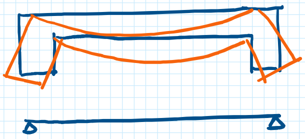
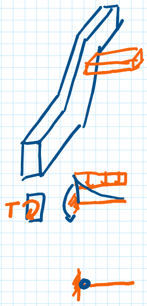
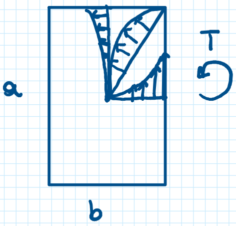
TORSIONE PER CONGRUENZA



TORSIONE PER EQUILIBRIO



SEZIONE OMOGENEA



$a > b$

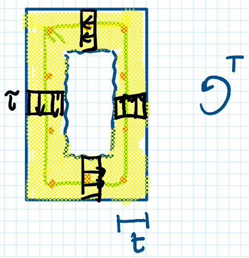
$$\tau_{MAX} = \frac{T}{a b^2} \psi$$

$$\psi = 3.0 + \frac{2.6}{0.45 + \frac{a}{b}}$$

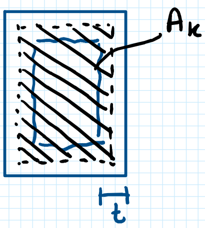
$a \to \infty \quad \psi \to 3.0$

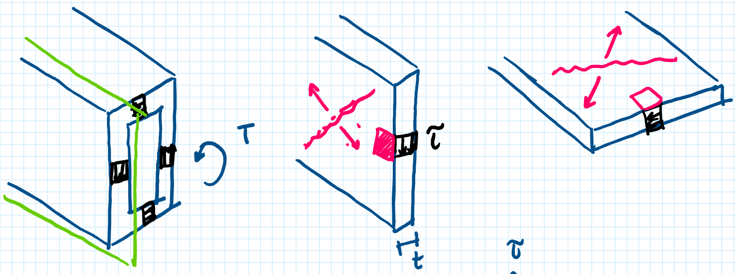
$a \to b \quad \psi \to 3.0 + \frac{2.6}{0.45 + 1}$

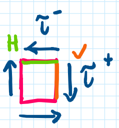
SEZ. IN C.A.



