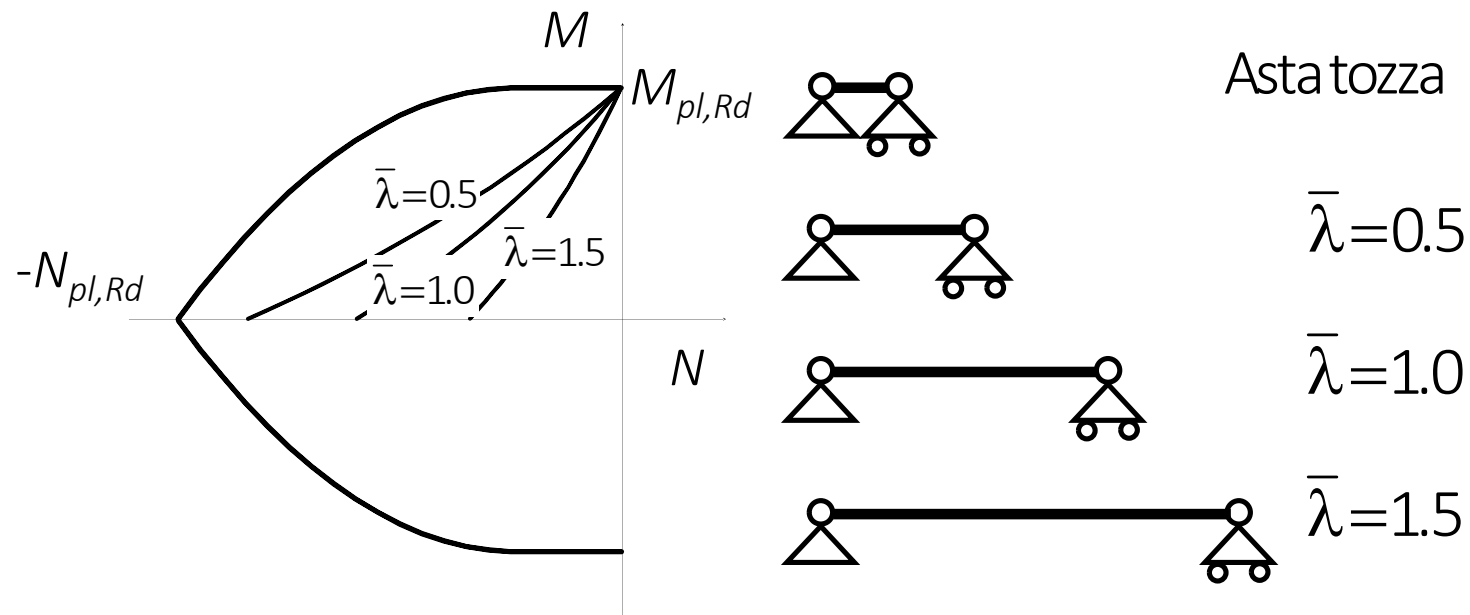
- L'ampiezza del dominio si riduce all'aumentare della snellezza;



Influenza della snellezza

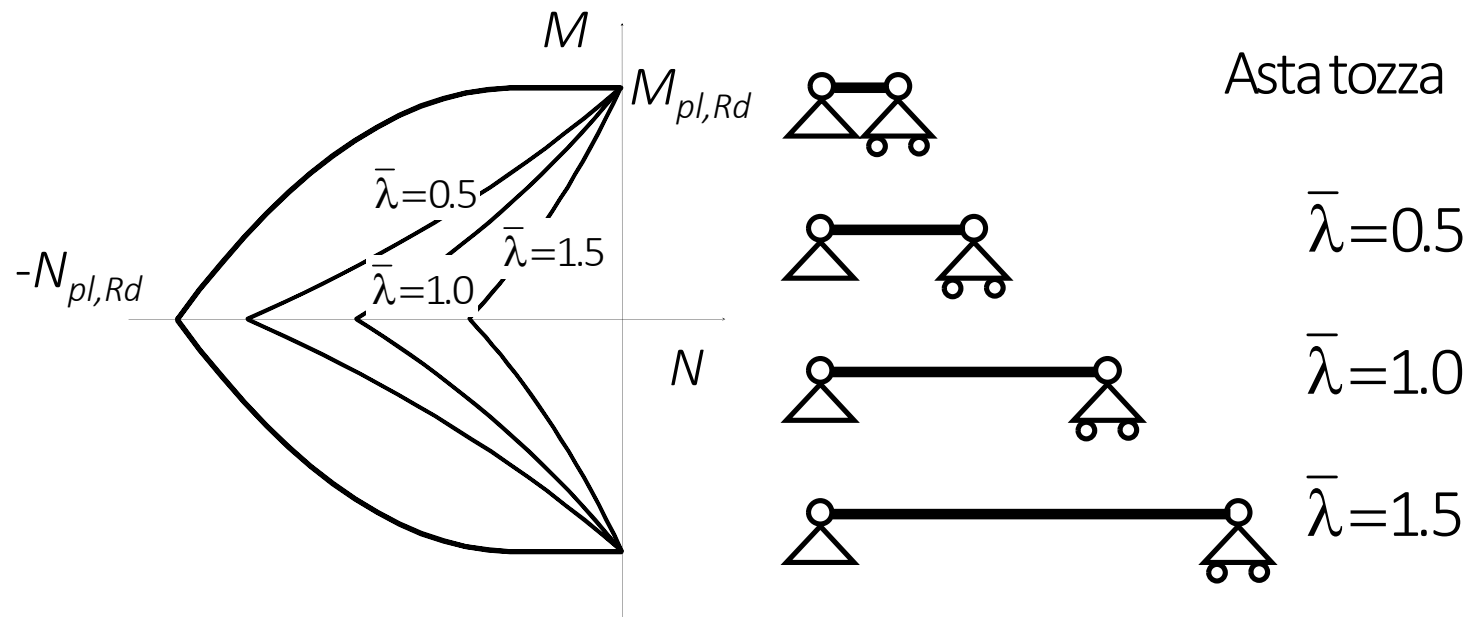
Il dominio dipende dalla snellezza dell'asta:

