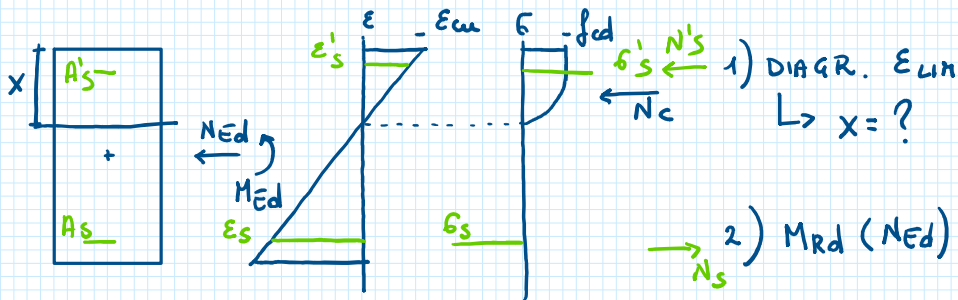


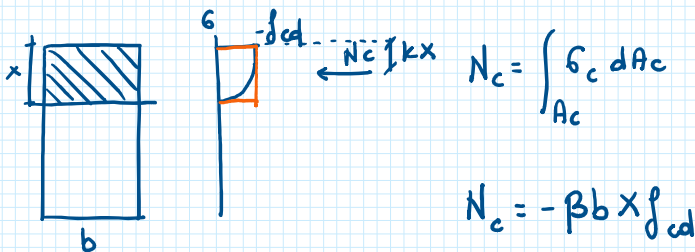
## SEZ. PARZIALIZZATA



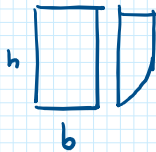
1) DIAGR.  $\epsilon_{LIN} \Leftrightarrow x$

$$N_c(x) + N'_s(x) + N_s(x) = N_{Ed} \Rightarrow x$$

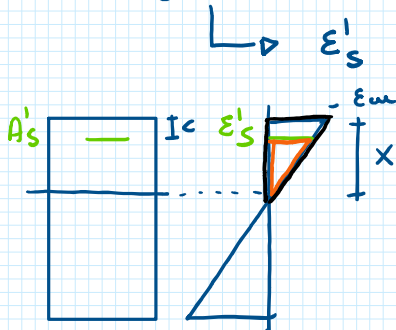
•  $N_c(x)$



$$\beta = \frac{\int \sigma_c dA_c}{b h f_{cd}} = 0.81 \quad k = 0.416$$

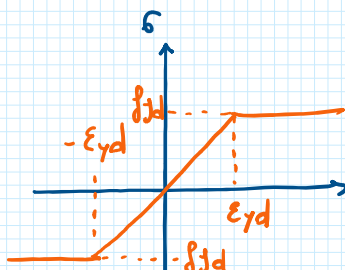


$$\bullet N'_s = A'_s \sigma'_s$$



$$\frac{\epsilon'_s}{-(x-c)} = \frac{\epsilon_{cu}}{x}$$

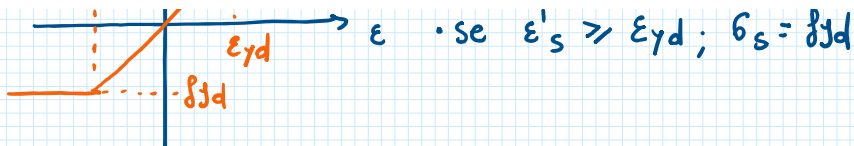
$$\epsilon'_s = -\frac{x-c}{x} \epsilon_{cu}$$



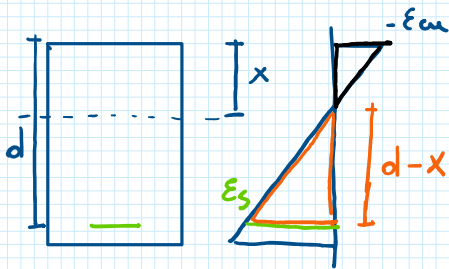
• se  $\epsilon'_s \leq -\epsilon_{yd}$ ;  $\sigma'_s = -f_{yd}$

•  $-\epsilon_{yd} \leq \epsilon'_s \leq \epsilon_{yd}$ ;  $\sigma'_s = \frac{\epsilon'_s}{\epsilon_{yd}} f_{yd}$

