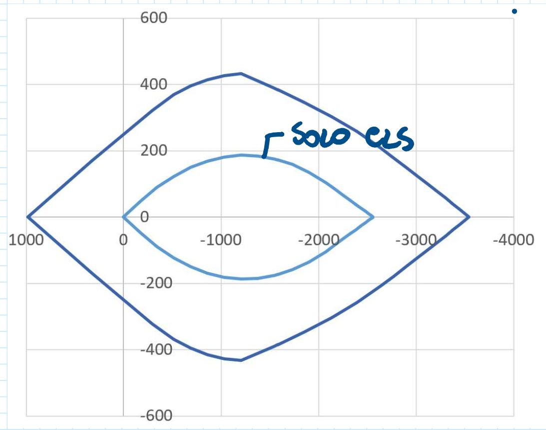
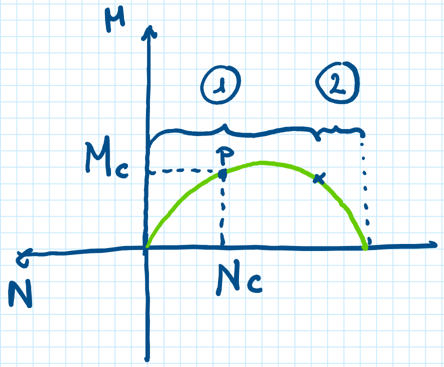
Flessione composta: domini MN

martedì 19 maggio 2026 14:01



GUARDO SOLO LA SEZ. IN CLS:

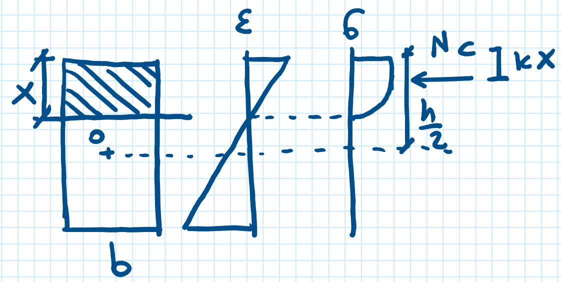


① SEZ. PARZIALIZZATA

② SEZ. TUTTA COMPRESSA

$P \in ①$

$$P \begin{cases} N_c = -\underset{=0.81}{\beta}\, b\, x\, f_{cd} & ① \\ M_c = -N_c\left(\dfrac{h}{2} - \underset{=0.416}{k}\, x\right) & ② \end{cases}$$



Da ① $\Longrightarrow$

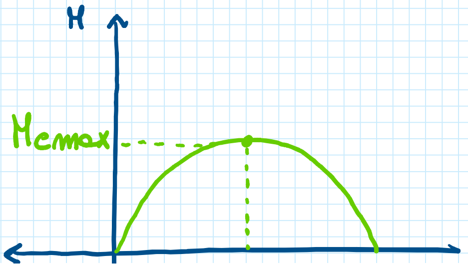
$$x = -\frac{N_c}{\beta\, b\, f_{cd}} \quad ③$$

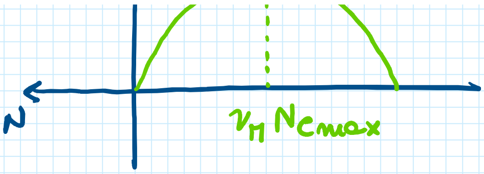
Sostituisco ③ in ② :

$$M_c = -N_c\left(\frac{h}{2} + k\,\frac{N_c}{\beta\, b\, f_{cd}}\right)$$

EQ. DI UNA PARABOLA

VERTICE DELLA PARABOLA:





$$\frac{dM_c}{dN_c} = 0 \qquad \frac{d}{dN_c}\left(-N_c\frac{h}{2} - \frac{k N_c^2}{\beta b f_{cd}}\right) = 0$$

$$-\frac{h}{2} - 2N_c\frac{k}{\beta b f_{cd}} = 0$$

$$N_c = -\frac{h}{2}\,\frac{\beta b f_{cd}}{2k}$$

$$N_c = \underbrace{-\frac{\overset{=0.81}{\beta}}{\underset{=0.416}{4k}}}_{\nu_M = 0.487}\;\underbrace{b h f_{cd}}_{N_{c\,max}} \qquad 4$$

