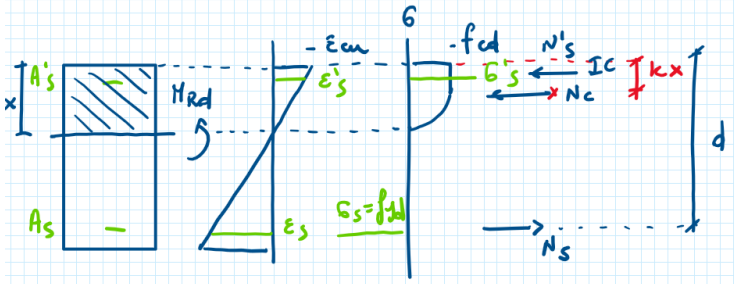
Flessione semplice: III stadio

martedì 1 aprile 2025 14:02



1) DETERMINARE ε_{LIM}

$\Rightarrow$ x

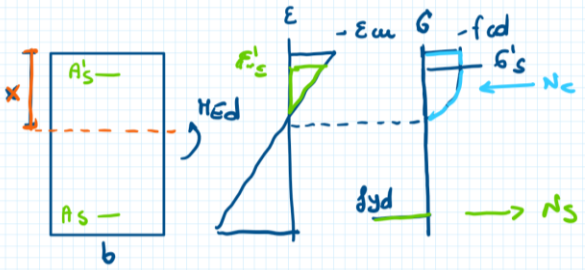
$$N_c(x) + N'_s(x) + N_s(x) = 0$$

$$-\beta b x f_{cd} + A'_s \sigma'_s(x) + A_s f_{yd} = 0 \quad \Rightarrow x$$

2) $M_{Rd} = -N'_s(kx - c) + N_s(d - kx)$

SEZ. RETTANGOLARE
- A SEMPLICE ARMATURA ($A'_s = 0$)
- A DOPPIA ARMATURA ($A'_s \neq 0$)
  1) SIA A_s CHE A'_s SONO SNERVATE
  2) A_s È SNERVATA, MA A'_s È ELASTICA

① SEZ. RETTANGOLARE, A'_s PLASTICIZZATA



1) DETERMINO x

$$N_c + N'_s + N_s = 0$$

$$-\beta b x f_{cd} - A'_s f_{yd} + A_s f_{yd} = 0$$

$$\Downarrow$$

$$\varepsilon'_s \leq -\varepsilon_{yd} \Rightarrow \sigma'_s = -f_{yd}$$

$$+\beta b x f_{cd} = -A'_s f_{yd} + A_s f_{yd}$$

$$x = \frac{(A_s - A'_s) f_{yd}}{\beta b f_{cd}}$$

VERIFICO CHE A'_s SIA SNERVATA $\Rightarrow$ $\varepsilon'_s \leq -\varepsilon_{yd}$

$$\frac{\varepsilon'_s}{-(x-c)} = \frac{-\varepsilon_{cu}}{-x} \Rightarrow \varepsilon'_s = -\frac{x-c}{x}\varepsilon_{cu}$$

$$-\frac{x-c}{x}\varepsilon_{cu} \leq -\varepsilon_{yd}$$

$$-(x-c)\varepsilon_{cu} \leq -x\varepsilon_{yd}$$

$$-x\varepsilon_{cu} + c\varepsilon_{cu} + x\varepsilon_{yd} \leq 0$$

$$-x\varepsilon_{cu} + c\varepsilon_{cu} + x\varepsilon_{yd} \leq 0$$

$$(-\varepsilon_{cu} + \varepsilon_{yd})x \leq -c\varepsilon_{cu}$$

$$(\varepsilon_{cu} - \varepsilon_{yd})x \geq c\varepsilon_{cu}$$

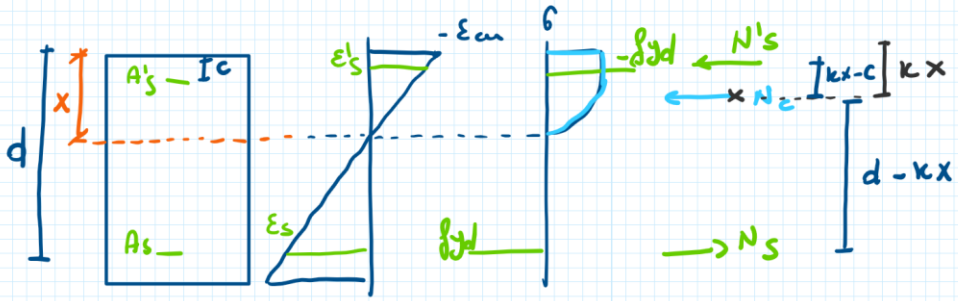
$$x \geq c \frac{\varepsilon_{cu}}{\varepsilon_{cu} - \varepsilon_{yd}}$$