• se  $\epsilon'_s \geq \epsilon_{yd}$ ;  $\sigma'_s = f_{yd}$



$$\bullet N_s = A_s \sigma_s$$



$$\frac{\epsilon_{cu}}{x} = \frac{\epsilon_s}{d-x}$$

$$\epsilon_s = \frac{d-x}{x} \epsilon_{cu}$$

$$\bullet \text{ se } \epsilon_s \leq -\epsilon_{yd}; \sigma_s = -\sigma_{yd}$$

$$\bullet -\epsilon_{yd} \leq \epsilon_s \leq \epsilon_{yd}; \sigma_s = \frac{\epsilon_s}{\epsilon_{yd}} \sigma_{yd}$$

$$\bullet \text{ se } \epsilon_s > \epsilon_{yd}; \sigma_s = \sigma_{yd}$$

$$\text{QUINDI: } -\beta b x f_{cd} + A'_s \sigma'_s(x) + A_s \sigma_s(x) = N_{Ed}$$

$$\Rightarrow \text{SE } \sigma'_s = -\sigma_{yd} \text{ E } \sigma_s = \sigma_{yd} \text{ ALLORA:}$$

$$-\beta b x f_{cd} - A'_s \sigma_{yd} + A_s \sigma_{yd} = N_{Ed}$$

$$-\beta b x f_{cd} = N_{Ed} + A'_s \sigma_{yd} - A_s \sigma_{yd}$$

$$x = \frac{N_{Ed} + (A'_s - A_s) \sigma_{yd}}{-\beta b f_{cd}}$$

$$x = \frac{-N_{Ed} + (A_s - A'_s) \sigma_{yd}}{\beta b f_{cd}}$$

$$\Rightarrow \text{SE } A'_s \text{ OPPURE } A_s \text{ NON SONO SNERVATE}$$

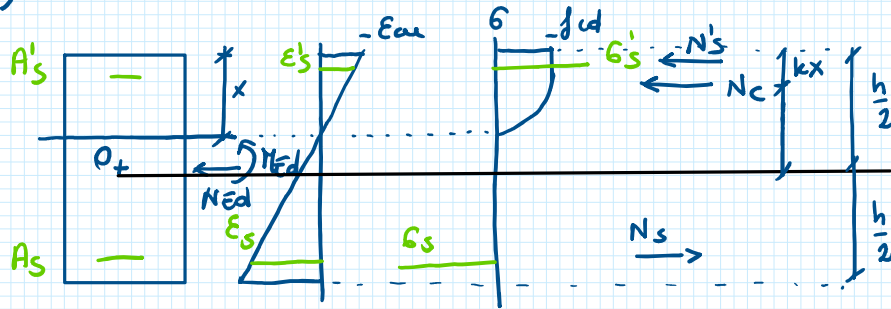
$$-\beta b x f_{cd} + A'_s \sigma'_s(x) + A_s \sigma_s(x) = N_{Ed}$$

DA RISOLVERE PER TENTATIVI

2) CALCOLARE  $M_{rd}(N_{Ed})$  RISPETTO "0"

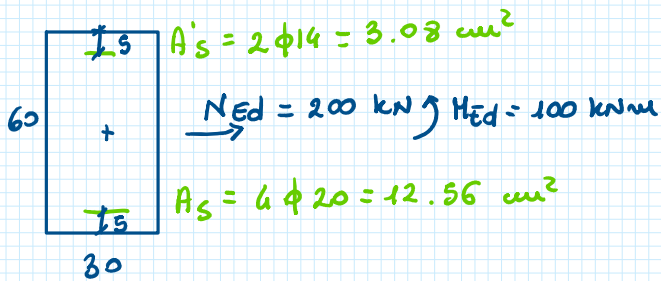
$$A'_s \text{ --- } * \text{ --- } \epsilon'_k \text{ --- } -\epsilon_{cu} \quad \sigma'_s \text{ --- } -\sigma_{yd} \quad \sigma_s \text{ --- } \sigma_{yd} \quad \epsilon_s \text{ --- } \epsilon_{yd}$$

2) CALCOLARE  $M_{Rd}$  ( $N_{Ed}$ ) RISPETTO A



$$M_{Rd}(N_{Ed}) = N_c \left[ -\left( \frac{h}{2} - kx \right) \right] + N'_s \left[ -\left( \frac{h}{2} - c \right) \right] + N_s \left( \frac{h}{2} - c \right)$$

### ESEMPIO 1



VERIFICA SLV

0) SEZ. PARZ. O TUTTA COMPRESSA?