$$\tau = \frac{T}{2 A_k t}$$

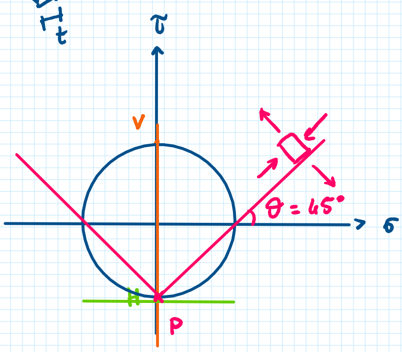




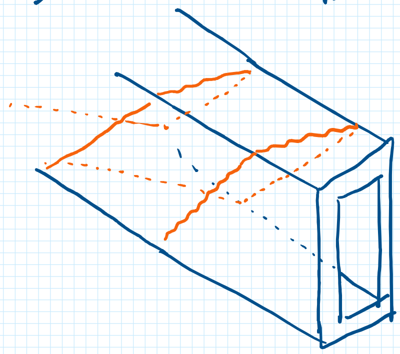


$V(0; \tau^+)$

$H(0; \tau^-)$



QUINDI => LESIONI A 45° A SPIRALE

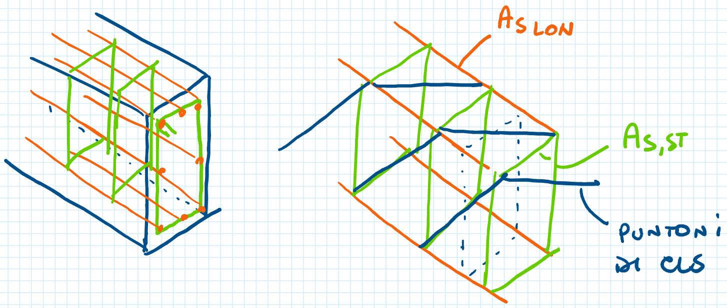


=> ARMATURA PER PORTARE σ_ξ

1) ARMATURA A SPIRALE 2) STAFFE + ARM. LONGITUDINALE

MODELLI DI CALCOLO

1) MODELLO DI (TRALICCIO) RAUSH



2) MODELLO DEI CAMPI DI TENSIONE

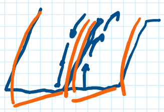
PASSAGGIO ALLO SW

APPLICO UN T...

... PORTA ALLO SNERVAMENTO DELLE ARMATURE...

... POSSO ULTERIORMENTE AUMENTARE T...

... PUNTONI DI CLS SI INFLETTONO ...



... PER EFFETTO DELL'INGRANAMENTO DEGLI INERTI NASCONO τ SULLA SUPERFICIE DI CONTATTO TRA I PUNTONI...

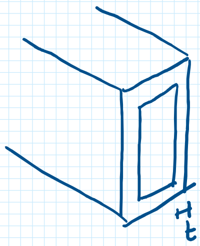
... IL CERCHIO DEL MOHR SI MODIFICA...

... LE DIREZIONI PRINCIPALI DI COMPRESSIONE

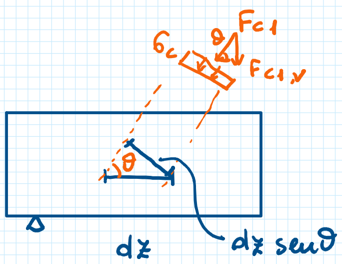
... IL CERCHIO DEL MOHR SI MODIFICA ...

... LE DIREZIONI PRINCIPALI DI COMPRESSIONE SONO INCLINATE $\vartheta < 45°$

MODELLO DEI CAMPI DI TENSIONE A SW

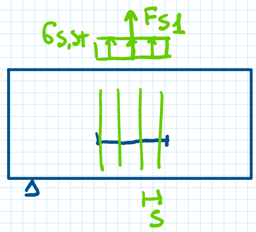


SEZIONE ORIZZONTALE



$$F_{c1} = \sigma_c \, t \, dz \, \text{sen}\vartheta$$

$$F_{c1_v} = F_{c1} \, \text{sen}\vartheta$$



$$F_{s1} = \sigma_{s,st} \, A_{s,st} \, \frac{dz}{s}$$

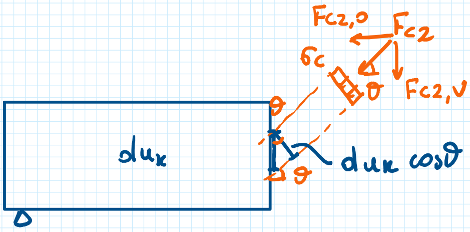
EQ. TRASLAZIONE VERTICALE:

$$F_{c1,v} = F_{s1}$$

$$\sigma_c \, t \, dz \, \text{sen}^2\vartheta = \sigma_{s,st} \, A_{s,st} \, \frac{dz}{s}$$

$$\boxed{\sigma_c \, t \, \text{sen}^2\vartheta = \sigma_{s,st} \, \frac{A_{s,st}}{s}} \quad (1)$$

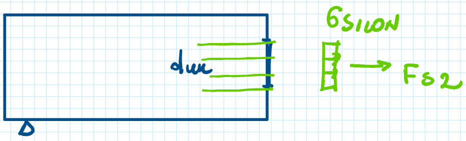
SEZIONE VERTICALE



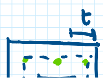
$$F_{c2} = \sigma_c \, t \, du_x \cos\vartheta$$

$$F_{c2_o} = F_{c2} \cos\vartheta$$

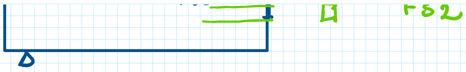
$$F_{c2_v} = F_{c2} \, \text{sen}\vartheta$$