$\varepsilon_{cu} = 0.0035$

$\varepsilon_{yd} = 0.00196$

$$x \geq 2.27\, c$$

2) M_{Rd}

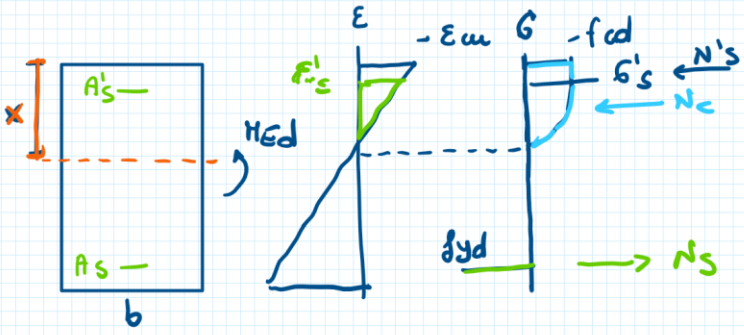


$$M_{Rd} = -N'_s (kx - c) + N_s (d - kx)$$

$$M_{Rd} = -(-A'_s f_{yd})(kx - c) + A_s f_{yd} (d - kx)$$

$$M_{Rd} = A'_s f_{yd} (kx - c) + A_s f_{yd} (d - kx)$$

② SEZ. RETTANGOLARE, A'_s ELASTICA



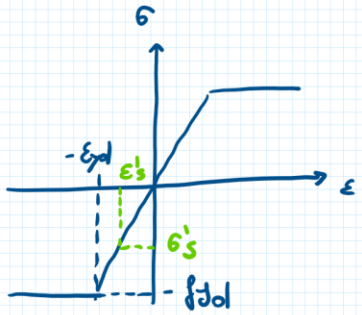
1) DETERMINARE x

$$N_c + N'_s + N_s = 0$$

$$-\beta b x f_{cd} + A'_s \sigma'_s + A_s f_{yd} = 0$$

$$\varepsilon'_s = -\frac{x-c}{x}\varepsilon_{cu}$$

$$\sigma'_s = \frac{\varepsilon'_s}{\varepsilon_{yd}} f_{yd}$$



$$-\beta b x f_{cd} + A'_s\left(-\frac{x-c}{x}\varepsilon_{cu}\right)\frac{f_{yd}}{\varepsilon_{yd}} + A_s f_{yd} = 0$$

$$-\beta b x f_{cd} - \frac{x-c}{x}\frac{\varepsilon_{cu}}{\varepsilon_{yd}} f_{yd} A'_s + A_s f_{yd} = 0$$

$$-\beta b x^2 f_{cd} - (x-c)\frac{\varepsilon_{cu}}{\varepsilon_{yd}} f_{yd} A'_s + A_s f_{yd} x = 0 \qquad \mu = \frac{A'_s}{A_s} \Rightarrow A'_s = \mu A_s$$

$$-\beta b x^2 f_{cd} - (x-c)\underbrace{\frac{\varepsilon_{cu}}{\varepsilon_{yd}}}_{\mu_1} f_{yd}\,\mu A_s + A_s f_{yd} x = 0 \qquad \mu_1 = \mu\frac{\varepsilon_{cu}}{\varepsilon_{yd}}$$

$$-\beta b x^2 f_{cd} - (x-c)\mu_1 A_s f_{yd} + A_s f_{yd} x = 0$$

$$\frac{-\beta b x^2 f_{cd}}{b d f_{cd}} - (x-c)\underbrace{\frac{\mu_1 A_s f_{yd}}{b d f_{cd}}}_{\omega} + \underbrace{\frac{A_s f_{yd} x}{b d f_{cd}}}_{\omega} = 0 \qquad \omega = \frac{A_s f_{yd}}{b d f_{cd}}$$