LA CORRISPONDENTE POSIZIONE DELL'ASSE NEUTRO x:

SOSTITUISCO 4 IN 3: $x = -\dfrac{N_c}{\beta b f_{cd}}$ 3

$$x = +\frac{\cancel{\beta}}{4k}\,\frac{\cancel{b} h \cancel{f_{cd}}}{\cancel{\beta}\,\cancel{b}\,\cancel{f_{cd}}}$$

$$x = \frac{h}{\underset{=0.416}{4k}} \quad 5 \quad = 0.61\,h$$

PER TROVARE $M_{c\,max}$ SOSTITUISCO 5 E 4 IN 2:

$$M_c = -N_c\left(\frac{h}{2} - kx\right) \quad 2$$

$$M_{c\,max} = +\frac{\beta}{4k}\,b h f_{cd}\left(\frac{h}{2} - \cancel{k}\frac{h}{4\cancel{k}}\right)$$

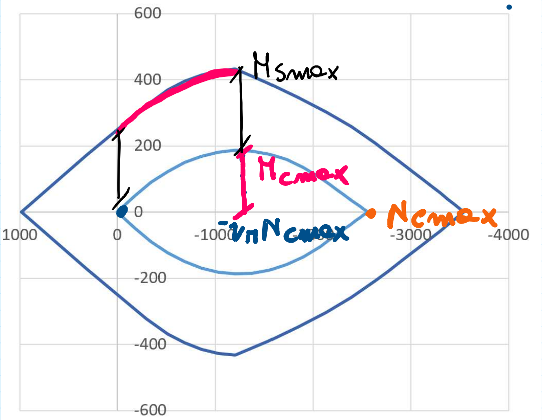
$$M_{c\,max} = \frac{\beta}{4k}\,b h f_{cd}\,\frac{h}{4}$$

$$M_{c\,max} = \frac{\beta}{16k}\, b h^2 f_{cd} = 0.122\, b h^2 f_{cd}$$

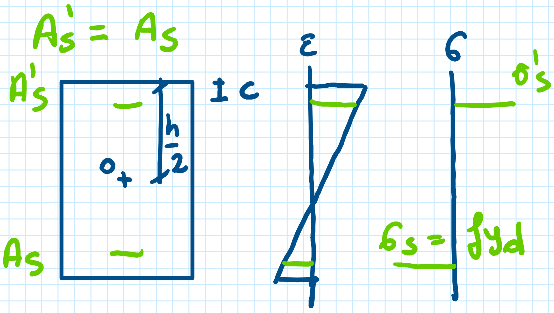
QUINDI:

$$\boxed{M_{Rd_c} = M_{c\,max}\left[1 - \left(\frac{N_{Ed} + \nu_M N_{c\,max}}{\nu_M N_{c\,max}}\right)^2\right]}$$

PER CONSIDERARE LE ARMATURE:

- 

$A'_s = A_s$



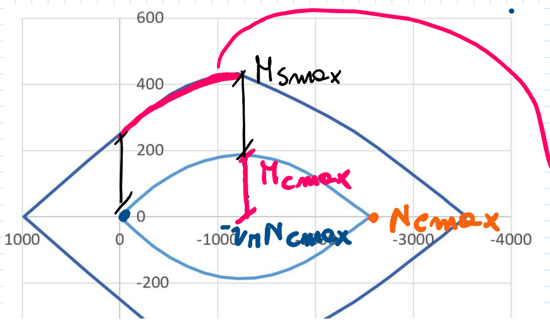
$\sigma'_s = -f_{yd}$ $\quad N'_s = -A'_s f_{yd}$