- L'ampiezza del dominio si riduce all'aumentare della snellezza;
- Nel caso di aste tozze coincide con quello per presso-flessione della sezione
- Nel caso di snellezza infinita si schiaccia sull'asse del momento

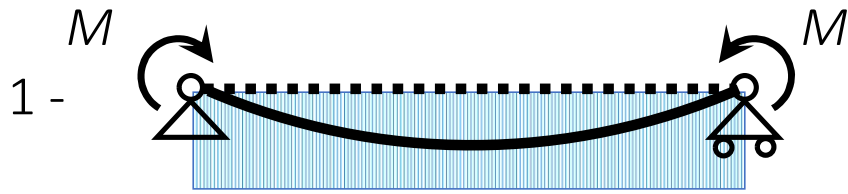


Influenza della snellezza

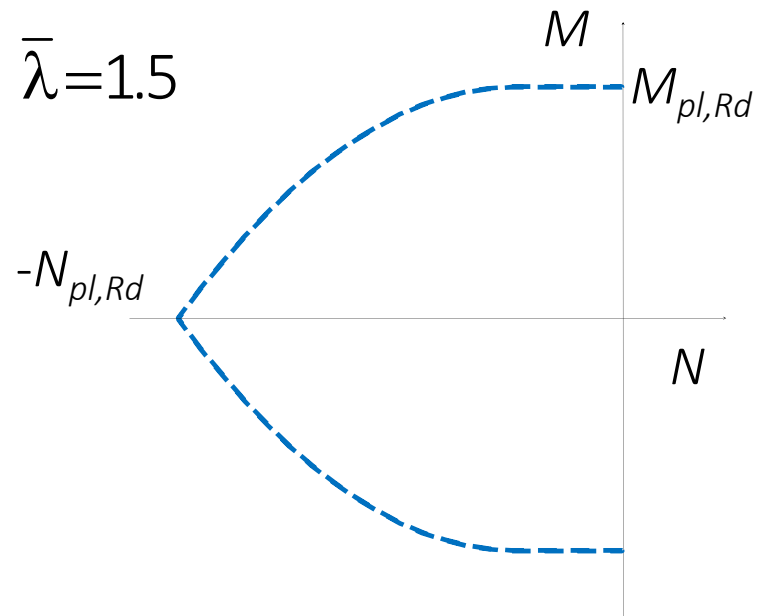
Specchiando le curve rispetto all'asse x si ottiene il dominio completo



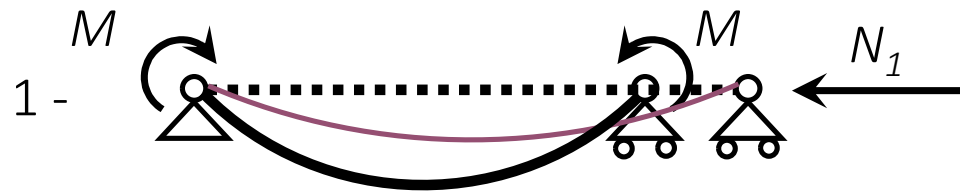
Influenza del diagramma del momento flettente



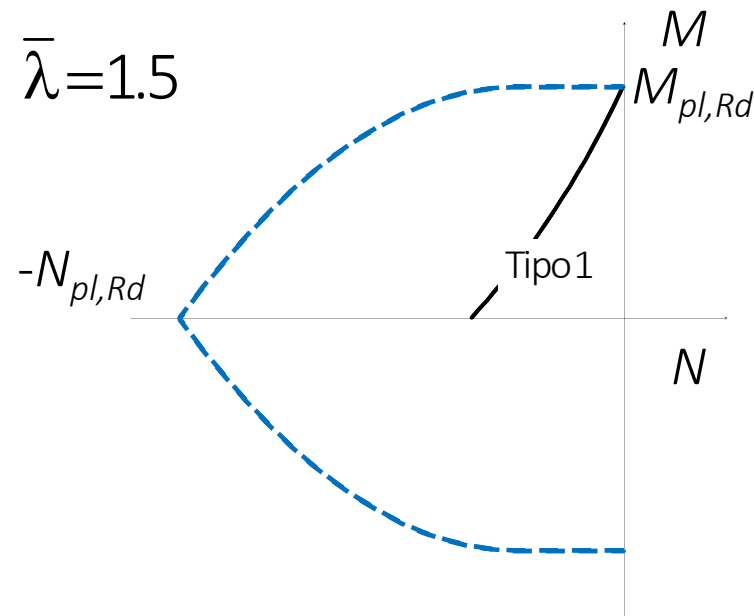
Distribuzione di
momenti tipo 1



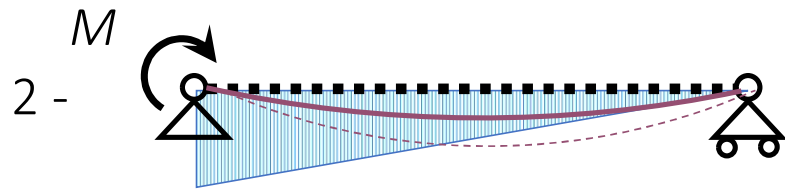
Influenza del diagramma del momento flettente



