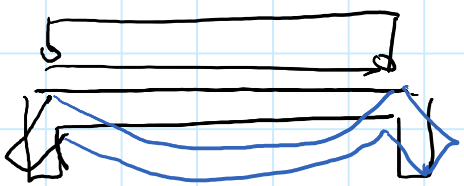
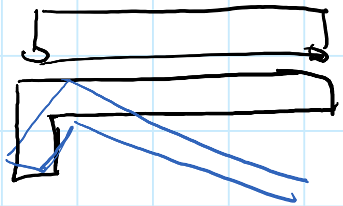
TORSIONE



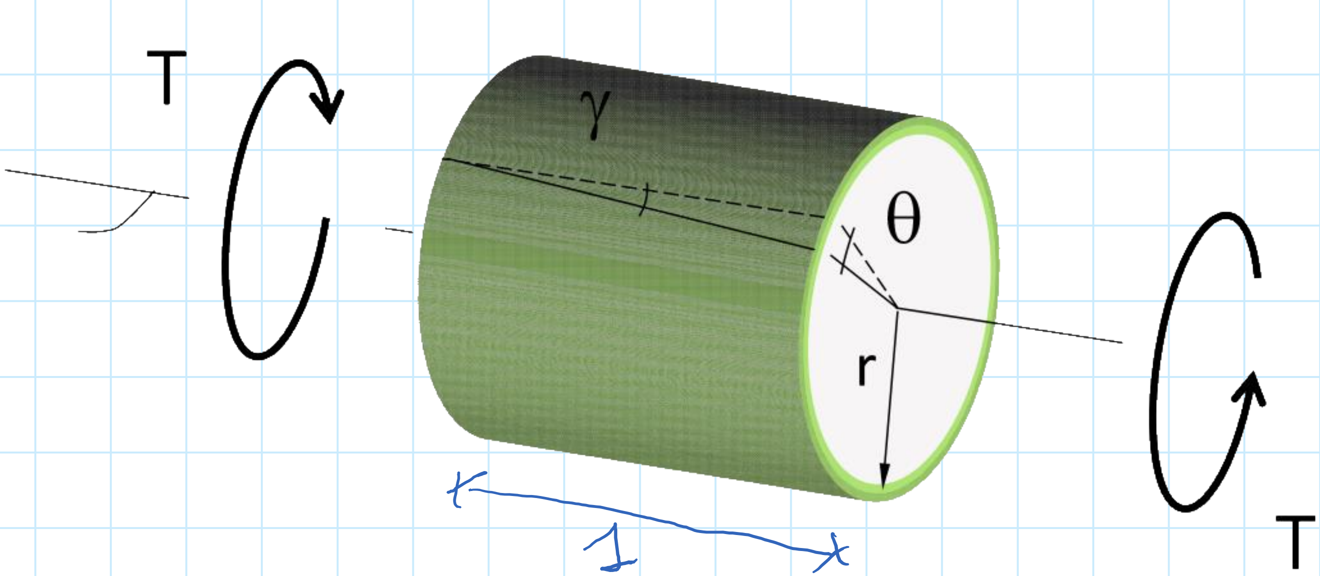
⟹ TORSIONE PER CONGRUENZA



→ SE NON CI FOSSE TORSIONE NELLA TRAVE LO SCHEMA SAREBBE LABILE

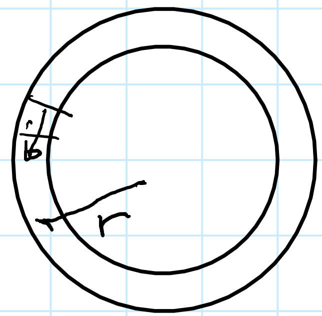
TORSIONE PER EQUILIBRIO

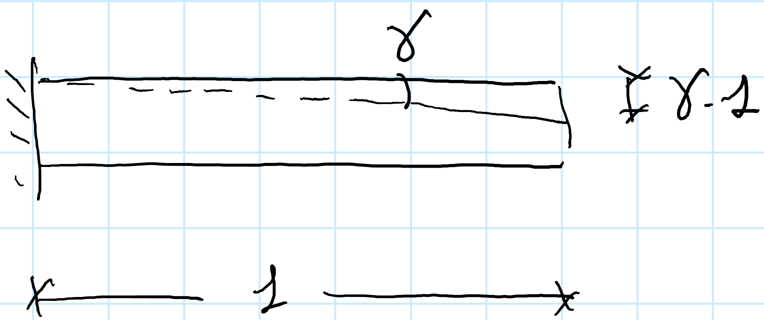
MATERIALE OMOGENEO E ISOTROPO



$\tau = \text{cost}$

$$\gamma = \frac{\tau}{G}$$