$N_s = A_s f_{yd}$

$$M_{s\,max} = -N'_s\left(\frac{h}{2} - c\right) + N_s\left(\frac{h}{2} - c\right)$$

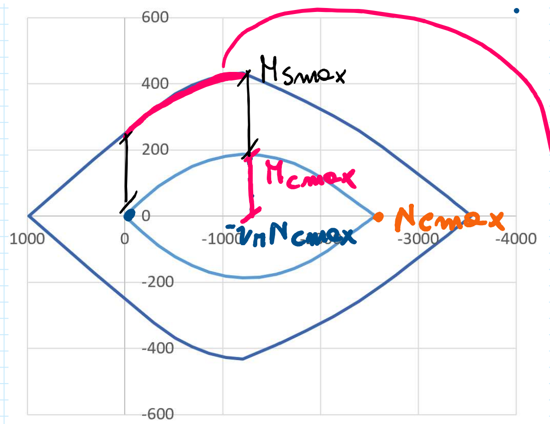
$$M_{s\,max} = +A'_s f_{yd}\left(\frac{h}{2} - c\right) + A_s f_{yd}\left(\frac{h}{2} - c\right)$$

$$M_{s\,max} = 2 A_s f_{yd}\left(\frac{h}{2} - c\right)$$

- 

$$M_{Rd} = M_{CLS} + M_{ARM}$$

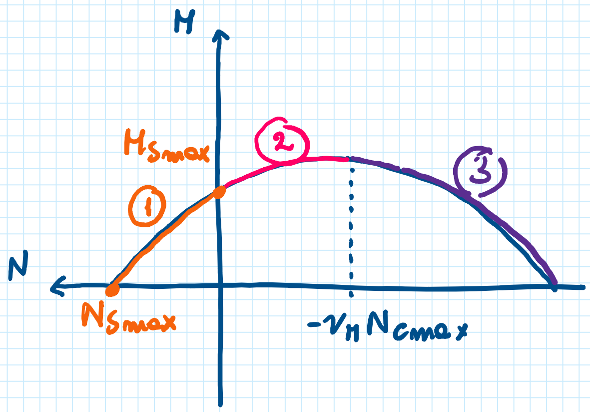
$$M_{Rd} = M_{c\,max}\left[1 - \left(\frac{N_{Ed} + \nu_M N_{c\,max}}{\nu_M N_{c\,max}}\right)^2\right] + M_{s\,max}$$



$$M_{Rd} = M_{cls} + M_{ARM}$$

$$M_{Rd} = M_{cmax}\left[1-\left(\frac{N_{Ed}+\nu_M N_{cmax}}{\nu_M N_{cmax}}\right)^2\right] + M_{smax}$$

EQUAZIONE DEL DOMINIO MN - 3 TRATTI



① $N_{Ed} \geq 0$

$$\boxed{M_{Rd} = M_{smax}\left[1-\frac{N_{Ed}}{N_{smax}}\right]}$$

$$N_{smax} = 2 A_s f_{yd}$$

② $-\nu_M N_{cmax} \leq N_{Ed} \leq 0$

$$\boxed{M_{Rd} = M_{cmax}\left[1-\left(\frac{N_{Ed}+\nu_M N_{cmax}}{\nu_M N_{cmax}}\right)^2\right] + M_{smax}}$$

③ $N_{Ed} \leq -\nu_M N_{cmax}$

$$M_{Rd} = \left(M_{cmax} + M_{smax}\right)\left[1 - \frac{\left|N_{Ed}+\nu_M N_{cmax}\right|^m}{(1-\nu_M)N_{cmax}+N_{smax}}\right]$$

$$m = 1 + \left(\frac{\nu_M N_{cmax}}{(1-\nu_M)N_{cmax} + N \ldots}\right)^2$$

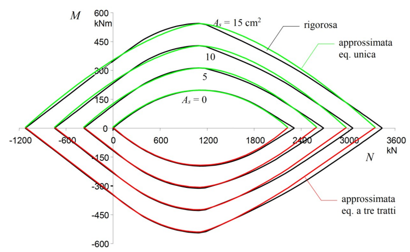
$$m = 1 + \left(\frac{\nu_M N_{c\,max}}{(1-\nu_M) N_{c\,max} + N_{s\,max}} \right)^2$$