$$-\frac{\beta}{d}x^2 - (x-c)\mu_1\omega + \omega x = 0$$

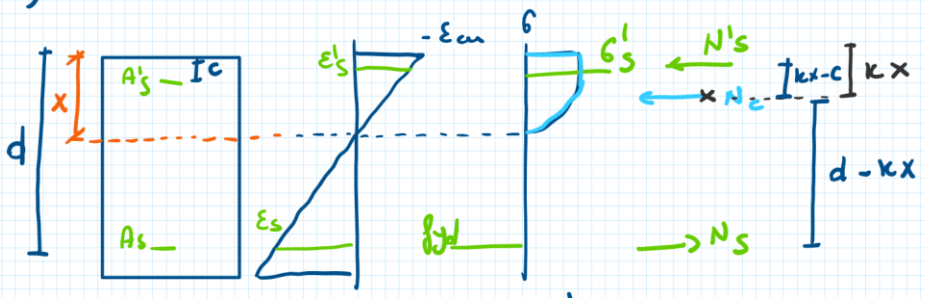
$$-\frac{\beta}{d}x^2 - \mu_1\omega x + \mu_1\omega c + \omega x = 0$$

$$-\frac{\beta}{d}x^2 + \omega(1-\mu_1)x + \mu_1\omega c = 0$$

$$\frac{\beta}{d}x^2 + \omega(\mu_1 - 1)x - \mu_1\omega c = 0$$

$$\boxed{\beta x^2 + \omega d(\mu_1 - 1)x - \mu_1\omega c d = 0} \Rightarrow x$$

2) M_{Rd}



$$M_{Rd} = -N'_S (kx - c) + N_S (d - kx)$$

$N_S = A_s f_{yd}$

PROCEDIMENTO:

IPOTIZZO A'_S SNERVATA

↓

TROVO x

$$x = \frac{(A_S - A'_S) f_{yd}}{\beta b f_{cd}}$$

→ VERIFICO $x \geq 2.27\, c$

SI → M_{Rd}

$$M_{Rd} = + A'_s f_{yd} (kx - c) + A_s f_{yd} (d - kx)$$

NO ↓

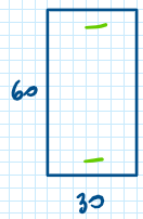
A'_S NON SNERVATA

$$\beta x^2 + (\mu_1 - 1)\, \omega\, d\, x - \mu_1\, \omega\, c\, d = 0$$

↓

$$M_{Rd} = - N'_S (kx - c) + N_S (d - kx)$$

ESEMPIO



$A'_s = 2\phi 14 = 2 \times 1.54 = 3.08\ cm^2$

$A_s = 4\phi 20 = 4 \times 3.14 = 12.56\ cm^2$

SW $M_{Rd} \geq M_{Ed}$

$M_{Ed} = 100\ kNm$

$c = 5\ cm$

1) Ipotizzo A'_s SNERVATA

$$x = \frac{(A_s - A'_s)\, f_{yd}}{\beta\, b\, f_{cd}} = \frac{(12.56 - 3.08)\, 391.3}{0.81 \cdot 30 \cdot 14.2} = 10.75\ cm$$

VERIFICO CHE A'_s SIA SNERVATA: $x \geq 2.27\, c$ (c = 5 cm)

$10.75 \leq 11.35 \Rightarrow A'_s$ NON È SNERVATA

2) RIVALUTO x CON EQ. DI 2° GRADO PER A'_s ELASTICA

$$\underbrace{\beta x^2}_{A} + \underbrace{(\mu_1 - 1)\, \omega\, d\, x}_{B} \underbrace{- \mu_1 \omega c d}_{C} = 0$$

$A = \beta = 0.81$

$B = (\mu_1 - 1)\, \omega\, d = (0.435 - 1) \cdot 0.209 \cdot 55 = -6.49$

$d = 60 - 5 = 55\ cm$

$$\omega = \frac{A_s f_{yd}}{b\, d\, f_{cd}} = \frac{12.56 \times 391.3}{30 \cdot 55 \times 14.2} = 0.209$$