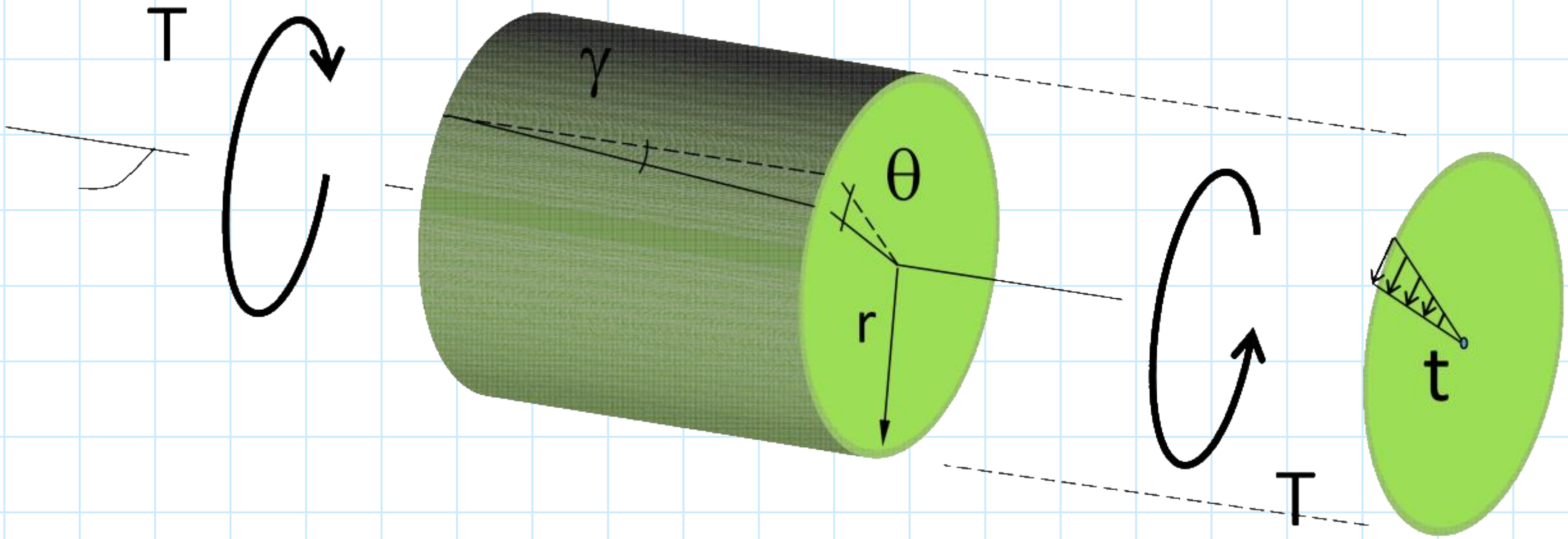


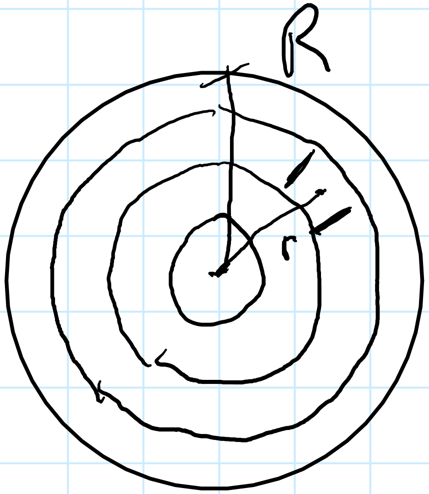


$$\theta = \frac{\gamma}{r}$$

ANGOLO UNITARIO DI ROTAZIONE

SEZIONE CIRCOLARE PIENA





$0 \leq r \leq R$

$$\vartheta = \frac{\gamma_{max}}{R}$$

$$\gamma = \vartheta \cdot r$$

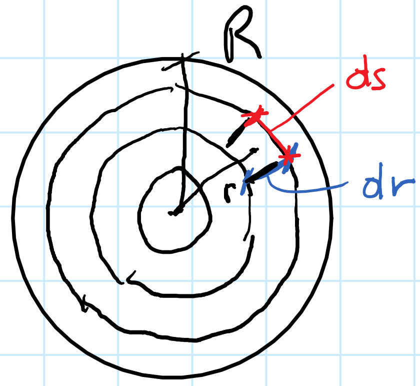
$$\gamma = \frac{\gamma_{max}}{R} r$$

$$\tau = \frac{\gamma_{max}}{R} r \cdot G \rightarrow \tau = \tau_{max} \cdot \frac{r}{R}$$