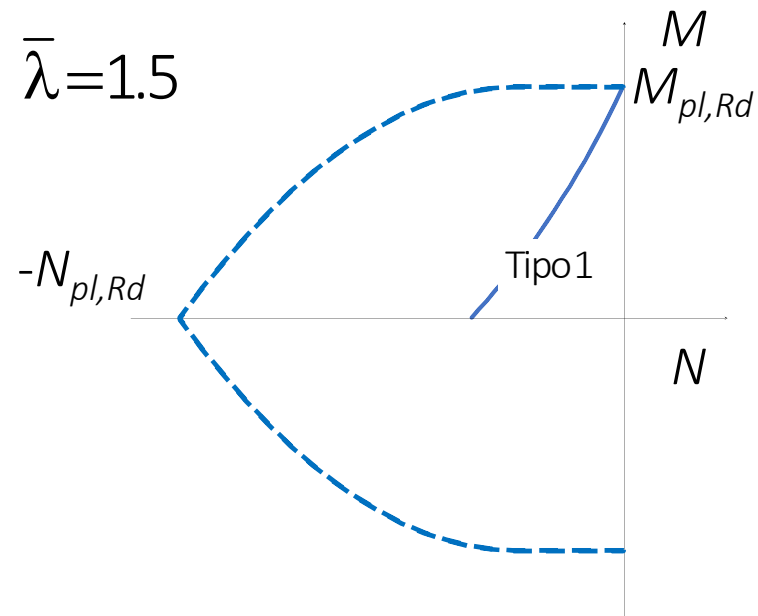
Distribuzione di
momenti tipo 1



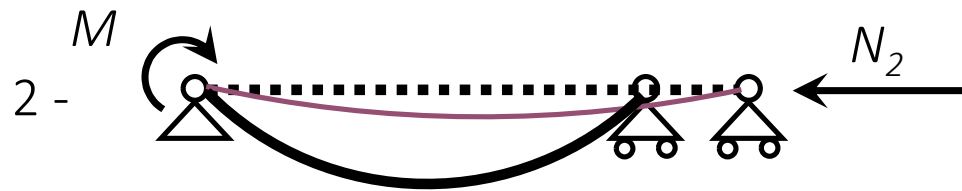
Influenza del diagramma del momento flettente



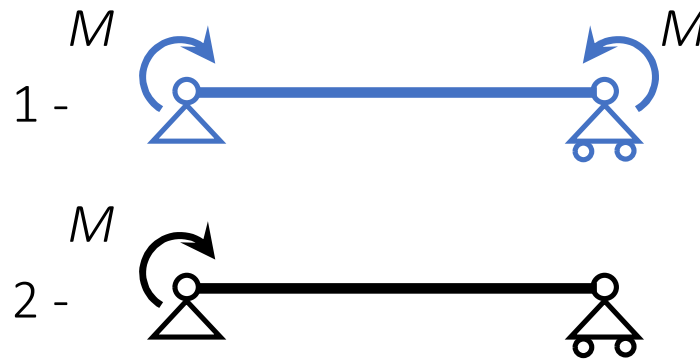
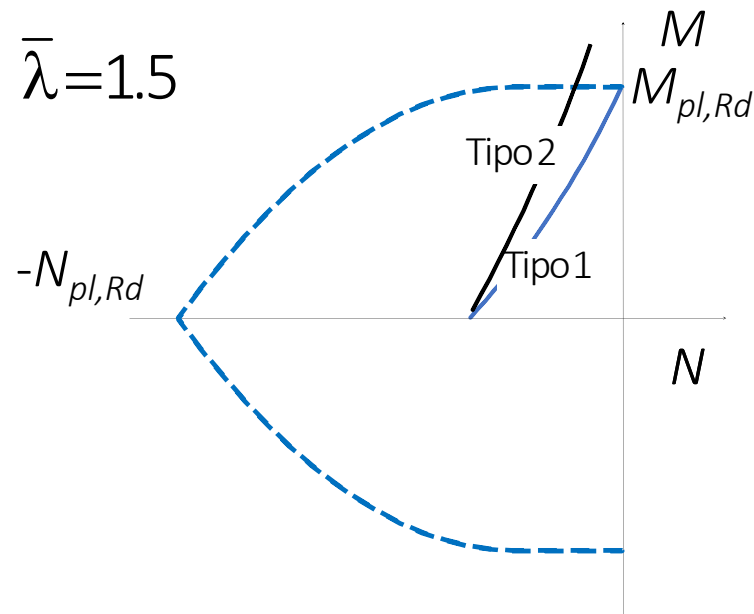
Distribuzione di
momenti tipo 2



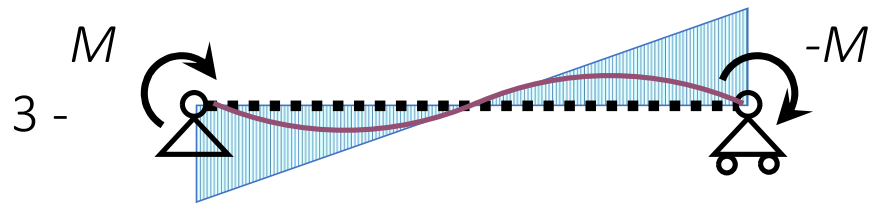
Influenza del diagramma del momento flettente



