Torsione

martedì 3 giugno 2025 14:01

$$\boxed{\sigma_c\, t \,\operatorname{sen}^2\vartheta = \sigma_{sw} \frac{A_{sw}}{s}} \quad (1)$$

$$\boxed{\sigma_c\, t \cos^2\vartheta = \sigma_{s\,lon} \frac{A_{s\,lon}}{u_k}} \quad (2)$$

$$\boxed{T = 2 A_k\, t\, \sigma_c \cos\vartheta \operatorname{sen}\vartheta} \quad (3)$$

$$\boxed{T_{Rd\,max} = 2 A_k\, t\, f'_{cd} \frac{\cot\vartheta}{1+\cot^2\vartheta}}$$

PER RICAVARE $T_{Rd,st}$:

DALLA (1) RICAVO σ_c: $\boxed{\sigma_c\, t \,\operatorname{sen}^2\vartheta = \sigma_{s,st} \frac{A_{s,st}}{s}}$ (1)

$$\sigma_c = \sigma_{s,st} \frac{A_{s,st}}{s} \frac{1}{t \operatorname{sen}^2\vartheta}$$

SOSTITUISCO IN (3)

$$\boxed{T = \sigma_c\, t \operatorname{sen}\vartheta \cos\vartheta\, 2A_k} \quad (3)$$

$$T = \sigma_{s,st} \frac{A_{s,st}}{s} \frac{1}{\cancel{t} \operatorname{sen}^{\cancel{2}}\vartheta} \cancel{t} \cancel{\operatorname{sen}\vartheta} \cos\vartheta\, 2A_k$$

$$T = \sigma_{s,st} \frac{A_{s,st}}{s} 2A_k \cot\vartheta$$

$T_{Rds,st} \Rightarrow \sigma_{s,st} = f_{yd}$ $\boxed{T_{Rd,st} = 2A_k \frac{A_{s,st}}{s} f_{yd} \cot\vartheta}$ T_{Rd} STAFFE

PER RICAVARE $T_{Rds,lon}$:

DALL' EQUAZ. (2) RICAVO σ_c: $\boxed{\sigma_c\, t \cos^2\vartheta = \sigma_{s,lon} \frac{A_{s,lon}}{u_k}}$ (2)

$$\sigma_c = \sigma_{s,lon} \frac{A_{s,lon}}{u_k} \frac{1}{t \cos^2\vartheta}$$

LA SOSTITUISCO IN 3 :

$$\boxed{T = \sigma_c \, t \sin\vartheta \cos\vartheta \, 2A_k} \quad 3$$

$$T = \sigma_{s,lon} \frac{A_{s,lon}}{u_k} \frac{1}{\cancel{t} \cos^2\vartheta} \cancel{t} \sin\vartheta \cancel{\cos\vartheta} \, 2A_k$$

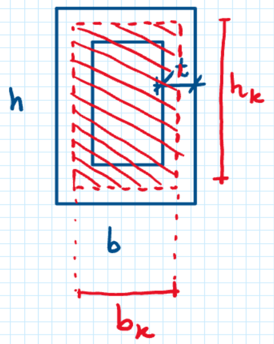
$$T = \sigma_{s,lon} \frac{A_{s,lon}}{u_k} 2A_k \frac{1}{\cot\vartheta}$$

$T_{Rds,lon} \rightarrow \sigma_{s,lon} = f_{yd}$

$$\boxed{T_{Rds,lon} = 2A_k \frac{A_{s,lon}}{u_k} \frac{f_{yd}}{\cot\vartheta}}$$

TRd ARM. LONG.

PER VERIFICARE ALLO SW :



$$T_{Rd\,max} = 2A_k \, t \, f'_{cd} \frac{\cot\vartheta}{1+\cot^2\vartheta}$$

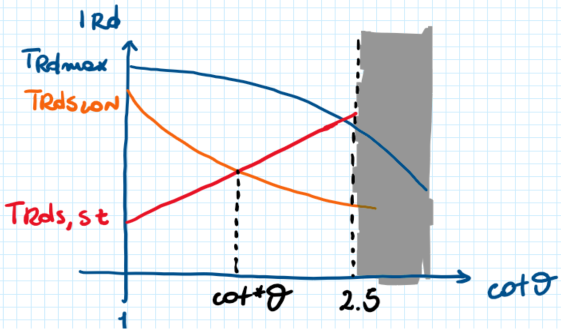
$$T_{Rds,st} = 2A_k \frac{A_{s,st}}{s} f_{yd} \cot\vartheta \qquad 1 \leq \cot\vartheta \leq 2.5$$