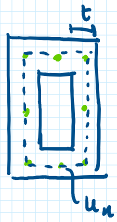


$$F_{s2} = \sigma_{s,lon} \, \frac{A_{s,lon}}{u_x} \, du_x$$



$$A_{s,lon} = \frac{x}{\ }$$





$$\frac{A_{s,lon}}{u_k} = \frac{x}{du_k}$$

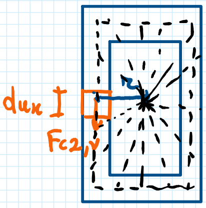
$$x = \frac{A_{s,lon}}{u_k}\, du_k$$

EQ. TRASLAZIONE ORIZZONTALE: $F_{c2,o} = F_{s2}$

$$\sigma_c\, t\, \cancel{du_k} \cos^2\theta = \sigma_{s,lon} \frac{A_{s,lon}}{u_k} \cancel{du_k}$$

$$\boxed{\sigma_c\, t \cos^2\theta = \sigma_{s,lon} \frac{A_{s,lon}}{u_k}} \quad (2)$$

EQUILIBRIO ALLA ROTAZIONE:



$$F_{c2,v} \cdot r = dT$$

$$T = \int_{u_k} F_{c2,v} \cdot r$$

$$T = \int_{u_k} \sigma_c\, t\, du_k \cos\theta \sin\theta \cdot r$$

$r\, du_k$ = 2·AREA DEL TRIANGOLINO

$$T = \sigma_c\, t \sin\theta \cos\theta \underbrace{\int_{u_k} r\, du_k}_{2A_k}$$

$$\boxed{T = 2A_k\, t\, \sigma_c \sin\theta \cos\theta} \quad (3)$$

T_{Rd} DELLA SEZIONE IN CLS:

$$\sigma_c = f'_{cd} \Rightarrow T_{Rd\,max}$$

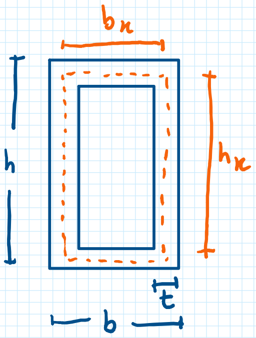
DA (3) $\Rightarrow$ $T_{Rd\,max} = 2A_k\, t\, f'_{cd} \sin\theta \cos\theta$

$$T_{Rd\,max} = 2A_k\, t\, f'_{cd} \sin\theta \cos\theta \frac{\sin\theta}{\sin\theta} \quad (\cot\theta)$$

$$T_{Rd\,max} = 2A_k\, t\, f'_{cd} \frac{\sin^2\theta \cot\theta}{\sin^2\theta + \cos^2\theta}$$

$$T_{Rd\,max} = 2A_k\, t\, f'_{cd} \frac{\cot\theta\ \cancel{\sin^2\theta}}{\cancel{\sin^2\theta}(1 + \cot^2\theta)}$$

$$T_{Rd\,max} = 2 A_k\, t\, f'_{cd} \frac{\cot\theta}{1+\cot^2\theta}$$



$f'_{cd} = \nu f_{cd} = 0.5\, f_{cd}$

$A_k = b_k \cdot h_k \qquad u_k = 2\,(h_k + b_k)$

$$t = MAX \begin{cases} 2c \\ \dfrac{A}{u} = \dfrac{bh}{2(b+h)} \end{cases}$$

$b_k = b - \frac{t}{2} - \frac{t}{2} \qquad h_k = h - \frac{t}{2} - \frac{t}{2}$

TRd DELLE STAFFE

$$\sigma_c\, t\, sen^2\theta = \sigma_{s,st} \frac{A_{s,st}}{s} \quad (1)$$

DA (1) → σ_c

$$\sigma_c = \frac{\sigma_{s,st} \dfrac{A_{s,st}}{s}}{t\, sen^2\theta}$$ => SOSTITUISCO IN (3)