EQUAZIONE DEL DOMINIO : UNICO TRATTO

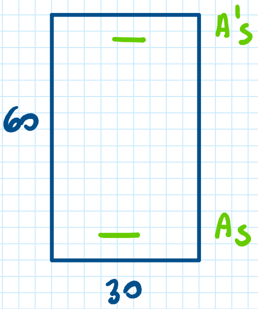
$$M_{Rd} = \left(M_{c\,max} + M_{s\,max} \right) \left[1 - \left| \frac{N_{Ed} + \nu_M N_{c\,max}}{\nu_M N_{c\,max} + N_{s\,max}} \right|^m \right]$$

$$m = 1 + \frac{\nu_M N_{c\,max}}{\nu_M N_{c\,max} + N_{s\,max}}$$

Confronto dominio rigoroso Vs. Equazioni



ESEMPIO



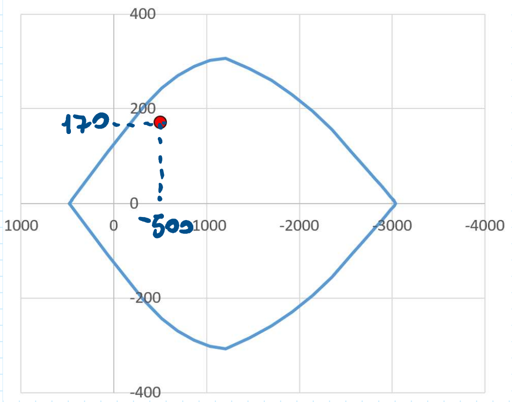
$c = 5$ cm

$A'_s = A_s = 6.16$ cm^2 = 4 ϕ14

$N_{Ed} = -500$ kN

$M_{Ed} = 170$ kNm

1) DOMINI PER PUNTI



2) DOMINIO 3 TRATTI

$$\nu_M N_{cmax} = 0.487 \times \underbrace{30 \times 60 \times \frac{14.17}{10}}_{N_{cmax} = 2550.6 \text{ kN}} = 1242.14 \text{ kN}$$

$N_{Ed} = -500$ $\qquad |N_{Ed}| < |\nu_M N_{cmax}| \Rightarrow$ 2

$$\boxed{M_{Rd} = M_{cmax}\left[1 - \left(\frac{N_{Ed} + \nu_M N_{cmax}}{\nu_M N_{cmax}}\right)^2\right] + M_{smax}}$$

$$M_{cmax} = 0.122\, b h^2 f_{cd} = 0.122 \cdot 30 \cdot 60^2 \cdot \frac{14.17}{10^3} = 186.7 \text{ kNm}$$

$$M_{smax} = 2 A_s f_{yd} \left(\frac{h}{2} - c\right) = 2 \times 6.16 \times \frac{391.3}{10^3} \times \left(\frac{60}{2} - 5\right) = 120.5 \text{ kNm}$$

$$M_{Rd} = 186.7 \left[1 - \left(\frac{-500 + 1242.14}{1242.14}\right)^2\right] + 120.5 = 240.55 \text{ kNm}$$

$M_{Rd} > M_{Ed}$ ok!