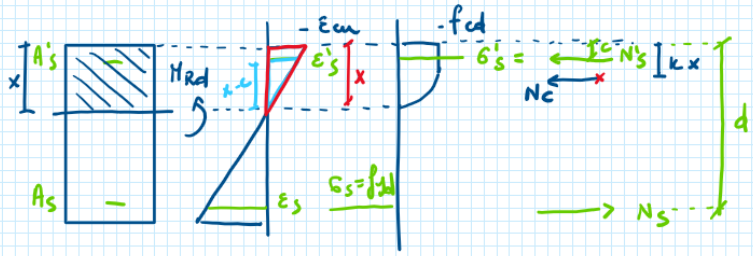
$$\mu_1 = \mu \frac{\varepsilon_{cu}}{\varepsilon_{yd}} = \frac{A'_s}{A_s} \frac{\varepsilon_{cu}}{\varepsilon_{yd}} = \frac{3.08}{12.56} \frac{0.0035}{0.00196} = 0.435$$

$C = -\mu_1 \omega c d = -0.435 \cdot 0.209 \cdot 5 \cdot 55 = -25.0$

$\Rightarrow 0.81 x^2 - 6.49 x - 25 = 0$

$$x = \frac{6.49 \pm \sqrt{6.49^2 + (4 \cdot 0.81 \cdot 25)}}{2 \times 0.81} = \begin{cases} + \ 10.85 \\ - \ 2.84 \end{cases}$$

3) M_{Rd}



$$M_{Rd} = -N'_s\,(kx - c) + N_s\,(d - kx)$$

$$N'_s = A'_s\,\sigma'_s$$

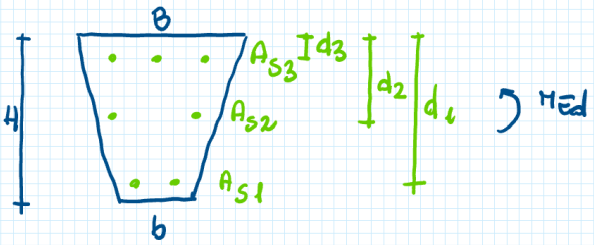
Da $\varepsilon'_s = -\dfrac{x-c}{x}\,\varepsilon_{cu} = -\dfrac{10.85 - 5}{10.85}\,0.0035 = -0.00189$

$$\sigma'_s = \frac{\varepsilon'_s}{\varepsilon_{yd}}\,f_{yd} = -\frac{0.00189}{0.00196}\,391.3 = -377.3\,MPa$$

$$M_{Rd} = +\underbrace{\left(3.08 \times \frac{377.3}{10}\right)}_{N'_s}\left(0.416 \times 10.85 - 5\right)\frac{1}{10^2}$$

$$+\underbrace{\left(12.56 \times \frac{391.3}{10}\right)}_{N_s}\left(55 - 0.416 \times 10.85\right)\frac{1}{10^2} = 247.56\ kN\,m$$

SEZIONE NON RETTANGOLARE

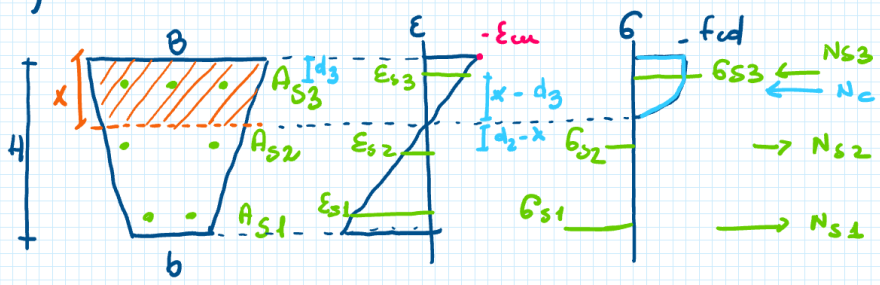


$M_{Rd} \geq M_{Ed}$

1) DET. ε_{LIM} → x

2) M_{Rd}

1) DETERMINARE $\varepsilon_{LIM} \Leftrightarrow x$

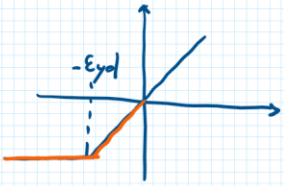


$$\Sigma N_i(x) = 0$$

$$N_c(x) + N_{s3}(x) + N_{s2}(x) + N_{s1}(x) = 0$$

- $N_{s3} = A_{s3} \cdot \sigma_{s3}$
  - ε_{s3}: $\dfrac{\varepsilon_{s3}}{-(x-d_3)} = \dfrac{-\varepsilon_{cu}}{x}$