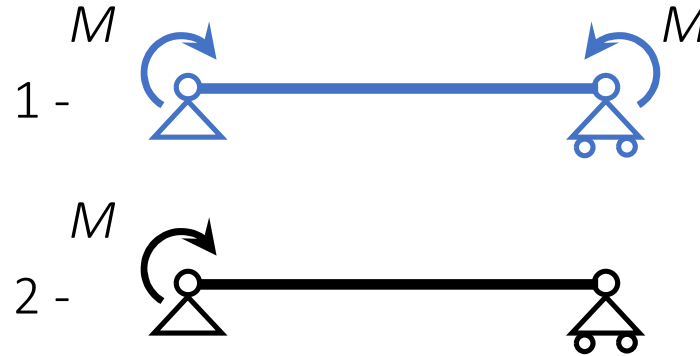
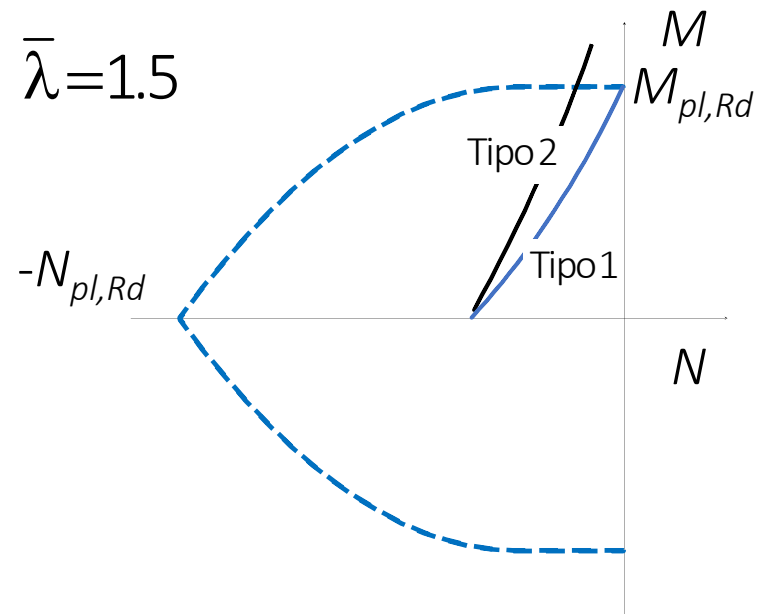
Distribuzione di
momenti tipo 2



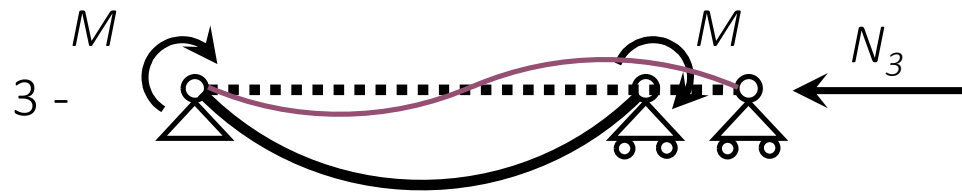
Influenza del diagramma del momento flettente



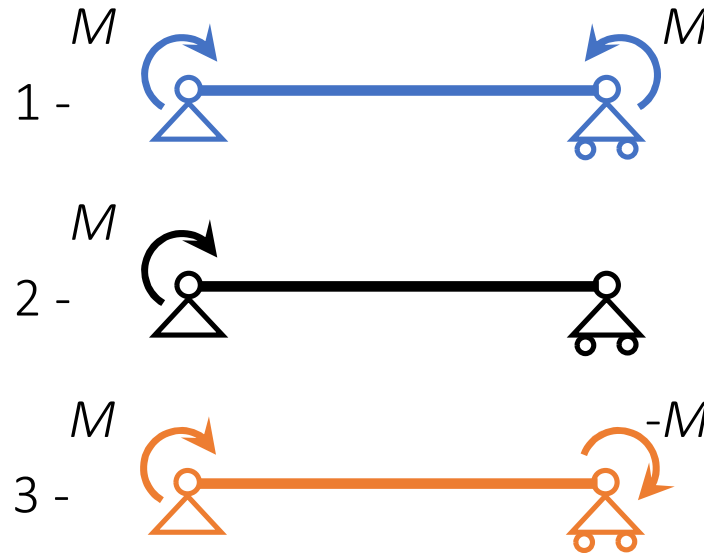
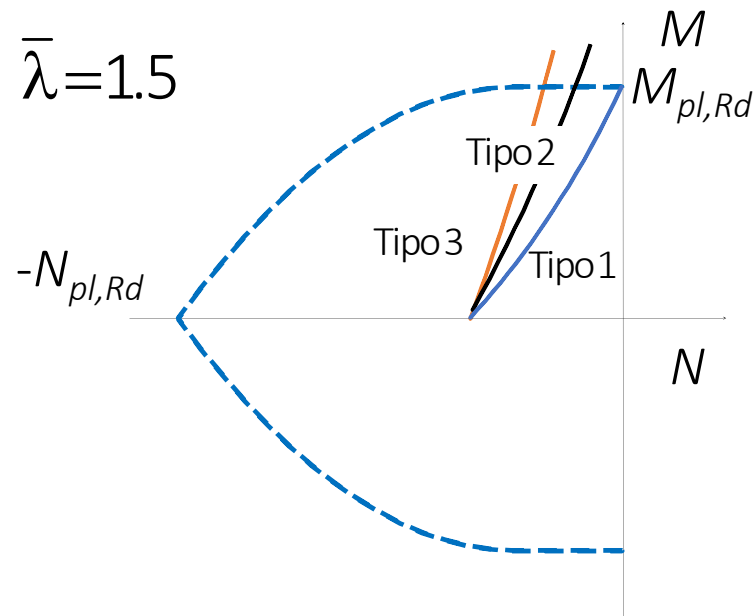
Distribuzione di
momenti tipo 3