3) EQ. UNICO TRATTO

$$M_{Rd} = (M_{cmax} + M_{smax})\left[1 - \left|\frac{N_{Ed} + \nu_M N_{cmax}}{\nu_M N_{cmax} + N_{smax}}\right|^m\right]$$

$$m = 1 + \frac{\nu_M N_{cmax}}{\nu_M N_{cmax} + N_{smax}}$$

$$N_{s\,max} = 2 A_s f_{yd} = 2 \times 6.16 \times \frac{391.3}{10} = 482.1 \text{ kN}$$

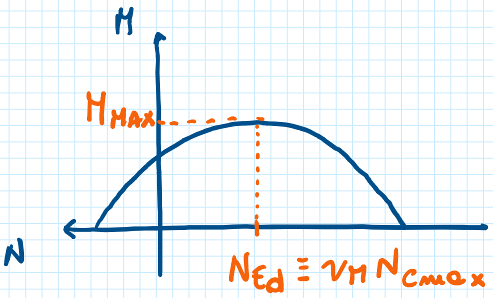
$$m = 1 + \frac{1242.14}{1242.14 + 482.1} = 1.72$$

$$M_{Rd} = (186.7 + 120.5)\left[1 - \left|\frac{-500 + 1242.14}{1242.14 + 482.1}\right|^{1.72}\right] = 235.14 \text{ kNm}$$

$\boxed{\text{PROGETTO A FLESSIONE COMPOSTA}}$

1) PROG. DELLA SEZ. IN CLS

$\Rightarrow N_{Ed}$



$$N_{Ed} = \nu_M N_{c\,max}$$

$$N_{Ed} = \nu_M\, b h f_{cd}$$

$$bh = \frac{N_{Ed}}{\nu_M f_{cd}}$$

$\Rightarrow M_{Ed}$

$$M_{Rd} = \frac{b d^2}{r'^2} = M_{Ed}$$

2) PROG. LE ARMATURE :

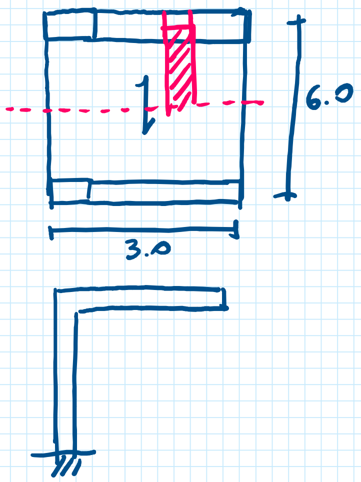
$$M_{Rd} = M_{c\,max}\left[1 - \left(\frac{N_{Ed} + \nu_M N_{c\,max}}{\nu_M N_{c\,max}}\right)^2\right] + M_{s\,max}$$

$$M_{s\,max} = \underbrace{M_{Ed} - M_{c\,max}\left[1 - \left(\frac{N_{Ed} + \nu_M N_{c\,max}}{\nu_M N_{c\,max}}\right)^2\right]}_{\Delta M}$$

$$\begin{cases} M_{s\,max} = 2 A_s f_{yd} \left(\frac{h}{2} - c\right) \\ M_{s\,max} = \Delta M \end{cases} \Rightarrow \Delta M = 2 A_s f_{yd} \left(\frac{h}{2} - c\right)$$

$$\boxed{A_s = \frac{\Delta M}{2 f_{yd} \left(\frac{h}{2} - c\right)}}$$

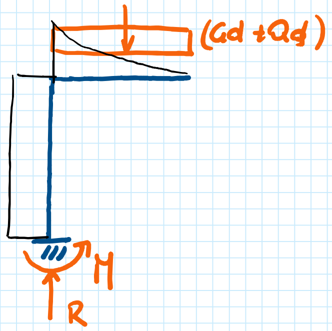
ESEMPIO



$g_d = 7 \ kN/m^2$

$q_d = 2 \ kN/m^2$

$P.P. = 3.0 \ kN/m$



$$G_d + Q_d \ (\text{TRAVE}) \begin{cases} \frac{6.0}{2} \times (7+2) = 27 \ \frac{kN}{m} \\ 1\,m \times 3 = 3 \ \frac{kN}{m} \end{cases} = 30 \ \frac{kN}{m}$$

$$N_{Ed} = R = 30 \times L_{TRA} = 30 \times 3 = 90 \ kN$$

$$M_{Ed} = 30 \times \frac{L_{TR}^2}{2} = 30 \times \frac{3^2}{2} = 135 \ kNm$$

$\Rightarrow$ PROG. SEZ. IN CLS CON M_{Ed}

$$M_{Ed} = \frac{b d^2}{r'^2} \Rightarrow d = r' \sqrt{\frac{M_{Ed}}{b}} = 0.019 \sqrt{\frac{135}{0.30}} = 0.40 \ m$$