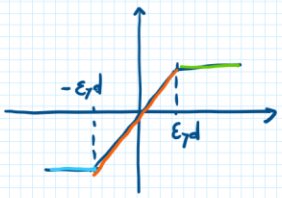
$$\varepsilon_{s3} = -\frac{x - d_3}{x}\,\varepsilon_{cu}$$



se $\varepsilon_{s3} \leq -\varepsilon_{yd} \Rightarrow \sigma_{s3} = -f_{yd}$

se $0 > \varepsilon_{s3} > -\varepsilon_{yd} \Rightarrow \sigma_{s3} = \dfrac{\varepsilon_{s3}}{\varepsilon_{yd}} f_{yd}$

- $N_{s2} = A_{s2}\,\sigma_{s2}$
  - ε_{s2}: $\dfrac{\varepsilon_{s2}}{d_2 - x} = \dfrac{-\varepsilon_{cu}}{-x} \Rightarrow \varepsilon_{s2} = \dfrac{d_2 - x}{x}\,\varepsilon_{cu}$



se $\varepsilon_{s2} \leq -\varepsilon_{yd} \Rightarrow \sigma_{s2} = -f_{yd}$

se $-\varepsilon_{yd} \leq \varepsilon_{s2} \leq \varepsilon_{yd} \Rightarrow \sigma_{s2} = \dfrac{\varepsilon_{s2}}{\varepsilon_{yd}} f_{yd}$

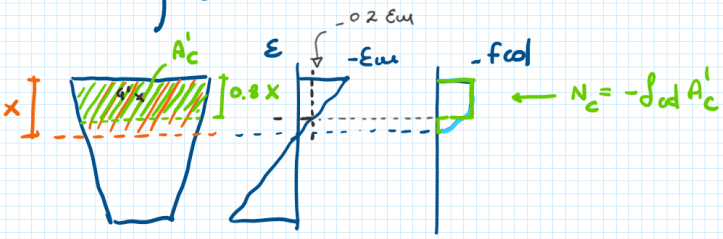
se $\varepsilon_{s2} \geq \varepsilon_{yd} \Rightarrow \sigma_{s2} = f_{yd}$

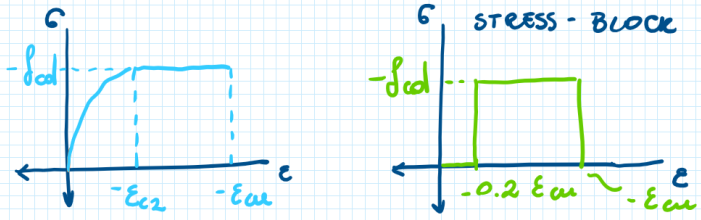
Per qualsiasi strato di armatura:

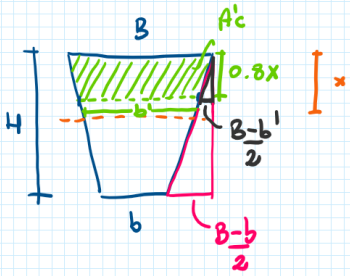
$$N_{si} = A_{si} * \sigma_{si}$$

- $\varepsilon_{si} = \dfrac{d_i - x}{x}\,\varepsilon_{cu} \Rightarrow \sigma_{si}$