$$T_{Rds,lon} = 2A_k \frac{A_{s,lon}}{u_k} \frac{f_{yd}}{\cot\vartheta}$$

$$t = MAX \begin{cases} 2c \\ \dfrac{A}{u} = \dfrac{b \times h}{(b+h)2} \end{cases}$$

$$b_k = b - \frac{t}{2} - \frac{t}{2} \qquad A_k = b_k \cdot h_k$$

$$h_k = h - \frac{t}{2} - \frac{t}{2} \qquad u_k = (b_k + h_k) \times 2$$



$$T_{Rd} = MIN \begin{cases} T_{Rd\,max} \\ T_{Rds,lon} \\ T_{Rds,st} \end{cases}$$

PER LA VERIFICA ALLO SLU:

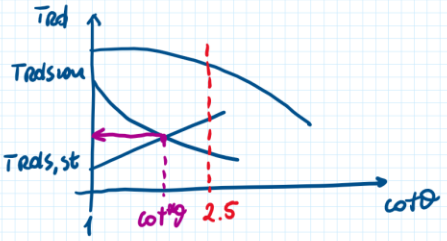
1) $\cot^*\vartheta \Rightarrow T_{Rds,lon} = T_{Rds,st}$

$$\cancel{2A_k}\,\frac{A_{s,lon}}{u_k}\,\cancel{f_{yd}}\,\frac{1}{\cot^*\vartheta} = \cancel{2A_k}\,\frac{A_{s,st}}{s}\,\cancel{f_{yd}}\,\cot^*\vartheta$$

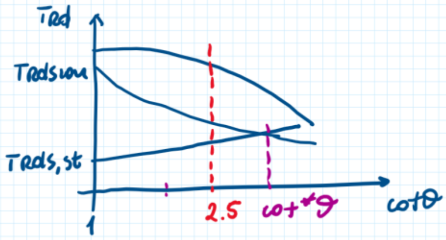
$$\cot^{*2}\vartheta = \frac{A_{s,lon}}{u_k}\,\frac{s}{A_{s,st}}$$

$$\boxed{\cot^*\vartheta = \sqrt{\frac{A_{s,lon}}{u_k}\,\frac{s}{A_{s,st}}}}$$

2) $\cot^*\vartheta < 2.5$



$\cot^*\vartheta > 2.5$



$$T_{Rds,st}(\cot^*\vartheta) = T_{Rd,lon}(\cot^*\vartheta) = T_{Rd,s}$$

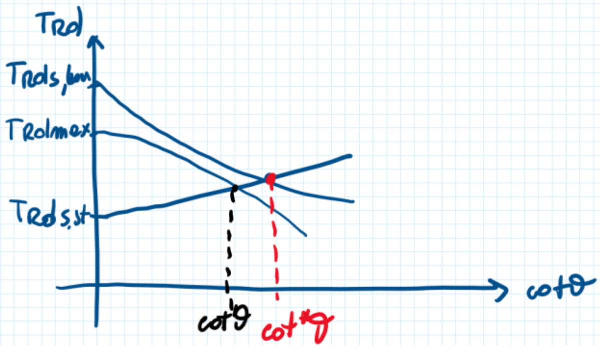
$$T_{Rd} = MIN \begin{cases} T_{Rdmax}(\cot^*\vartheta) \\ T_{Rds}(\cot^*\vartheta) \end{cases}$$

$$\left.\begin{matrix} T_{Rds,st}(2.5) \\ T_{Rds,lon}(2.5) \end{matrix}\right\} MIN = T_{Rds}$$

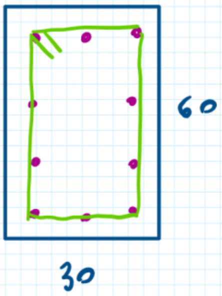
$$T_{Rd} = MIN \begin{cases} T_{Rdmax}(2.5) \\ T_{Rds}(2.5) \end{cases}$$

3) VERIFICARE $T_{Rdmax}(\cot^*\vartheta) > T_{Ed}$

Se $T_{Rdmax}(\cot^*\vartheta) < T_{Ed} \Rightarrow \cot'\vartheta : T_{Rds,st} = T_{Rdmax}$