$b = 0.30 \ m \qquad r' = 0.019$

$d = 40 \ cm \qquad h = 40 + 5 = \cancel{45} \ cm$ 30 x 50

⇒ PROG. L'ARMATURA:

$$\Delta M = M_{Ed} - M_{cmax}\left[1 - \left(\frac{N_{Ed} + \nu_M N_{cmax}}{\nu_M N_{cmax}}\right)^2\right]$$

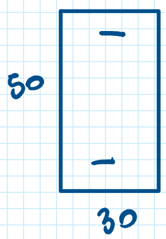
$$\nu_M N_{cmax} = 0.487 \cdot \underbrace{30 \times 50 \times \frac{14.17}{10}}_{N_{cmax} = 2125.5} = 1035.12 \text{ kN}$$

$$M_{cmax} = 0.122 \cdot 30 \times 50^2 \times \frac{14.17}{10^3} = 129.7 \text{ kNm}$$

$$\Delta M = 135 - 129.7\left[1 - \left(\frac{-90 + 1035.12}{1035.12}\right)^2\right] = 113.4 \text{ kNm}$$

$$A_s = \frac{\Delta M}{2 f_{yd}\left(\frac{h}{2} - c\right)} = \frac{113.4 \times 10^3}{2 \cdot 391.3\left(\frac{50}{2} - 5\right)} = 7.25 \quad cm^2$$

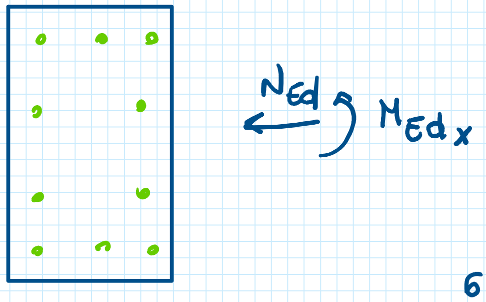
$$\frac{\cancel{kN}\,\cancel{m}}{\frac{\cancel{N}}{mm^2}\,\cancel{cm}} \times 10^3 \times \frac{\cancel{10^2}}{\cancel{10^2}}$$