- $N_c = \int \sigma_c \, dA_c$







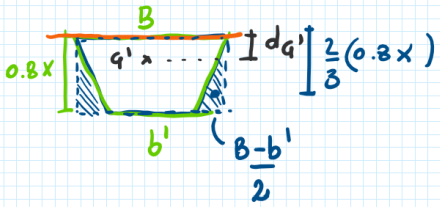
$$A'_c = \frac{(B + b')(0.8x)}{2}$$

$$\frac{\frac{B-b}{2}}{H} = \frac{\frac{B-b'}{2}}{0.8x}$$

$$\frac{B-b}{H} = \frac{B-b'}{0.8x}$$

$$\frac{B-b}{H}(0.8x) = B - b'$$

$$b' = B - \left[\frac{B-b}{H}(0.8x)\right]$$



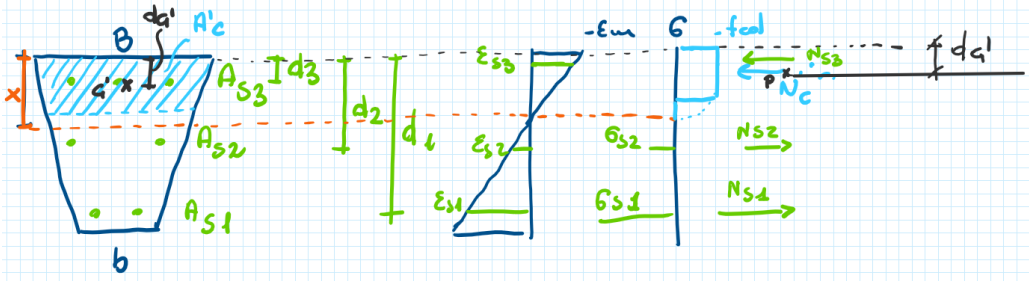
$$S_{sup} = \frac{B\,(0.8x)^2}{2} - 2\left[\frac{B-b'}{2} \times \frac{0.8x}{2} \times \frac{2}{3}(0.8x)\right]$$

(base: $\frac{B-b'}{2}$; altezza: $\frac{0.8x}{2}$)

$$S_{sup} = \frac{B\,(0.8x)^2}{2} - \left[\frac{(B-b')}{3}(0.8x)^2\right]$$

$$d_{G'} = \frac{S_{sup}}{A'_c}$$

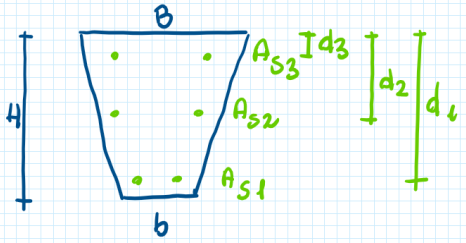
2) DETERMINARE M_{Rd}



$$M_{Rd} = -N_{S3}\left(d_{g'} - d_3\right) + N_{S2}\left(d_2 - d_{g'}\right) + N_{S1}\left(d_1 - d'_g\right)$$