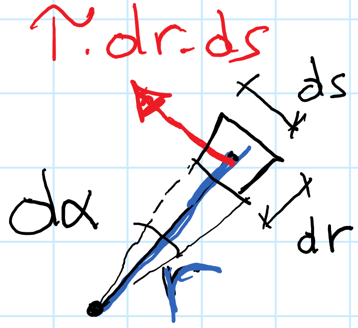
RELAZIONE TRA T E TENSIONI TANGENZIALI

$$dT = \tau \cdot dr \cdot ds \cdot r$$

$$T = \int dT$$



$$T = \int_0^R \int_0^{2\pi} \underbrace{\tau_{max} \frac{r}{R}}_{\tau} r^2 \, d\alpha \, dr$$



$$ds = r \, d\alpha$$

$$= \int_0^R 2\pi \frac{\tau_{max}}{R} r^3 \, dr$$

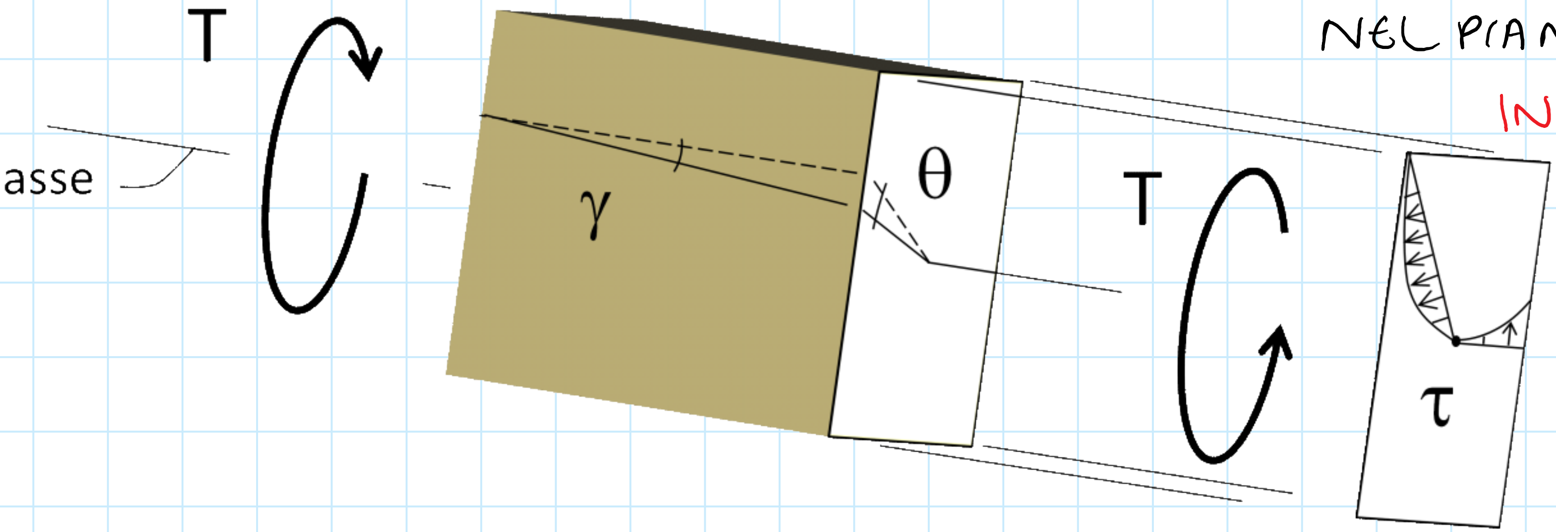
$$= 2\pi \frac{R^4}{4} \cdot \frac{\tau_{max}}{R}$$

$$I_p = \frac{\pi R^4}{2}$$ MOMENTO D'INERZIA POLARE

$$T = I_p \frac{\tau_{max}}{R} \Longrightarrow \tau_{max} = R \cdot \frac{T}{I_p}$$