Influenza del diagramma del momento flettente



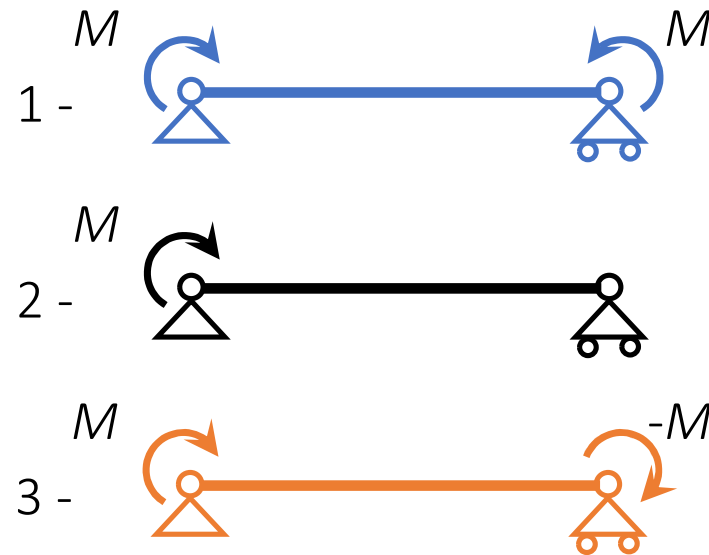
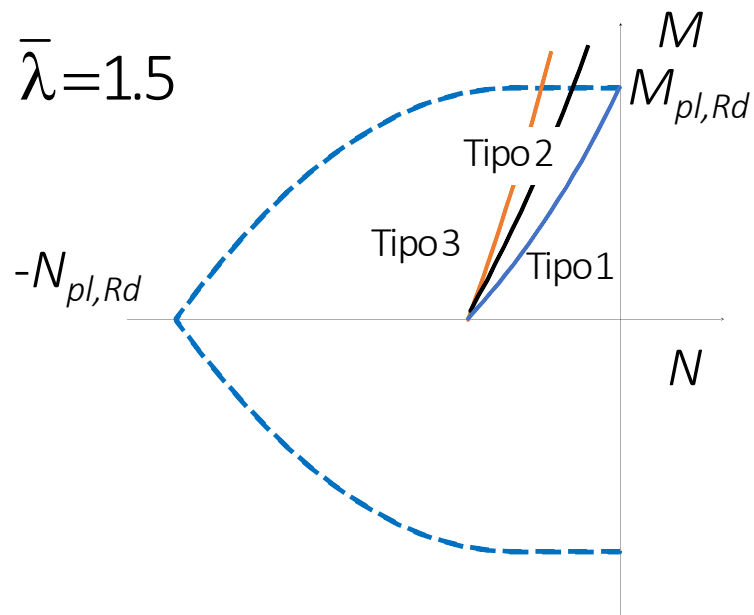
Distribuzione di
momenti tipo 3



Influenza del diagramma del momento flettente

Il dominio dipende dall'andamento del diagramma del momento flettente:

- L'ampiezza del dominio aumenta passando dal diagramma di tipo 1 a quello di tipo 3.



Verifica - stato limite ultimo

E' possibile usare un approccio semplificato.

Se non vi è rischio di instabilità flessione-torsionale, occorre controllare che:

$$\frac{N_{Ed} \cdot \gamma_{M1}}{\chi_{min} \cdot f_{yk} \cdot A} + \frac{M_{y,eq,Ed} \cdot \gamma_{M1}}{f_{yk} \cdot W_y \cdot \left(1 - \frac{N_{Ed}}{N_{cr,y}}\right)} + \frac{M_{z,eq,Ed} \cdot \gamma_{M1}}{f_{yk} \cdot W_z \cdot \left(1 - \frac{N_{Ed}}{N_{cr,z}}\right)} \leq 1$$

o, in sostanza:

$$\frac{N_{Ed}}{N_{bRd}} + \frac{M_{y,eq,Ed}}{M_{y,Rd} \left(1 - \frac{N_{Ed}}{N_{cr,y}}\right)} + \frac{M_{z,eq,Ed}}{M_{z,Rd} \left(1 - \frac{N_{Ed}}{N_{cr,z}}\right)} \leq 1$$

N_{cr} è il carico critico
Euleriano

Verifica - stato limite ultimo

Il momento equivalente $M_{eq,Ed}$ tiene conto della variazione del momento nell'asta e può essere preso pari a

$$M_{eq,Ed} = 1.3 M_{m,Ed}$$

assumendo comunque $0.75 M_{max,Ed} \leq M_{eq,Ed} \leq M_{max,Ed}$

$M_{m,Ed}$ è il valore medio del momento nell'asta

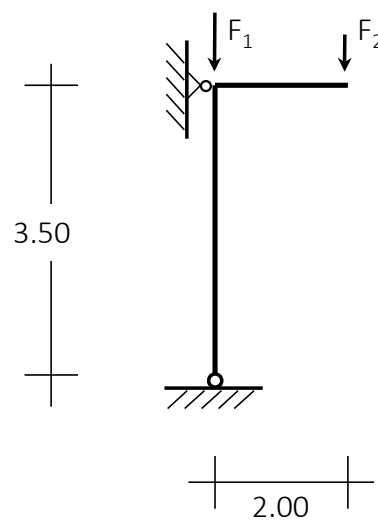
Per asta vincolata agli estremi con momento variabile linearmente si può assumere

$$M_{eq,Ed} = 0.6 M_a - 0.4 M_b$$

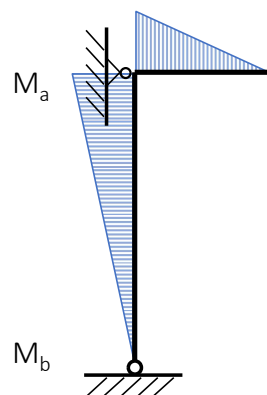
assumendo comunque $M_{eq,Ed} \geq 0.4 M_a$

M_a è il massimo tra i due ed il segno si riferisce al verso della coppia M_a (se $M_b = -M_a$ il diagramma di M è costante e $M_{eq,Ed} = M_a$)