POICHE' HO  $N_{Ed}^+$   $\Rightarrow$  SEZ. PARZIALIZZATA

1)  $\epsilon_{LIM} \Rightarrow x$

IPOTIZZO  $A'_s$  E  $A_s$  SNERVATE  $\Rightarrow$  EQ. 1° GRADO PER  $x$

$$x = \frac{-N_{Ed} + (A_s - A'_s) f_{yd}}{\beta b f_{cd}} = \frac{-200 + (12.56 - 3.08) \cdot \frac{391.3}{10}}{0.81 \cdot 30 \cdot \frac{14.17}{10}} = 4.96 \text{ cm}$$

DEVO VERIFICARE CHE  $A'_s$  E  $A_s$  SIANO SNERVATE:

$$\epsilon'_s = - \frac{x - c}{x} \epsilon_{cu} = - \frac{4.96 - 5}{4.96} 0.0035 = 2.32 \cdot 10^{-5}$$

$$\epsilon_{yd} = 0.00196 = 1.96 \cdot 10^{-3} \Rightarrow A'_s \text{ NON È SNERVATA} \\ \epsilon'_s < \epsilon_{yd}$$

⇒ DEVO RICALCOLARE X PER TENTATIVI

SEZ. PARZIALIZZATA						
X	6.80					
eps's	-0.0009249	sigma's	-184.9826	MPa	β	0.81
eps s	0.024826	sigma s	391.3	MPa	k	0.416
$N_c = -\beta \times f_{cd}$						
Nc	-234.50	bc	-27.17	cm	$\epsilon'_s = -\frac{x-c}{x} \epsilon_{cu}$	
N's	-56.97	b's	-25.00	cm		
Ns	491.47	bs	25.00	cm	$\epsilon_s = \frac{d-x}{x} \epsilon_{cu}$	
Somma N	200.00	kN				

2)  $M_{Rd} (N_{Ed})$

$$b_c = -\left(\frac{h}{2} - kx\right) = -\left(\frac{60}{2} - 0.416 \cdot 6.8\right) = -27.17 \text{ cm}$$

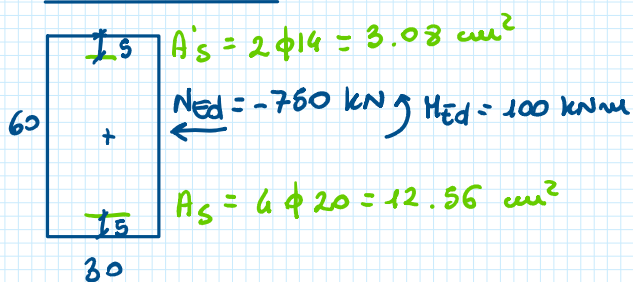
$$b'_s = -\left(\frac{h}{2} - c\right) = -\left(\frac{60}{2} - 5\right) = -25 \text{ cm}$$

$$b_s = \frac{h}{2} - c = \frac{60}{2} - 5 = 25 \text{ cm}$$

$$M_{Rd}(N_{Ed}) = 234.5 \left( \frac{27.17}{10^2} \right) + 56.97 \cdot \frac{25}{10^2} + 491.4 \cdot \frac{25}{10^2} = 200.8 \text{ kNm}$$

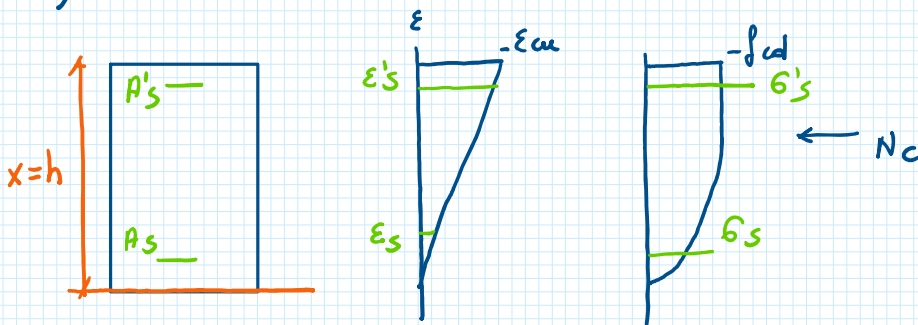
$$M_{Rd}(N_{Ed}) > M_{Ed} \quad \underline{\text{OK!}}$$

## ESEMPIO 2



VERIFICA SLV

0) SEZ. PARZIALIZZATA O TUTTA COMPRESSA ?