SEZIONE RETTANGOLARE

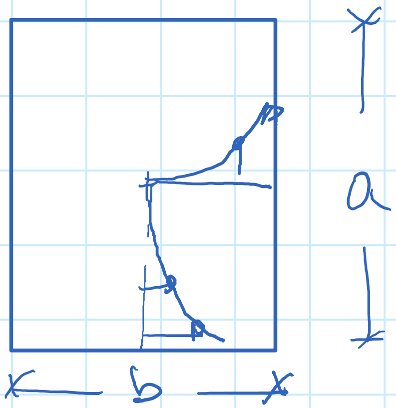
NON HO SOLO ROTAZIONE NEL PIANO MA ANCHE INGOBBIMENTO



Il valore massimo della tensione tangenziale è :

$$\tau_{max} = \psi \frac{T}{a\,b^2} \quad \text{dove} \quad \psi = 3 + \frac{2.6}{0.45 + a/b}$$

a dim. lato maggiore
b dim. lato minore

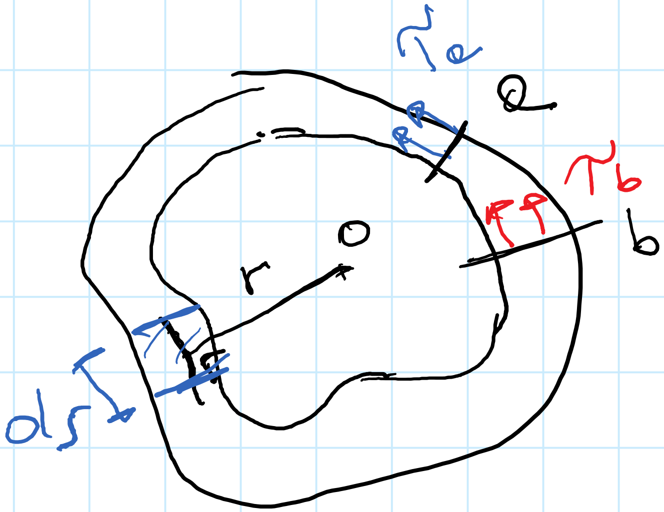


SEZIONE QUADRATA $a = b \Rightarrow \psi = 4{,}8$

" MOLTO ALLUNGATA $a >> b$

$\Longrightarrow \psi \rightarrow 3$

FORMULA DI BREDT



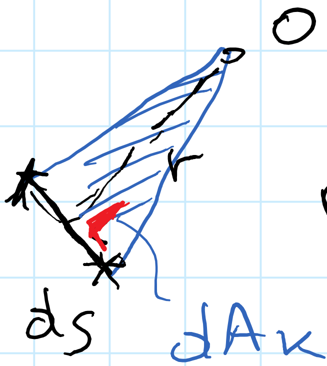
τ = COSTANTE LUNGO LA CORDA (SE SPESSORE È PICCOLO)

$$\tau_b t_b = \tau_a t_a \rightarrow$$

$\tau \cdot t =$ COSTANTE

$$dT = r\,\tau \cdot t \cdot ds$$

$$dT = \tau t \cdot 2 dA_k$$



$$r\,ds = 2 dA_k$$

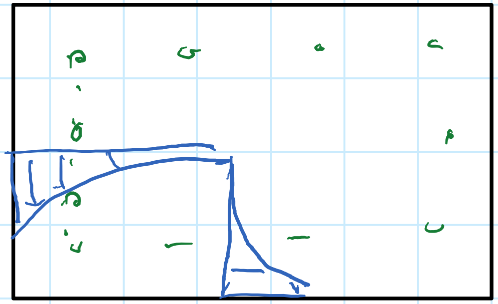
$$T = \int dT = \int \tau t\, 2 dA_k$$

$$= 2\tau \cdot t \int dA_k$$

$$T = 2\tau \cdot t A_k \Longrightarrow \tau_{max} = \frac{T}{2 t_{min} A_k}$$

A_k = AREA RACCHIUSA DALLA LINEA MEDIA

CEMENTO ARMATO (COMPORTAMENTO LINEARE)