Esempio



$$F_1 = 800 \text{ kN}$$
$$F_2 = 60 \text{ kN}$$

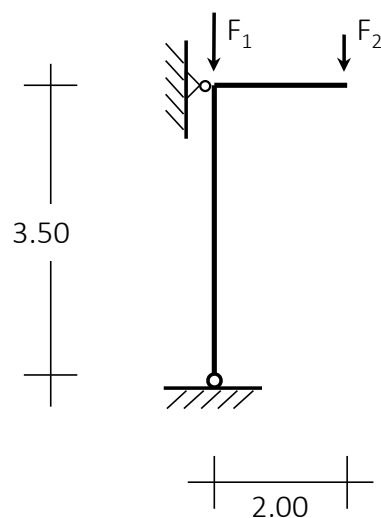


$$M_a = 120 \text{ kNm}$$
$$M_b = 0 \text{ kN}$$

$$N_{Ed} = 860 \text{ kN}$$

$$M_{eq,Ed} = 0.6 \times 120 - 0.4 \times 0 = 72 \text{ kNm}$$

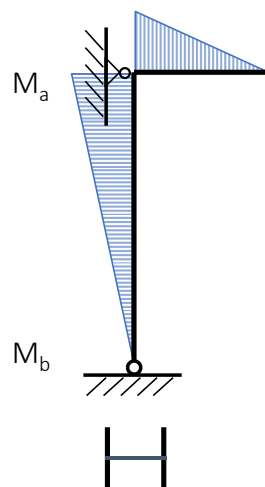
Esempio



$$N_{cr} = \frac{\pi^2 E I_y}{l_0^2}$$



instabilità intorno a x



Sezione	HEB300
A	149 cm ²
W _{pl}	1868 cm ³
Acciaio	S235

$$M_{pl,Rd} = 418.1 \text{ kNm}$$

$$N_{bRd} = 2828.6 \text{ kN}$$

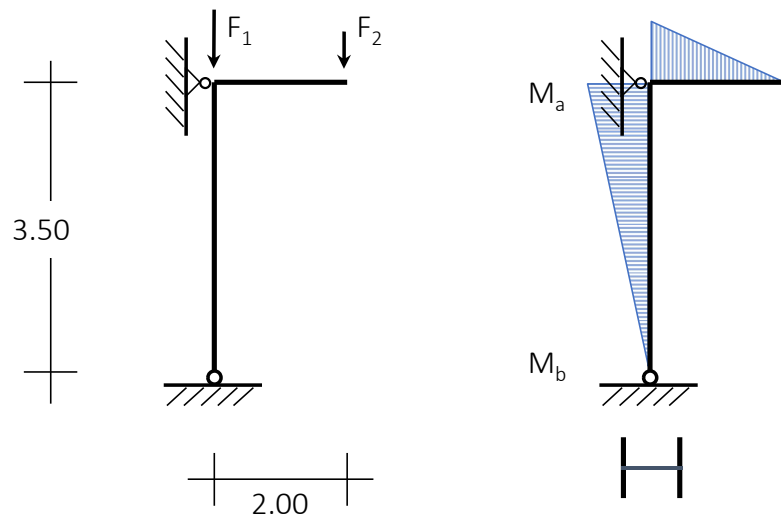


instabilità intorno a y

$$l_0 = 3.50 \text{ m}$$

$$N_{cr} = 42586 \text{ kN}$$

Esempio



Sezione	HEB300
A	149 cm ²
W_{pl}	1868 cm ³
Acciaio	S235

$$M_{pl,Rd} = 418.1 \text{ kNm}$$

$$N_{bRd} = 2828.6 \text{ kN}$$

$$N_{bRd,x} = 3233.2 \text{ kN}$$

$$\frac{N_{Ed}}{N_{bRd}} + \frac{M_{y,eq,Ed}}{M_{x,Rd} \left(1 - \frac{N_{Ed}}{N_{cr,x}} \right)} = \frac{860}{2828.6} + \frac{72}{418.1 \left(1 - \frac{860}{42586} \right)} =$$

$$= 0.304 + 0.175 = 0.483 \leq 1$$

Verifica - stato limite ultimo

In alternativa, occorre controllare che:

$$\frac{N_{Ed} \gamma_{M1}}{\chi_y A f_{yk}} + k_{yy} \frac{M_{y,Ed} \gamma_{M1}}{W_y f_{yk}} + k_{yz} \frac{M_{z,Ed} \gamma_{M1}}{W_z f_{yk}} \leq 1$$

$$\frac{N_{Ed} \gamma_{M1}}{\chi_z A f_{yk}} + k_{zy} \frac{M_{y,Ed} \gamma_{M1}}{W_y f_{yk}} + k_{zz} \frac{M_{z,Ed} \gamma_{M1}}{W_z f_{yk}} \leq 1$$

espressione analoga alla precedente ($k M = M_{eq}$)

ma con espressioni complicate per il calcolo di k

Si noti che N_{bRd} è calcolato separatamente nelle due direzioni

Verifica - stato limite ultimo

In alternativa, occorre controllare che:

$$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy} \frac{M_{y,Ed}}{M_{y,Rd}} + k_{yz} \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1$$

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{y,Rd}} + k_{zz} \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1$$

espressione analoga alla precedente ($k M = M_{eq}$)

ma con espressioni complicate per il calcolo di k

Si noti che N_{bRd} è calcolato separatamente nelle due direzioni

Verifica - stato limite ultimo

In alternativa, occorre controllare che:

$$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy} \frac{M_{y,Ed}}{M_{y,Rd}} + k_{yz} \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1$$

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{y,Rd}} + k_{zz} \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1$$

$k_{yy}, k_{zz}, k_{yz}, k_{zy}$ dipendono da:

- snellezza dell'asta
- diagramma del momento
- sforzo normale

Fattori d'interazione k_{yy} , k_{zz} , k_{yz} , k_{zy} per sezioni a doppio T di classe 1 e 2

$$k_{yy} = C_{my} \left(1 + (\bar{\lambda}_y - 0.2) \frac{N_{Ed}}{N_{b,Rd,y}} \right) \leq C_{my} \left(1 + 0.8 \frac{N_{Ed}}{N_{b,Rd,y}} \right)$$

$$k_{yz} = 0.6 k_{zz}$$

$$k_{zy} = 0.6 k_{yy}$$


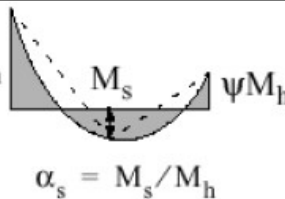
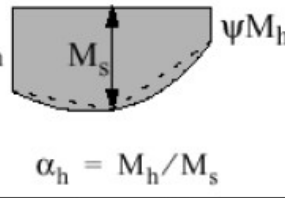
$$k_{zz} = C_{mz} \left(1 + (2\bar{\lambda}_z - 0.6) \frac{N_{Ed}}{N_{b,Rd,z}} \right) \leq C_{mz} \left(1 + 1.4 \frac{N_{Ed}}{N_{b,Rd,z}} \right)$$