$$M_{Rd} = \Sigma N_{Si}\left(d_i - d'_g\right)$$

ESEMPIO



$\curvearrowright M_{Ed}$

$A_{S3} = 2\phi 20$ $\quad d_3 = 5$ cm

$A_{S2} = 2\phi 20$ $\quad d_2 = 45$ cm

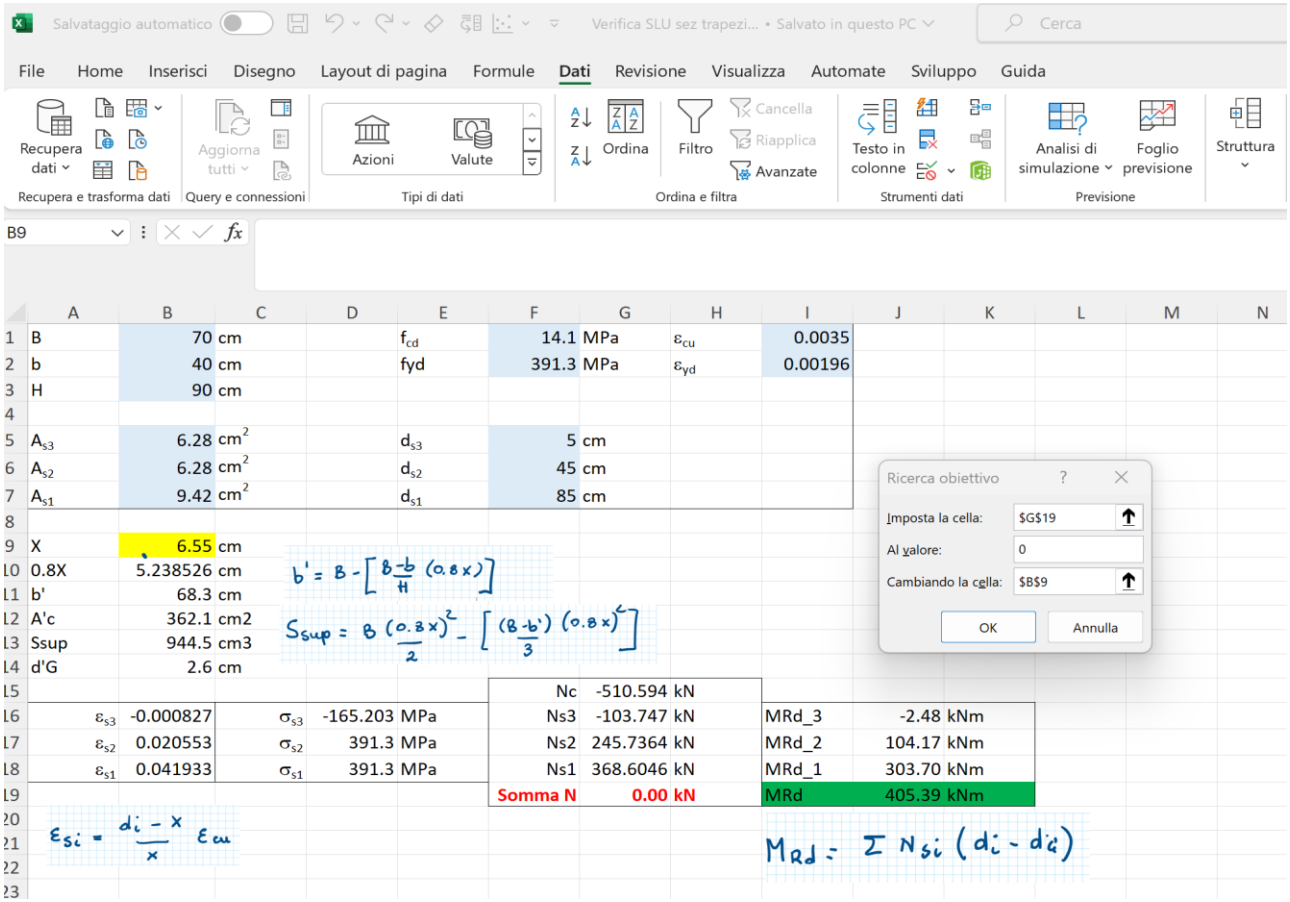
$A_{S1} = 3\phi 20$ $\quad d_1 = 85$ cm

$B = 70$ cm

$b = 40$ cm

$H = 90$ cm

$M_{Ed} = 300$ KNm



B	70	cm		f_{cd}	14.1	MPa	ε_{cu}	0.0035	
b	40	cm		fyd	391.3	MPa	ε_{yd}	0.00196	
H	90	cm							
A_{s3}	6.28	cm^2		d_{s3}	5	cm			
A_{s2}	6.28	cm^2		d_{s2}	45	cm			
A_{s1}	9.42	cm^2		d_{s1}	85	cm			

X	6.55	cm							
0.8X	5.238526	cm							
b'	68.3	cm							
A'c	362.1	cm2							
Ssup	944.5	cm3							
d'G	2.6	cm							

$b' = B - \left[\frac{B-b}{H}(0.8x)\right]$

$S_{sup} = B\frac{(0.8x)^2}{2} - \left[\frac{(B-b')}{3}(0.8x)^2\right]$

				Nc	-510.594	kN	
ε_{s3}	-0.000827	σ_{s3}	-165.203	MPa	Ns3	-103.747	kN
ε_{s2}	0.020553	σ_{s2}	391.3	MPa	Ns2	245.7364	kN
ε_{s1}	0.041933	σ_{s1}	391.3	MPa	Ns1	368.6046	kN
					Somma N	0.00	kN

MRd_3	-2.48 kNm
MRd_2	104.17 kNm
MRd_1	303.70 kNm
MRd	405.39 kNm

$\varepsilon_{si} = \frac{d_i - x}{x}\varepsilon_{cu}$

$M_{Rd} = \sum N_{si}(d_i - d_i')$