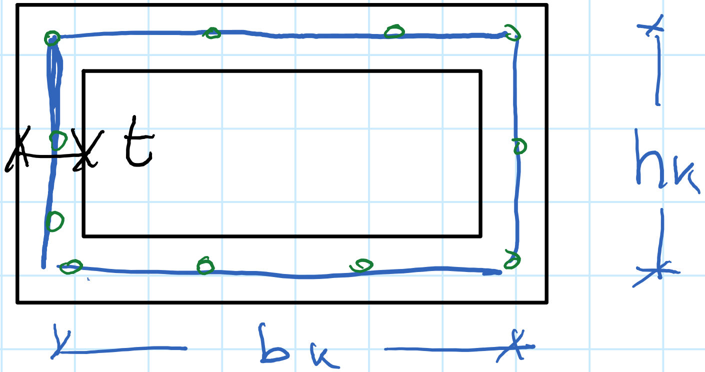
SCUOLA NAPOLETANA

$$\tau = \psi \frac{T}{a b^2}$$

MODELLO VEROSIMILE FINO A QUANDO NON HO FESSURAZIONE

EVIDENZA SPERIMENTALE

CONTRIBUTO SOSTANZIALE FORNITO DALLA PARTE ESTERNA DELLA SEZIONE



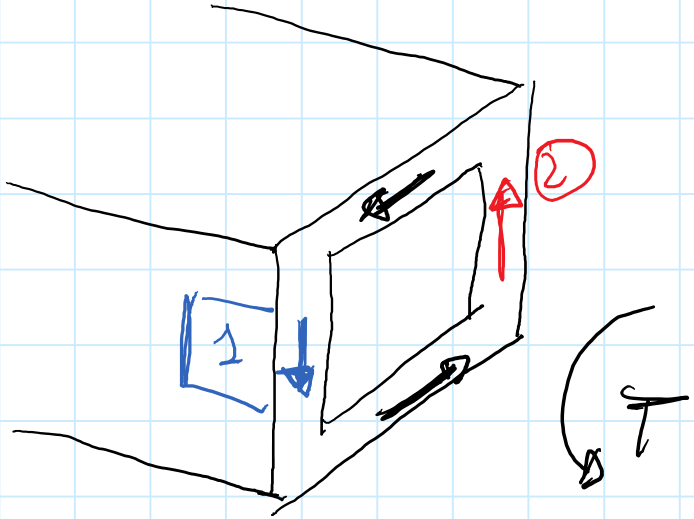
SEZ. TUBOLARE EQUIVALENTE (BREDT)

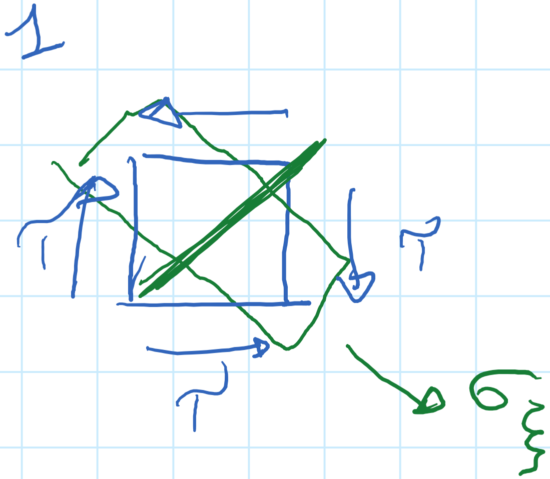
$$t = 2c \qquad h_k = h - t \; ; \quad b_k = b - t$$

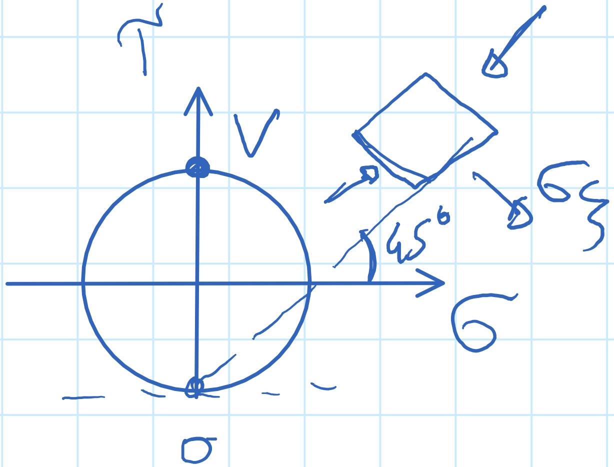
NUCLEO CERCHIATO = PARTE RACCHIUSA DA LINEA MEDIA

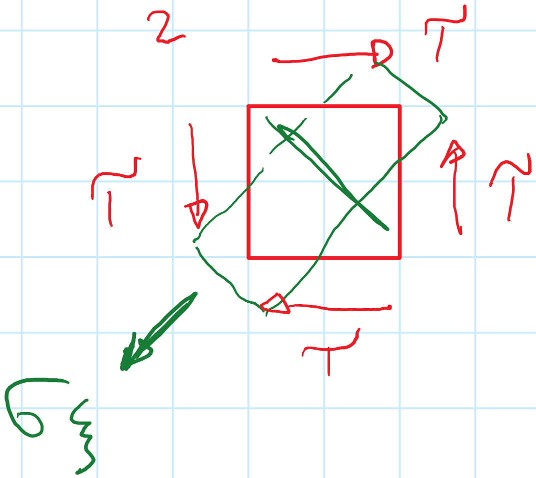
$$A_k = b_k h_k \; ; \quad u_k = 2(b_k + h_k)$$