ESEMPIO



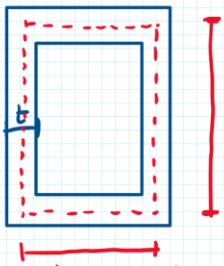
$c = 5\ cm$

$\phi 8/15$

$10\ \phi 14$

$T_{Ed} = 30\ kNm$

0) u_k , A_k



$$t = MAX \begin{cases} 2c = 2 \times 5 = 10 \\ \dfrac{A}{u} = \dfrac{30 \times 60}{30+30+60+60} = \dfrac{1800}{180} = 10 \end{cases}$$

$$h_k = h - \frac{t}{2} - \frac{t}{2} = 60 - \frac{10}{2} - \frac{10}{2} = 50$$

$$b_k = b - \frac{t}{2} - \frac{t}{2} = 30 - \frac{10}{2} - \frac{10}{2} = 20$$

$$A_k = 50 \times 20 = 1000\ cm^2$$

$$u_k = 50 + 50 + 20 + 20 = 140\ cm$$

1) $\cot^*\vartheta$

$$\cot^*\vartheta = \sqrt{\frac{A_{s\,lon}}{u_k} \frac{s}{A_{s,st}}} = \sqrt{\frac{15.4}{140} \times \frac{15}{0.5}} = 1.82 < 2.5$$

$A_{s\,lon} = 10 \times 1.54 = 15.4\ cm^2$

$s = 15\ cm$

$A_{s,st} = 0.5\ cm^2$

2) $$T_{Rd\,max} = 2\, A_k\, t\, f'_{cd} \frac{\cot^*\vartheta}{1 + \cot^*\vartheta^2} =$$

$$= 2 \times 1000 \times 10 \times 0.5 \times 14.17 \times \frac{1.82}{1 + 1.82^2} \frac{1}{10^3} = 91.5\ kNm$$

$T_{Rd\,max}\,(\cot^*\vartheta) > T_{Ed}$ <u>ok</u>!

$$T_{Rd_s} = T_{Rd_{s,lon}} = T_{Rd_{s,st}} = 2 A_k \frac{A_{s,st}}{s} f_{yd} \cot^*\vartheta =$$

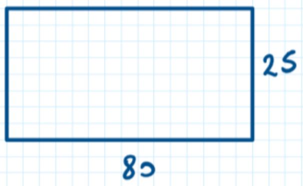
$$= 2 \times 1000 \times \frac{0.5}{15} \times 391.3 \times \frac{1.82}{10^3} = 47.5\ kNm$$

QUINDI $\quad T_{Rd\,max} = 91.5\ kNm$

$T_{Rd\,s} = 47.5\ kNm \implies T_{Rd} = 47.5\ kNm > 30\ kNm$

<u>ok</u>!

ESEMPIO



$c = 4\ cm$

$T_{Ed} = 40\ kNm$

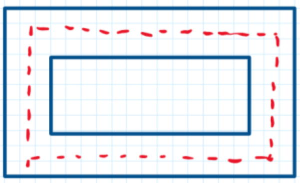
$\frac{A_{s,st}}{s} = ?$

$A_{s,lon} = ?$

1) VERIFICO LA SEZ IN CLS

$$T_{Rd\,max} = 2 A_k\, t\, f'_{cd} \frac{\cot\vartheta}{1 + \cot^2\vartheta}$$

$$t = \text{MAX} \begin{cases} 2 \times 4 = 8 \\ \frac{A}{u} = \frac{80 \times 25}{(80 + 25) \times 2} = 9.5 \end{cases} \implies t = 9.5\ cm$$



$h_k = 25 - \frac{9.5}{2} - \frac{9.5}{2} = 15.5$

$b_k = 80 - \frac{9.5}{2} - \frac{9.5}{2} = 70.5$

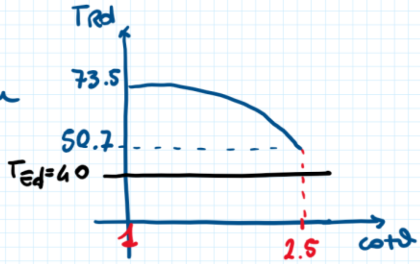
$A_k = 70.5 \times 15.5 = 1092.75\ cm^2$

$u_k = (70.5 + 15.5) \times 2 = 172\ cm$

$$T_{Rd_{max}} = 2 \times 1092.75 \times 9.5 \times 0.5 \times \frac{14.17}{10^3} \; \frac{\cot\vartheta}{1+\cot^2\vartheta} = 147 \; \frac{\cot\vartheta}{1+\cot^2\vartheta} \; \text{kNm}$$

se $\cot\vartheta = 1.0 \Rightarrow T_{Rd_{max}} = 73.5$ kNm

se $\cot\vartheta = 2.5 \Rightarrow T_{Rd_{max}} = 50.7$ kNm



$\forall \cot\vartheta$ PER IL PROG. DELLE ARMATURE ANDRA' BENE PER LA SEZ. IN CLS.