9

$$T = 2 A_k\, t\, \sigma_c\, sen\theta \cos\theta \quad (3)$$

$$T = 2 A_k \cancel{t} \frac{\sigma_{s,st} A_{s,st}}{\cancel{t}\, sen^{\cancel{2}}\theta\, s} \cancel{sen\theta} \cos\theta$$

$$T = 2 A_k\, \sigma_{s,st} \frac{A_{s,st}}{s} \cot\theta$$

SE $\sigma_{s,st} = f_{yd}$ => $$T_{Rds,st} = 2 A_k\, f_{yd} \frac{A_{s,st}}{s} \cot\theta$$

TRd DELLE ARM. LONGITUDINALI

DA (2) $$\sigma_c\, t \cos^2\theta = \sigma_{s,lon} \frac{A_{s,lon}}{u_k} \quad (2)$$ => σ_c

$$\sigma_c = \sigma_{s,lon} \frac{A_{s,lon}}{u_k} \frac{1}{t \cos^2\theta}$$ => SOSTITUISCO IN (3)

$$T = 2 A_k\, t\, \sigma_c\, sen\theta \cos\theta \quad (3)$$

$$T = 2 A_k \cancel{t}\, \sigma_{s,lon} \frac{A_{s,lon}}{u_k} \frac{1}{\cancel{t} \cos^{\cancel{2}}\theta}\, sen\theta\, \cancel{\cos\theta}$$

$$T = 2A_k \, \sigma_{s,lon} \, \frac{A_{s,lon}}{u_k} \, \frac{1}{\cot\theta}$$

SE $\sigma_{s,lon} = f_{yd}$ $\Rightarrow$ $T_{Rds,lon} = 2 A_k \frac{A_{s,lon}}{u_k} \frac{f_{yd}}{\cot\theta}$

PER LA VERIFICA ALLO SLU:

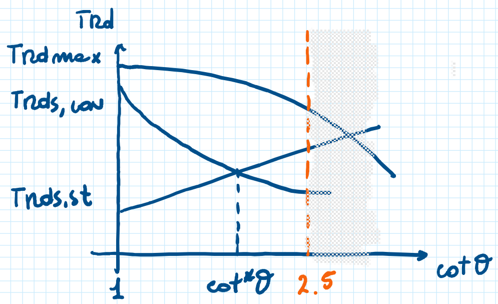
$$T_{Rd\,max} = 2 A_k \, t \, f'_{cd} \frac{\cot\theta}{1+\cot^2\theta}$$

$$T_{Rds,st} = 2 A_k \frac{A_{s,st}}{s} f_{yd} \cot\theta$$

$$T_{Rds,lon} = 2 A_k \frac{A_{s,lon}}{u_k} \frac{f_{yd}}{\cot\theta}$$

MIN = T_{Rd}

$1 \leq \cot\theta \leq 2.5$



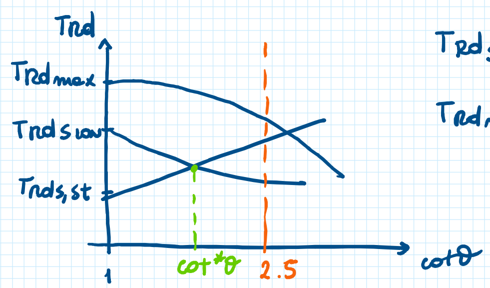
$\cot^*\theta \Rightarrow T_{Rds,st} = T_{Rds,lon}$

$$2A_k \frac{A_{s,st}}{s} f_{yd} \cot\theta = 2A_k \frac{A_{s,lon}}{u_k} \frac{f_{yd}}{\cot\theta}$$