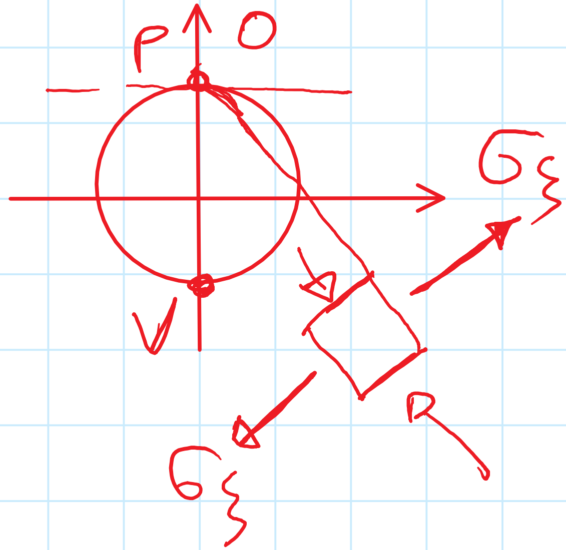
QUADRO FESSURATIVO



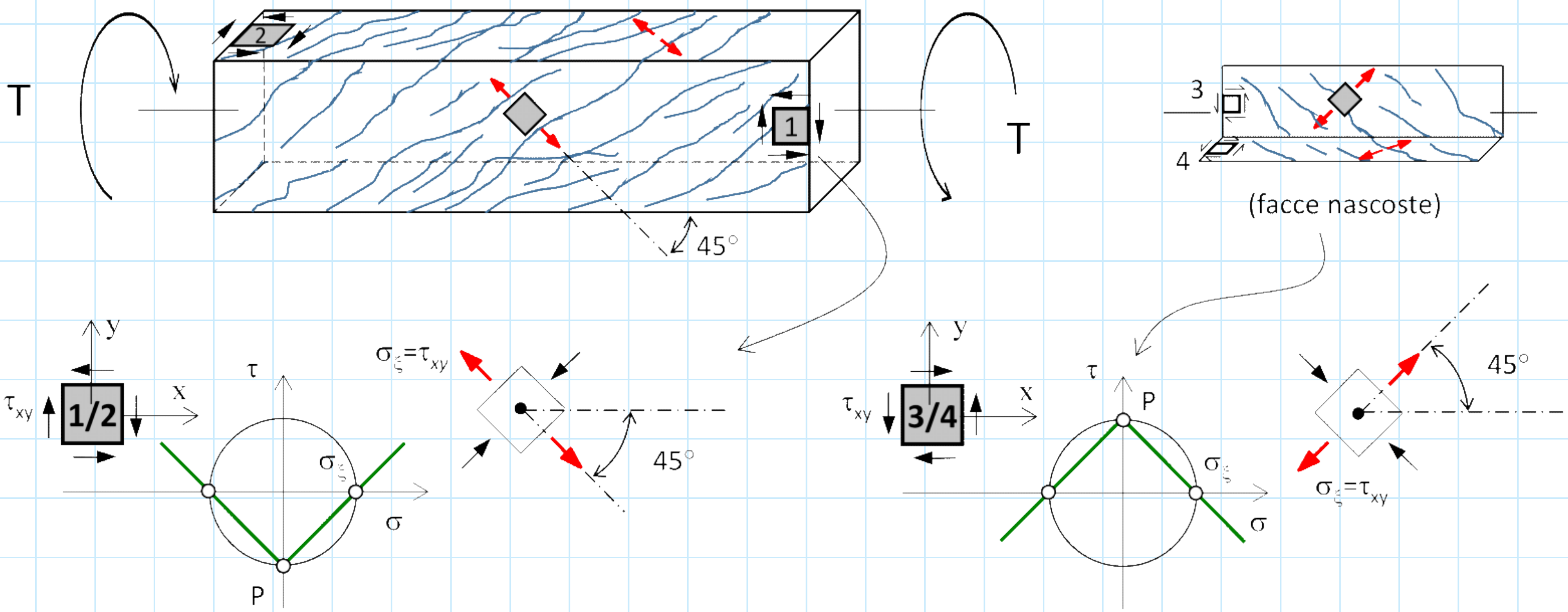






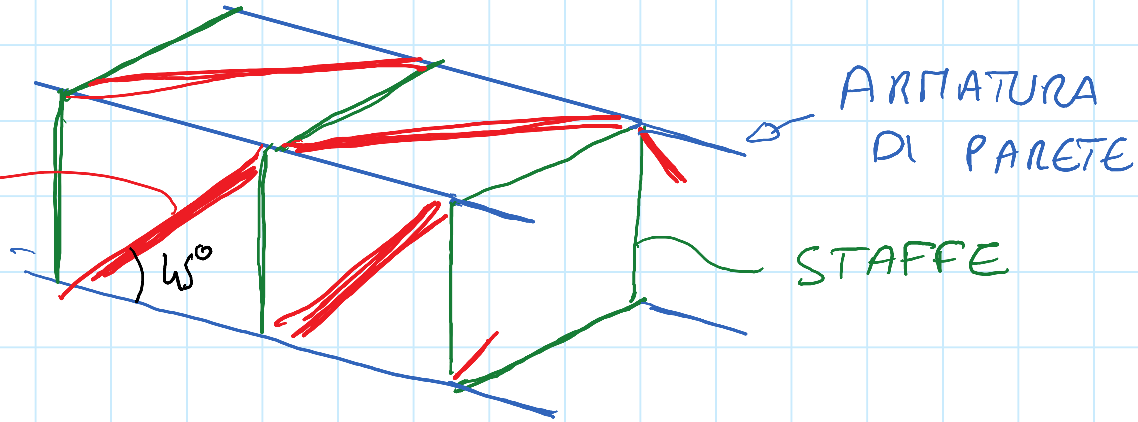


LESIONI A SPIRALE



MODELLI DI TRAVE ARMATA A TORSIONE

CAMPO LINEARE
- CAMPI DI TENSIONE (σ_c INCLINATE A 45°)
- TRALICCIO SPAZIALE DI RAUSCH