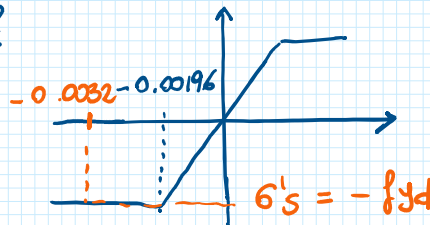


$$N_c = -\beta b \bar{x} f_{cd} = -0.81 \cdot 30 \cdot 60 \cdot \frac{14.17}{10} = -2070 \text{ kN}$$

$$N'_s = A'_s \sigma'_s = 3.08 \cdot \left( \frac{-391.3}{10} \right) = -120.52 \text{ kN}$$

$$\hookrightarrow \varepsilon'_s = -\frac{x-c}{x} \varepsilon_{uc} = -\frac{60-5}{60} 0.0035 = -0.0032$$

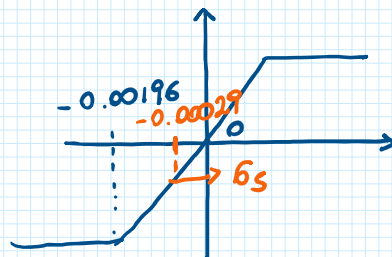
$$\hookrightarrow \sigma'_s = ?$$



$$N_s = A_s \sigma_s = 12.56 \cdot \left( \frac{-58.3}{10} \right) = -73.27 \text{ kN}$$

$$\hookrightarrow \varepsilon_s = \frac{55-60}{60} 0.0035 = -0.00029$$

$$\varepsilon_s \geq -\varepsilon_{yd} = -0.00196$$



$$\sigma_s = \frac{-0.00029}{0.00196} \cdot 391.3 = -58.3 \text{ MPa}$$

$$\text{QUINDI } |2070 + 120.52 + 73.27| = |2264.15|$$

$$\begin{array}{c} N_c + N'_s + N_s \\ |2264.15| \end{array} > \begin{array}{c} N_{Ed} \\ |750| \end{array} \Rightarrow \text{SEZ. PARZIALIZZATA}$$

$$1) \varepsilon_{lin} \Leftrightarrow x$$

IPOTIZZO  $A'_s$  e  $A_s$  SNERNATE:

$$x = \frac{-N_{Ed} + (A_s - A'_s) f_{yd}}{\beta b f_{cd}} = \frac{+750 + (12.56 - 3.08) \cdot \frac{391.3}{10}}{0.81 \cdot 30 \cdot \frac{14.17}{10}} = 32.55 \text{ cm}$$

VERIFICO LE ARMATURE :  $\varepsilon'_s = -\frac{x-c}{x} \varepsilon_{cu} = -\frac{32.55-5}{32.55} 0.0035 = -0.00296$

$\hookrightarrow \varepsilon'_s < -\varepsilon_{yd} \Rightarrow \sigma'_s = -f_{yd}$

$\varepsilon_s = \frac{d-x}{x} \varepsilon_{cu} = \frac{55-32.55}{32.55} 0.0035 = 0.00241$

$\hookrightarrow \varepsilon_s > \varepsilon_{yd} \Rightarrow \sigma_s = f_{yd}$

2)  $M_{rd} (N_{ed})$

$b_c = -\left(\frac{h}{2} - kx\right) = -\left(\frac{60}{2} - 0.416 \cdot 32.55\right) = -16.46 \text{ cm}$

$b'_s = -\left(\frac{h}{2} - c\right) = -\left(\frac{60}{2} - 5\right) = -25 \text{ cm}$

$b_s = \frac{h}{2} - c = \frac{60}{2} - 5 = 25 \text{ cm}$

$N_c = -\beta b x f_{cd} = -0.81 \cdot 30 \cdot 32.55 \cdot \frac{14.17}{10} = -1123.17 \text{ kN}$