$$\cot^2\theta = \frac{A_{s,lon}}{u_k} \frac{s}{A_{s,st}}$$

$$\cot^*\theta = \sqrt{\frac{A_{s,lon}}{u_k} \frac{s}{A_{s,st}}}$$

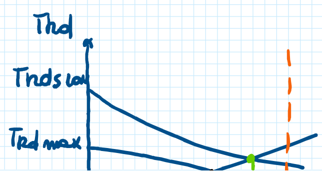
1) $\cot^*\theta \leq 2.5$



$T_{Rds} = T_{Rds,st}(\cot^*\theta) = T_{Rds,lon}(\cot^*\theta)$

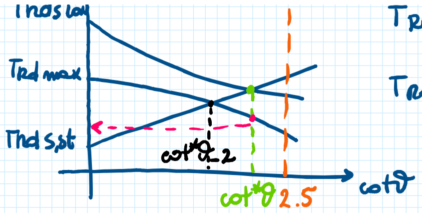
$T_{Rd\,max} = T_{Rd\,max}(\cot^*\theta)$

$T_{Rd} = \text{MIN} \begin{cases} T_{Rds} \\ T_{Rd,max} \end{cases}$



$T_{Rds} = T_{Rds,st}(\cot^*\theta) = T_{Rds,lon}(\cot^*\theta)$

$T_{Rd\,max} = T_{Rd\,max}(\cot^*\theta)$



$T_{Rd_s} = T_{Rd_{s,st}}(\cot^*\vartheta) = T_{Rd_{s,con}}(\cot^*\vartheta)$

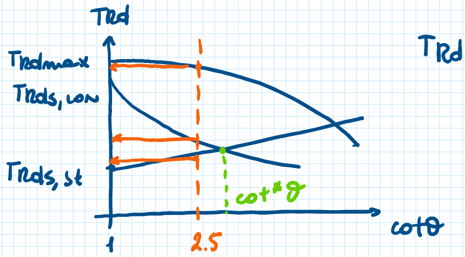
$T_{Rd_{max}} = T_{Rd_{max}}(\cot^*\vartheta)$

SE $T_{Rd_{max}}(\cot^*\vartheta) < T_{Rd,s}$

⇓

$\cot^*\vartheta_{-2}$: $T_{Rd_{s,st}} = T_{Rd,max}$

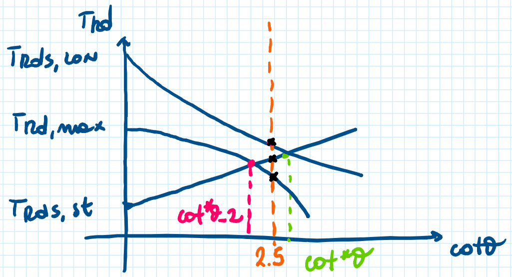
2) $\cot^*\vartheta > 2.5 \Rightarrow \cot\vartheta = 2.5$



$$T_{Rd_s} = MIN \begin{cases} T_{Rd_{s,st}}(\cot\vartheta = 2.5) \\ T_{Rd_{s,con}}(\cot\vartheta = 2.5) \end{cases}$$

$T_{Rd_{max}} = T_{Rd_{max}}(\cot\vartheta = 2.5)$

$$T_{Rd} = MIN \begin{cases} T_{Rd_s} \\ T_{Rd_{max}} \end{cases}$$



$$T_{Rd_s} = MIN \begin{cases} T_{Rd_{s,st}}(\cot\vartheta = 2.5) \\ T_{Rd_{s,con}}(\cot\vartheta = 2.5) \end{cases}$$

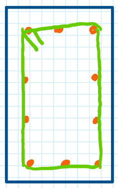
$T_{Rd_{max}} = T_{Rd_{max}}(\cot\vartheta = 2.5)$