CAMPO NON LINEARE
- CAMPI DI TENSIONE (σ_c INCLINATE DI ϑ)
- TRALICCIO SPAZIALE CON INCLINAZIONE ϑ DEL PUNTONE

CAMPI DI TENSIONE (COMPORTAMENTO NON LINEARE)

CONSIDERO AGENTI:

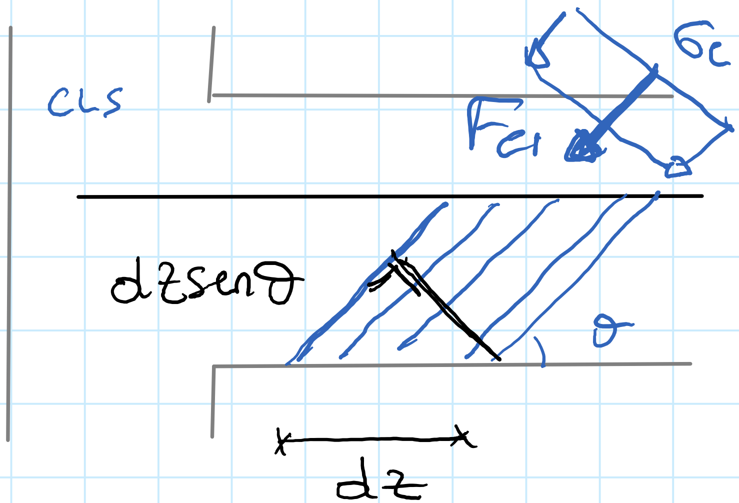
σ_c INCLINATE DI UN ANGOLO θ

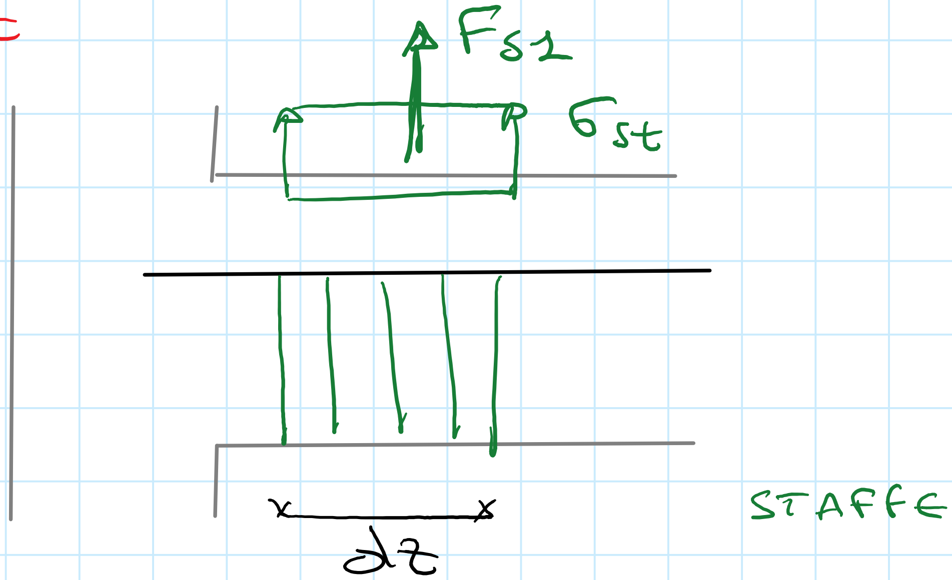
σ_{st} VERTICALI (STAFFE)