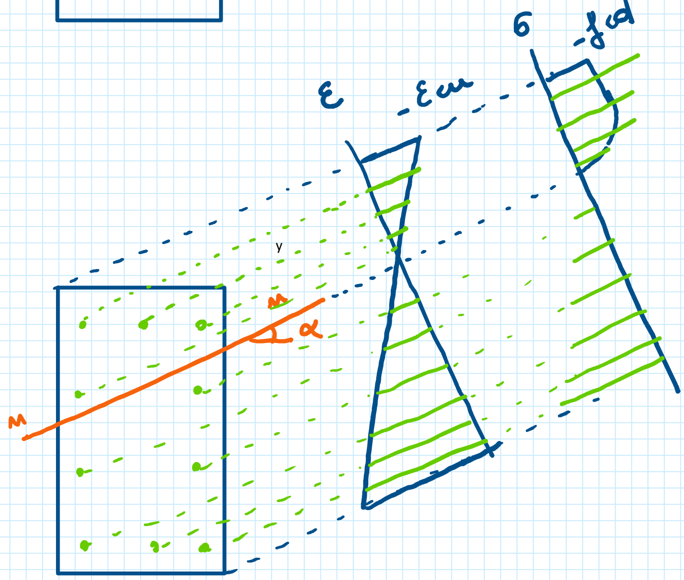


$A'_s = 2\phi 20 + 1\phi 14$

$A_s = 2\phi 20 + 1\phi 14$

FLESSIONE COMPOSTA DEVIATA

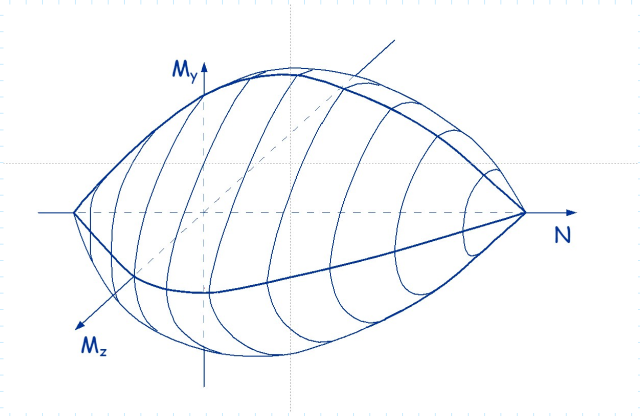




$$N = \int \sigma \, dA$$

$$M_x = \int \sigma y \, dA$$

$$M_y = -\int \sigma x \, dA$$



PER LA VERIFICA $\Rightarrow$ FISSO N_{Ed} $\Rightarrow$ $\left(\dfrac{M_{Ed\,y}}{M_{Rd,y}}\right)^{\alpha} + \left(\dfrac{M_{Ed\,x}}{M_{Rd,x}}\right)^{\alpha} \leq 1$

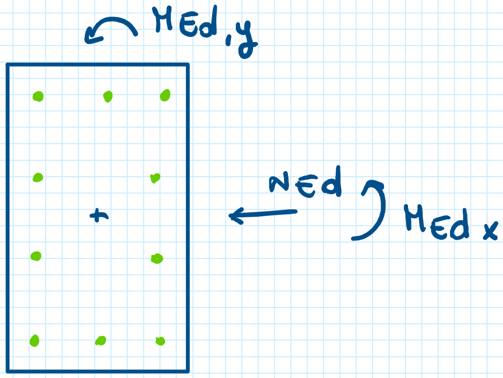
In mancanza di una specifica valutazione, può assumersi:

- per sezioni rettangolari:

N_{Ed}/N_{Rcd}	0,1	0,7	1,0
α	1,0	1,5	2,0

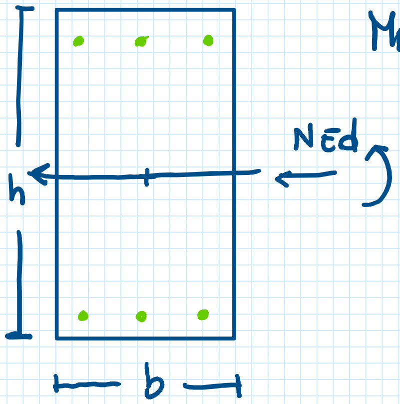
CAUTELATIVAMENTE $\Rightarrow$ $\alpha = 1.5$

OPERATIVAMENTE:



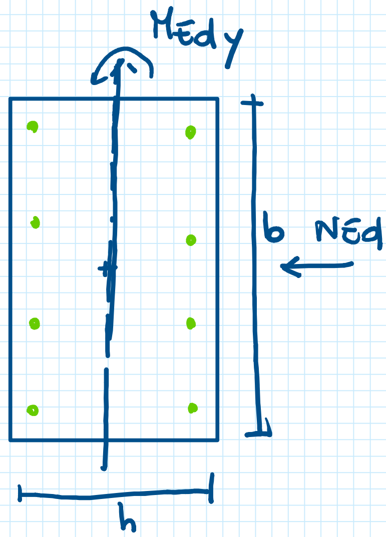
$$\left(\frac{M_{Ed\,y}}{M_{Rd,y}}\right)^{\alpha} + \left(\frac{M_{Ed\,x}}{M_{Rd,x}}\right)^{\alpha} \leq 1$$

PER $M_{Rd,x}$ =>



$M_{Rd\,x}(N_{Ed})$

PER $M_{Rd\,y}$ =>



$M_{Rd\,y}(N_{Ed})$