SE $T_{Rd_{max}} < T_{Rd_s} \Rightarrow \cot^*\vartheta_{-2}$

⇓

$T_{Rd_{s,st}} = T_{Rd,max}$

ESEMPIO



30 x 60

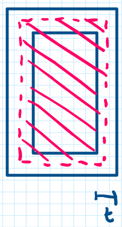
c = 5 cm

ϕ 8/15

10 ϕ14

$T_{Ed} = 30$ kNm

0) t, A_k, u_k



$$t = MAX \begin{cases} 2c = 2 \times 5 = 10 \text{ cm} \\ \frac{A}{u} = \frac{30 \times 60}{2(30+60)} = \frac{1800}{180} = 10 \text{ cm} \end{cases}$$

$b_k = 30 - \frac{10}{2} - \frac{10}{2} = 20$ cm

$h_k = 60 - \frac{10}{2} - \frac{10}{2} = 50$ cm

$u_k = 2(20 + 50) = 140$ cm

$A_k = 20 \times 50 = 1000$ cm^2

1) $\cot^*\vartheta$

$$\cot^*\vartheta = \sqrt{\frac{A_{s,lon}}{u_k}\,\frac{s}{A_{s,st}}} = \sqrt{\frac{10\times1.54}{140}\cdot\frac{15}{0.5}} = 1.82$$

$A_{s,st} = 0.5\ cm^2$

Poiché $\cot^*\vartheta < 2.5 \Rightarrow \cot\vartheta = 1.82$

2) $T_{Rd,s} = T_{Rd,s,st}\,(\cot\vartheta = 1.82) = T_{Rd,s,lon}\,(\cot\vartheta = 1.82)$

$$T_{Rd,s,st} = 2\,A_k\,\frac{A_{s,st}}{s}\,f_{yd}\,\cot\vartheta$$

$$= 2\cdot1000\cdot\frac{0.5}{15}\cdot391.3\cdot\frac{1.82}{10^3} = 47.5\ kNm$$

$$cm^2\ \ \frac{cm^2}{cm}\ \ \frac{kN}{mm^2}\ \ \frac{10^2}{10^3}\ \ \overline{10^2}$$

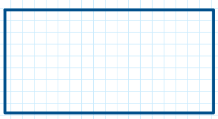
$$T_{Rd,max} = 2\,A_k\,t\,f'_{cd}\,\frac{\cot\vartheta}{1+\cot^2\vartheta}$$

$$= 2\cdot1000\cdot10\cdot\frac{0.5\cdot14.17}{10^3}\cdot\frac{1.82}{1+1.82^2} = 59.8\ kNm$$

$$T_{Rd} = MIN\begin{cases} T_{Rd,s} = 47.5\ kNm \\ T_{Rd,max} = 59.8\ kNm \end{cases} = 47.5\ kNm > T_{Ed} = 30\ kNm$$

OK!

ESEMPIO



80×25

$e = 4\ cm$

$T_{Ed} = 40\ kNm$

$\frac{A_{s,st}}{s} = ?$

$A_{s,lon} = ?$

0) $$t = MAX\begin{cases} 2e = 2\times4 = 8\ cm \\ \frac{A}{u} = \frac{80\times25}{2(80+25)} = 9.52\ cm \end{cases} \Rightarrow t = 9.52\ cm$$

$$b_k = 80 - \frac{9.52}{2} - \frac{9.52}{2} = 70.48\ cm$$

$$h_k = 25 - \frac{9.52}{2} - \frac{9.52}{2} = 15.48\ cm$$

$$A_k = 70.48\times15.48 = 1091.03\ cm^2$$

$u_k = 2\left(70.48 + 15.48\right) = 171.9$ cm

1) VERIFICO LA SEZ. IN CLS:

$$T_{Rd\,max} = 2\, A_k\, t\, f'_{cd}\, \frac{\cot\theta}{1+\cot^2\theta}$$

$\cot\theta = 2.5 \Rightarrow T_{Rd\,max} = 2 \cdot 1091.03 \cdot 9.52 \cdot \frac{0.5 \cdot 14.17}{10^3} \cdot \frac{2.5}{1+2.5^2} = 50.75$ kNm