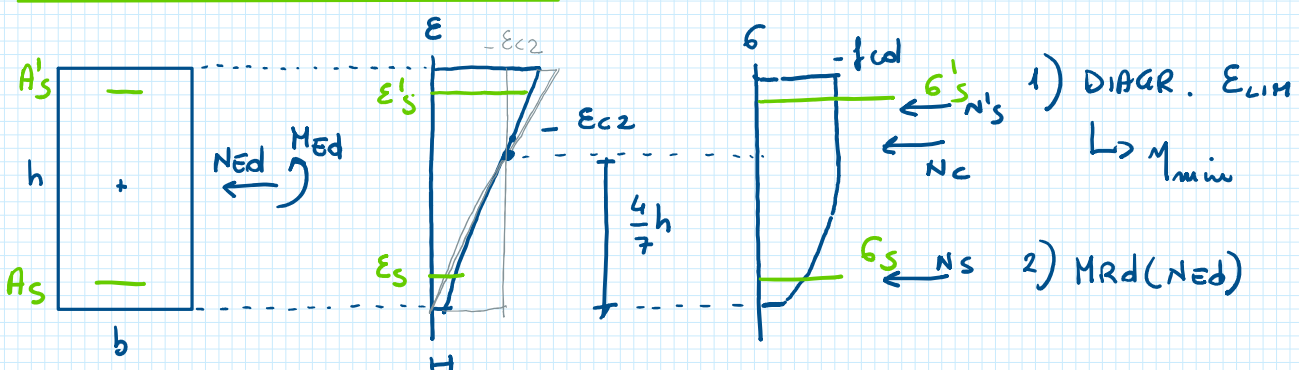
$N'_s = A'_s \sigma'_s = 3.08 \cdot \left(-\frac{391.3}{10}\right) = -120.52$

$N_s = A_s \sigma_s = 12.56 \cdot \frac{391.3}{10} = +491.47$

$M_{rd} (N_{ed}) = -1123.17 \cdot \left(-\frac{16.46}{10^2}\right) - 120.52 \left(-\frac{25}{10^2}\right) + 491.47 \left(\frac{25}{10^2}\right) = 337.86 \text{ kNm}$

$M_{rd} (N_{ed}) > M_{ed} \quad \underline{\text{OK!}}$

SEX. TUTTA COMPRESSA



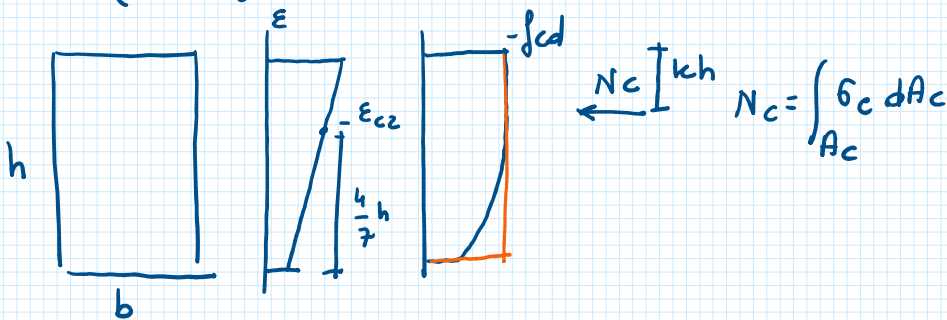
$$-\epsilon_{c2} \leq \epsilon_{cmin} \leq 0$$

$$0 \leq \eta_{min} = \frac{\epsilon_{cmin}}{-\epsilon_{c2}} \leq 1$$

1) DIAGR.  $\epsilon_{LIM} \Leftrightarrow \eta_{min}$

$$N_c(\eta_{min}) + N'_s(\eta_{min}) + N_s(\eta_{min}) = N_{Ed} \Rightarrow \eta_{min}$$

•  $N_c(\eta_{min})$

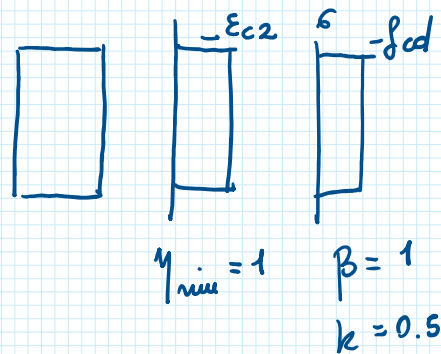
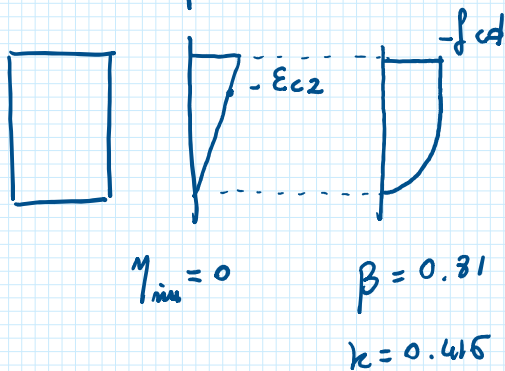