Per presso-flessione retta con $M_{Ed,y} \neq 0$ si assume $k_{zy} = 0$

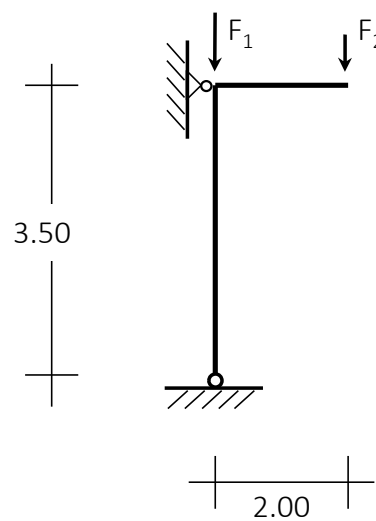
Table B.1: Interaction factors k_{ij} for members not susceptible to torsional deformations

Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
k_{yy}	I-sections RHS-sections	$C_{my} \left(1 + 0,6 \bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left(1 + 0,6 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$	$C_{my} \left(1 + (\bar{\lambda}_y - 0,2) \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left(1 + 0,8 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$
k_{yz}	I-sections RHS-sections	k_{zz}	$0,6 k_{zz}$
k_{zy}	I-sections RHS-sections	$0,8 k_{yy}$	$0,6 k_{yy}$
k_{zz}	I-sections	$C_{mz} \left(1 + 0,6 \bar{\lambda}_z \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 0,6 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$	$C_{mz} \left(1 + (2\bar{\lambda}_z - 0,6) \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 1,4 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$
	RHS-sections		$C_{mz} \left(1 + (\bar{\lambda}_z - 0,2) \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mz} \left(1 + 0,8 \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right)$
For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient k_{zy} may be $k_{zy} = 0$.			

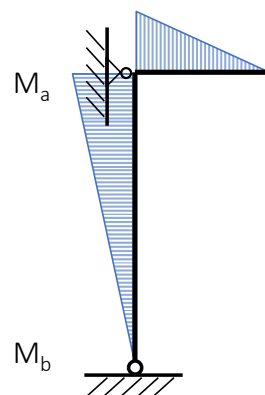
Fattori di momento uniforme equivalente C_{my} e C_{mz}

Moment diagram	range		C_{my} and C_{mz} and C_{mLT}	
			uniform loading	concentrated load
	$-1 \leq \psi \leq 1$		$0,6 + 0,4\psi \geq 0,4$	
 $\alpha_s = M_s/M_h$	$0 \leq \alpha_s \leq 1$	$-1 \leq \psi \leq 1$	$0,2 + 0,8\alpha_s \geq 0,4$	$0,2 + 0,8\alpha_s \geq 0,4$
	$-1 \leq \alpha_s < 0$	$0 \leq \psi \leq 1$	$0,1 - 0,8\alpha_s \geq 0,4$	$-0,8\alpha_s \geq 0,4$
		$-1 \leq \psi < 0$	$0,1(1-\psi) - 0,8\alpha_s \geq 0,4$	$0,2(-\psi) - 0,8\alpha_s \geq 0,4$
 $\alpha_h = M_h/M_s$	$0 \leq \alpha_h \leq 1$	$-1 \leq \psi \leq 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
	$-1 \leq \alpha_h < 0$	$0 \leq \psi \leq 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
		$-1 \leq \psi < 0$	$0,95 + 0,05\alpha_h(1+2\psi)$	$0,90 - 0,10\alpha_h(1+2\psi)$
For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{mx} = 0,9$ or $C_{Mz} = 0,9$ respectively.				
C_{my} , C_{mz} and C_{mLT} should be obtained according to the bending moment diagram between the relevant braced points as follows:				
moment factor	bending axis	points braced in direction		
C_{my}	y-y	z-z		
C_{mz}	z-z	y-y		
C_{mLT}	y-y	y-y		

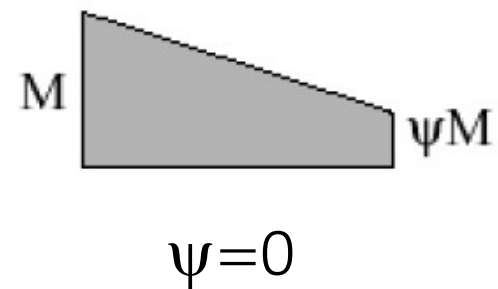
Esempio



$$F_1 = 800 \text{ kN}$$
$$F_2 = 60 \text{ kN}$$



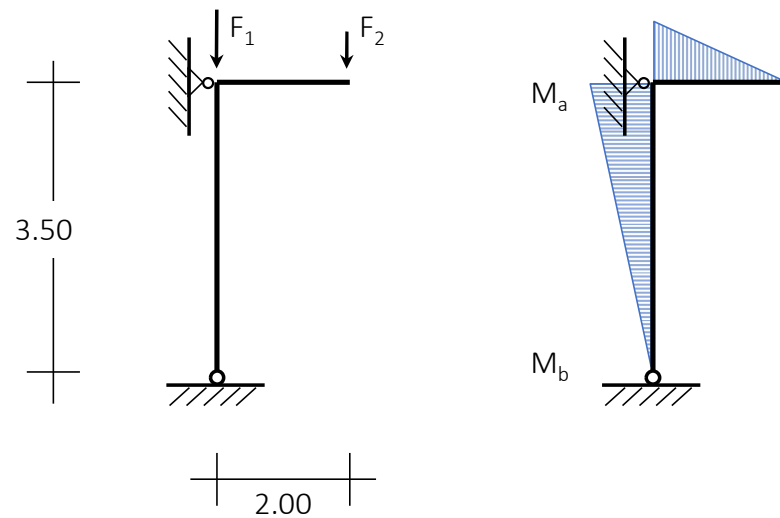
$$M_a = 120 \text{ kNm}$$
$$M_b = 0 \text{ kN}$$