$\cot\theta = 1.0 \Rightarrow T_{Rd\,max} = 2 \cdot 1091.03 \cdot 9.52 \cdot \frac{0.5 \cdot 14.17}{10^3} \cdot \frac{1}{1+1^2} = 73.6$ kNm

POICHÉ $T_{Rd\,max}(\cot\theta = 2.5) > T_{Ed} = 40$ kNm

↳ POSSO USARE $\forall\ 1 \leq \cot\theta \leq 2.5$ PER PROG. LE ARMATURE

2) PROG. LE ARMATURE:

$\cot\theta = 1.0$

$$\frac{A_{s,st}}{s} = \frac{T_{Ed}}{2A_k\, f_{yd}\, \cot\theta} \Rightarrow A_{s,st} = \frac{40 \times 1 \times 10^5}{2 \cdot 1091.03 \cdot 391.3 \cdot 1} = 4.68\ \frac{cm^2}{1m}$$

(s = 1 m; T_{Ed} → 1 m)

$$\frac{kNm \cdot m}{cm^2 \cdot \frac{N}{mm^2}} \times 10^3 \times 10^2 \times \frac{10^2}{10^2}$$

$$A_{sLON} = \frac{T_{Ed}\, u_k\, \cot\theta}{2A_k\, f_{yd}} = \frac{40 \cdot 171.9 \cdot 1 \times 10^3}{2 \cdot 1091.03 \cdot 391.3} = 8.05\ cm^2$$

3) LIMITI DA NORMATIVA

$s \leq \frac{1}{8} u = \frac{1}{8} \cdot 210 = \sout{26.25}$ 25

$d < 35$ cm

DA CALCOLO $A_{s,st} = 4.68\ \frac{cm^2}{1m}$

$A_{1\phi 8} = 0.5\ cm^2$ $\Rightarrow N_{STAFFE}/1m = \frac{4.68}{0.5} = \sout{9.36}$ 10

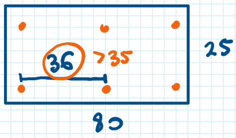
$s = \frac{100}{N.\,STAFFE} = \frac{100}{10} = 10$ cm $\Rightarrow \phi 8/10$

DA CALCOLO: $A_{sLON} = 8.05\ cm^2$

$A_{1\phi 14} = 1.54\ cm^2 \Rightarrow N_{\phi 14} = \frac{8.05}{1.54} = \sout{5.23}$ 6

DA CALCOLO: $A_{s,lon} = 8.05\ cm^2$

$A_{1\phi14} = 1.54\ cm^2 \Rightarrow N_{\phi14} = \frac{8.05}{1.54} = 5.23$ ~~5.23~~ 6



$\Rightarrow$ DEVO USARE $8\phi14$

PER OTTIMIZZARE IL PROG:

$A_{s,lon} = 8\phi14 \Rightarrow \cot\theta = ?$

$$T_{Rd,s,lon} = 2A_k \frac{A_{s,lon}}{u_k}\frac{f_{yd}}{\cot\theta}$$

$$\cot\theta = 2A_k \frac{A_{s,lon}}{u_k}\frac{f_{yd}}{T_{Ed}} = \frac{2\cdot1091.03\cdot8\cdot1.54\cdot391.3}{171.9\cdot40\cdot10^3} = 1.53$$

$$\Rightarrow \left.\frac{A_{s,st}}{s}\right|_{1m} = \frac{T_{Ed}}{2A_k f_{yd}\cot\theta} = \frac{40\times1\cdot10^5}{2\cdot1091.03\cdot391.3\times1.53} = 3.06\ \frac{cm^2}{m}$$

$\Downarrow$

STAFFE OTTIMIZZATE