$$N_c = -\beta b h f_{cd}$$

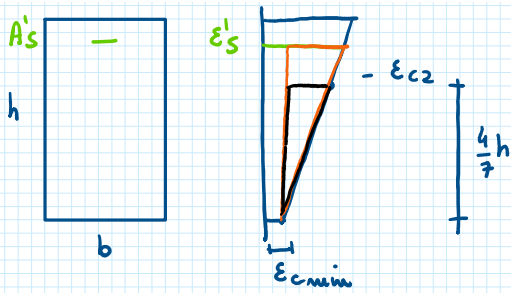
$$\beta = 1 - \frac{4}{21} (1 - \eta_{min})^2$$

$$k = \frac{1}{2} \frac{1 - \frac{16}{49} (1 - \eta_{min})^2}{1 - \frac{4}{21} (1 - \eta_{min})^2}$$

OSSERVIAMO  $\beta$  e  $k$ :



$$\bullet N'_s = A'_s \sigma'_s$$



$$\frac{\epsilon'_s - \epsilon_{cmin}}{d} = \frac{-\epsilon_{c2} - \epsilon_{cmin}}{\frac{4}{7}h}$$

$$\epsilon'_s - \epsilon_{cmin} = \frac{d}{\frac{4}{7}h} (-\epsilon_{c2} - \epsilon_{cmin})$$

$$\epsilon'_s = \frac{d}{\frac{4}{7}h} (-\epsilon_{c2} - \epsilon_{cmin}) + \epsilon_{cmin}$$

$$\epsilon'_s = -\epsilon_{c2} \left[ \frac{d}{\frac{4}{7}h} \left( 1 - \frac{\epsilon_{cmin}}{-\epsilon_{c2}} \right) + \frac{\epsilon_{cmin}}{-\epsilon_{c2}} \right]$$

$$\epsilon'_s = -\epsilon_{c2} \left[ \frac{d}{\frac{4}{7}h} (1 - \eta_{min}) + \eta_{min} \right]$$

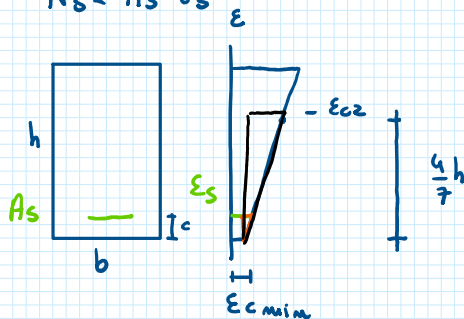
$$\text{se } \epsilon'_s \leq -\epsilon_{yd} \Rightarrow \sigma'_s = -f_{yd}$$

$$\text{se } -\epsilon_{yd} \leq \epsilon'_s \leq \epsilon_{yd} \Rightarrow \sigma'_s = \frac{\epsilon'_s}{\epsilon_{yd}} f_{yd}$$

$$\text{se } \epsilon'_s > \epsilon_{yd} \Rightarrow \sigma_s = f_{yd}$$

•  $N_s(\eta_{min})$

$$N_s = A_s \sigma_s$$



$$\frac{\epsilon'_s - \epsilon_{cmin}}{c} = \frac{-\epsilon_{c2} - \epsilon_{cmin}}{\frac{4}{7}h}$$

$$\epsilon'_s - \epsilon_{cmin} = \frac{c}{\frac{4}{7}h} (-\epsilon_{c2} - \epsilon_{cmin})$$



$$\epsilon_s = \frac{c}{\frac{4}{7}h} \left( -\epsilon_{c2} - \epsilon_{cmin} \right) + \epsilon_{cmin}$$

$$\epsilon_s = -\epsilon_{c2} \left[ \frac{c}{\frac{4}{7}h} \left( 1 - \frac{\epsilon_{cmin}}{-\epsilon_{c2}} \right) + \frac{\epsilon_{cmin}}{-\epsilon_{c2}} \right]$$