$$N_{Ed} = 860 \text{ kN}$$

$$\alpha_m = 0.6 + 0.4 \psi = 0.6$$

Esempio



$$\bar{\lambda}_x = 0.287$$

$$\chi_x = 0.9689$$

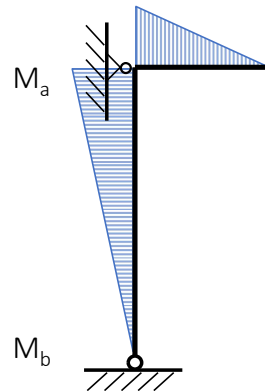
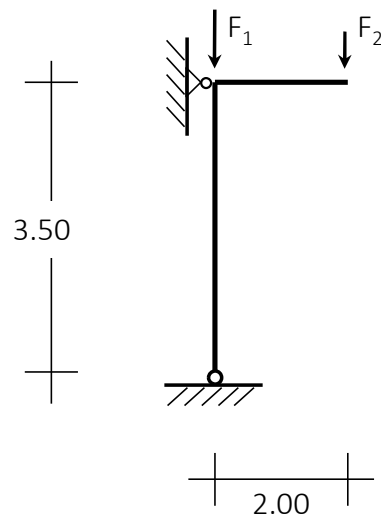
$$\bar{\lambda}_y = 0.492$$

$$\chi_y = 0.8476$$

$$k_{xx} = \alpha_m \left[1 + (\bar{\lambda}_x - 0.2) \frac{N_{Ed} \gamma_{M1}}{\chi_x A f_x} \right] = 0.6 \left[1 + (0.287 - 0.2) \frac{860}{3233.2} \right] = 0.614$$

$N_{bRd,x}$

Esempio



$$\bar{\lambda}_x = 0.287$$

$$\chi_x = 0.9689$$

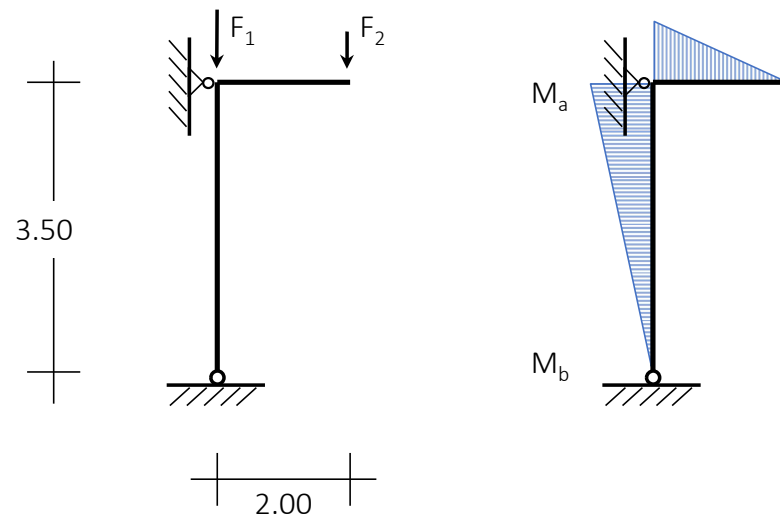
$$\bar{\lambda}_y = 0.492$$

$$\chi_y = 0.8476$$

$$k_{yy} \leq \alpha_m \left[1 + 0.8 \frac{N_{Ed} \gamma_{M1}}{\chi_y A f_y} \right] = 0.6 \left[1 + 0.8 \frac{860}{3233.2} \right] = 0.728$$

Ok, quindi $k_{yy} = 0.614$

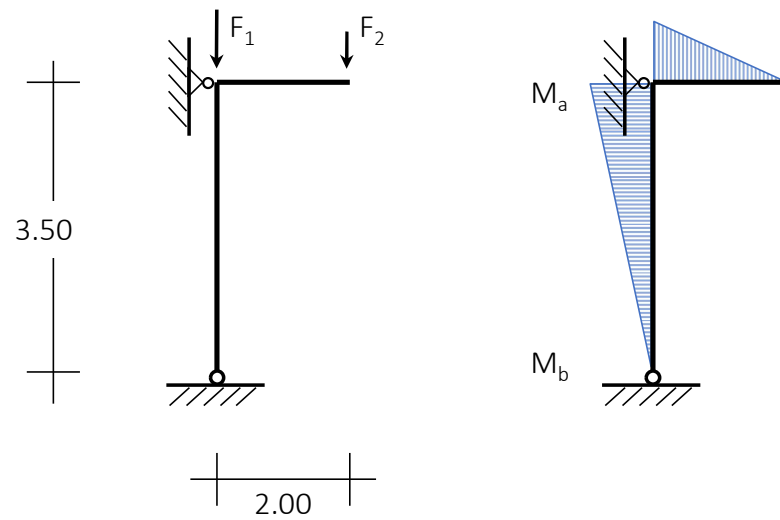
Esempio



$$\frac{N_{Ed} \gamma_{M1}}{\chi_y A f_y} + \frac{k_{yy} M_{y,Ed} \gamma_{M1}}{W_{pl,y} f_y} = \frac{860}{3233.2} + \frac{0.614 \times 120}{418.1} =$$

$$N_{bRd,y} \quad M_{pl,Rd,y} = 0.266 + 0.176 = 0.442 < 1$$

Esempio



Perché $k_{yx} = 0$

$$\frac{N_{Ed} \gamma_{M1}}{\chi_z A f_y} + \frac{k_{zy} M_{y,Ed} \gamma_{M1}}{W_{pl,y} f_y} = \frac{860}{2828.6} = 0.304$$

$N_{bRd,z}$