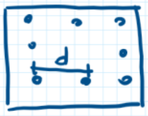
2) PROGETTO LE ARMATURE $\Rightarrow \cot\vartheta = 1.0$

$$A_{s,st} = \frac{T_{Ed} \times s^{=1m}}{2 A_k f_{yd} \cot\vartheta} = \frac{40 \times 1 \times 10^5}{2 \times 1092.75 \times 391.3 \times 1} = 4.68 \; \frac{cm^2}{1m}$$

$$A_{s\,lon} = \frac{T_{Ed}\, u_k \cot\vartheta}{2 A_k f_{yd}} = \frac{40 \times 172 \times 1.0 \times 10^3}{2 \times 1092.75 \times 391.3} = 8.05 \; cm^2$$

MINIMI DA NORMATIVA: STAFFE $s < \frac{u}{8}$

$A_{s\,lon} \Rightarrow d < 35$ cm



DOBBIAMO AVERE $s < \frac{(80+25) \times 2}{8} = \frac{210}{8} = \cancel{26.25}\; 25$ cm

DA CALCOLO $\Rightarrow 4.68 \; \frac{cm^2}{m}$

FISSO $\phi 8 \Rightarrow A_{1\phi 8} = 0.5 \; cm^2$

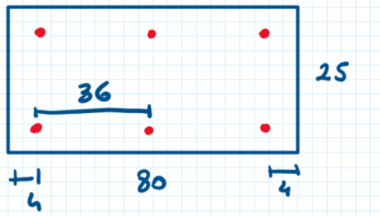
$$\frac{n_{STAFFE}}{1m} = \frac{4.68}{0.5} = \cancel{9.4} \; \frac{STAFFE}{1m} \; 10$$

$$s = \frac{100}{10} = 10 \; cm \Rightarrow \phi 8/10$$

DA CALCOLO $A_{s\,lon} = 8.05\ cm^2$

$A_{\phi 14} = 1.54\ cm^2$

$n_{\phi 14} = \dfrac{8.05}{1.54} = \cancel{5.23}^{\,6}\ \phi 14$



$d = 36 > 35$! NON VA BENE

$\Downarrow$

DEVO METTERE $8\phi 14$

POSSO OTTIMIZZARE IL PROGETTO:

$A_{s\,lon} = 8 \times 1.54 = 12.32\ cm^2$

$$\cot\theta = \frac{A_{s\,lon}}{u_k}\,\frac{2A_k f_{yd}}{T_{Ed}} = \frac{12.32}{172} \times \frac{2 \times 1092.72}{40} \times \frac{391.3}{10^3} = 1.53$$

RIPROGETTO LE STAFFE:

$$A_{s,st} = \frac{T_{Ed} \times s^{=1m}}{2A_k f_{yd} \cot\theta} = \frac{40 \times 1 \times 10^3}{2 \times 1092.75 \times 391.3 \times 1.53} = 3.06\ \frac{cm^2}{1m}$$

$$\frac{n_{STAFFE}}{1m} = \frac{3.06}{0.5} = \cancel{6.12}^{\,7.0}\ \text{STAFFE}/1m$$

$$s = \frac{100}{\cancel{6.12}_{\,7.0}} = \cancel{14.28}^{\,15} \qquad \phi 8/15$$

INTERAZIONE V + T

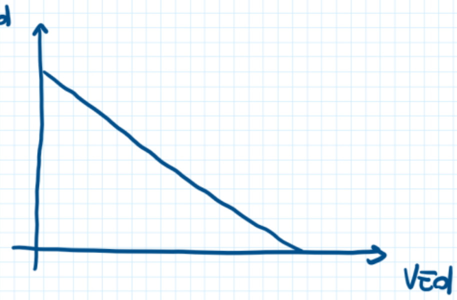
$$\left.\begin{array}{l} V \\ T \end{array}\right\} \tau$$

PER VERIFICARE ALLO SW :

1) PER LA SEZ. IN CLS

$$\frac{V_{Ed}}{V_{Rd\,max}} + \frac{T_{Ed}}{T_{Rd\,max}} < 1$$

↳ STESSO COTθ



2) PER LE ARMATURE

$$\left.\begin{array}{l} A_{s,st\,V} \\ A_{s,st\,T} \end{array}\right\} \qquad \left.\begin{array}{l} A_{par\,V} \\ A_{slon\,T} \end{array}\right\}$$