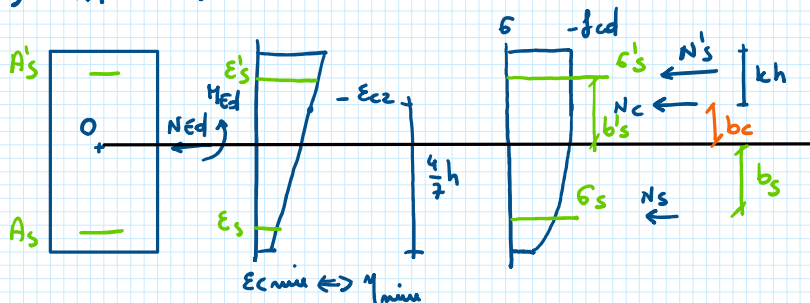
$$\epsilon_s = -\epsilon_{c2} \left[ \frac{c}{\frac{4}{7}h} (1 - \eta_{min}) + \eta_{min} \right]$$

$$\text{se } \epsilon_s \leq -\epsilon_{yd} \Rightarrow \sigma_s = -f_{yd}$$

$$\text{se } -\epsilon_{yd} \leq \epsilon_s \leq \epsilon_{yd} \Rightarrow \sigma_s = \frac{\epsilon_s}{\epsilon_{yd}} f_{yd}$$

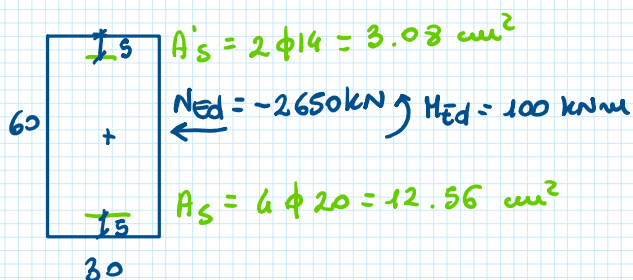
$$\text{se } \epsilon_s > \epsilon_{yd} \Rightarrow \sigma_s = f_{yd}$$

2)  $M_{Rd} (N_{Ed})$



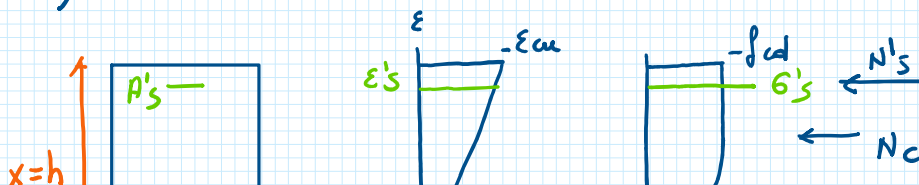
$$M_{Rd} (N_{Ed}) = -N_c \left( \frac{h}{2} - kh \right) - N'_s \left( \frac{h}{2} - c \right) + N_s \left( \frac{h}{2} - c \right)$$

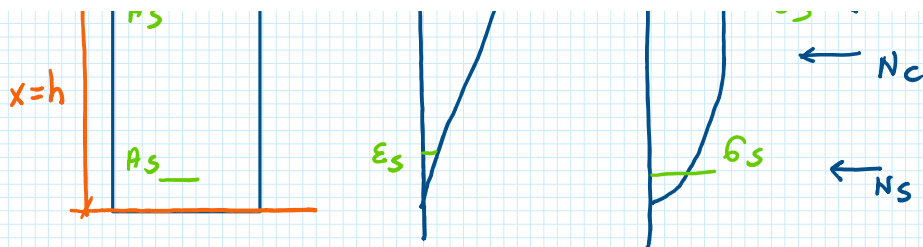
ESEMPIO



VERIFICA SLV

0) SEZ. PARZIALIZZATA O TUTTA COMPRESSA ?



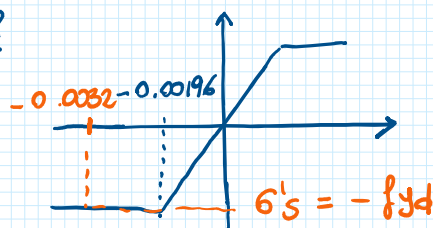


$$N_c = -\beta b \bar{x} f_{cd} = -0.81 \cdot 30 \cdot 60 \cdot \frac{14.17}{10} = -2070 \text{ kN}$$

$$N'_s = A'_s \sigma'_s = 3.08 \cdot \left( \frac{-391.3}{10} \right) = -120.52 \text{ kN}$$

$$\hookrightarrow \varepsilon'_s = -\frac{x-c}{x} \varepsilon_{uc} = -\frac{60-5}{60} 0.0035 = -0.0032$$

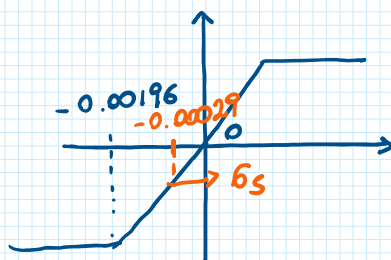
$$\hookrightarrow \sigma'_s = ?$$



$$N_s = A_s \sigma_s = 12.56 \cdot \left( \frac{-58.3}{10} \right) = -73.27 \text{ kN}$$

$$\hookrightarrow \varepsilon_s = \frac{55-60}{60} 0.0035 = -0.00029$$

$$\varepsilon_s \geq -\varepsilon_{yd} = -0.00196$$



$$\sigma_s = \frac{-0.00029}{0.00196} \cdot 391.3 = -58.3 \text{ MPa}$$

$$\text{QUINDI } |2070 + 120.52 + 73.27| = |2264.15|$$

$$|N_c + N'_s + N_s| \cdot |N_{ed}|$$

$$|2264.15| < |2650| \Rightarrow \text{SEZ. TUTTA COMPRESSA}$$

1) DIAGR  $\epsilon_{lim} \Rightarrow \eta_{min}$

SEZ. TUTTA COMPRESSA					
$\eta_{min}$	0.309				
$\epsilon_{ps's}$	-0.0028349	$\sigma_{ps's}$	-391.3	MPa	$\beta$
$\epsilon_{ps}$	-0.000820	$\sigma_{ps}$	-163.9318	MPa	$k$
$N_c = -\beta b h f_{cd}$					
$N_c$	-2323.58	$b_c$	-2.14	cm	$\epsilon'_s = -\epsilon_{c2} \left[ \frac{d}{\frac{4}{3}h} (1 - \eta_{min}) + \eta_{min} \right]$ $\epsilon_s = -\epsilon_{c2} \left[ \frac{c}{\frac{4}{3}h} (1 - \eta_{min}) + \eta_{min} \right]$
$N's$	-120.52	$b's$	-25.00	cm	
$N_s$	-205.90	$b_s$	25.00	cm	
Somma N	-2650.00	kN			

$$\beta = 1 - \frac{4}{21} (1 - \eta_{min})^2$$

$$k = \frac{1}{2} \frac{1 - \frac{16}{49} (1 - \eta_{min})^2}{1 - \frac{4}{21} (1 - \eta_{min})^2}$$

$$N_c(\eta_{min}) + N's(\eta_{min}) + N_s(\eta_{min}) = N_{Ed} \Rightarrow \eta_{min} = 0.309$$

2)  $M_{Rd}(N_{Ed})$

$$N_c = -\beta b h f_{cd} = -0.909 \cdot 30 \cdot 60 \cdot \frac{14.17}{10} = -2323.6 \text{ kN}$$

$$\beta = 1 - \frac{4}{21} (1 - \eta_{min})^2 = 0.909$$

$$N's = A's \sigma's = 2 \times 1.54 \times 391.3 / 10 = -120.52 \text{ kN}$$

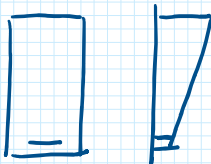
$$\hookrightarrow \epsilon'_s = -\epsilon_{c2} \left[ \frac{d}{\frac{4}{3}h} (1 - \eta_{min}) + \eta_{min} \right] = -0.0028$$

$$\epsilon'_s \leq -\epsilon_{yd} \Rightarrow \sigma'_s = -391.3 \text{ MPa}$$

$$-0.00196$$

$$N_s = A_s \sigma_s = 4 \times 3.14 \times (-163.9) / 10 = -205. \text{ kN}$$

$$\hookrightarrow \epsilon_s = -\epsilon_{c2} \left[ \frac{c}{\frac{4}{3}h} (1 - \eta_{min}) + \eta_{min} \right] = -0.00082$$



$$\sigma_s = \frac{-0.00082}{0.00196} 391.3 = -163.9 \text{ MPa}$$

$$k = \frac{1}{2} \frac{1 - \frac{16}{49} (1 - \eta_{min})^2}{1 - \frac{4}{21} (1 - \eta_{min})^2} = 0.464$$

$$M_{Rd}(N_{Ed}) = -2323.6 \left[ \left( \frac{60}{2} - 0.464 \times 60 \right) \right] \frac{1}{10^2}$$

$$-120.52 \left[ \left( \frac{60}{2} - 5 \right) \right] \frac{1}{10^2}$$

$$-209 \left( \frac{60}{2} - 5 \right) \frac{1}{10^2} = 23.46 \text{ kNm}$$