σ_{sl} ORIZZ. (ARM. DI PARETE)

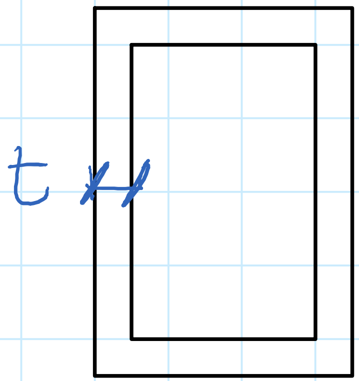
T (MOMENTO TORCENTE)

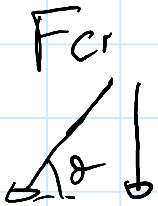
SEZIONE ORIZZONTALE





SEZ TRASVERSALE





$$F_{c1} = \sigma_c \, dz \, \mathrm{sen}\theta \, t$$

$$F_{s1} = \sigma_{st} \, A_{st} \, \frac{dz}{s}$$

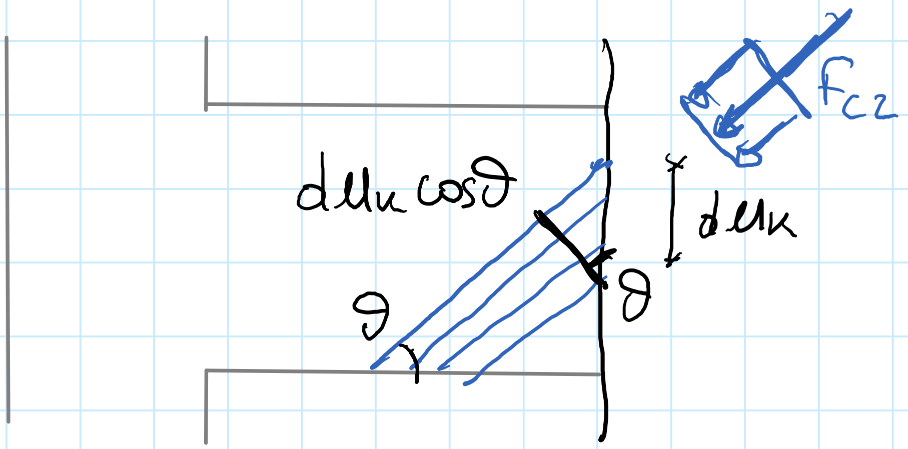
Eq. TRASLAZ. VERTICALE

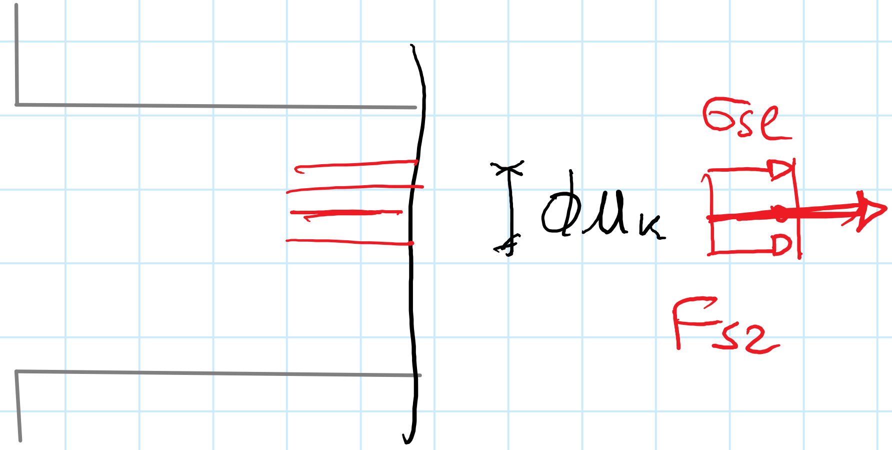
$$F_{c1v} = F_{s1} \rightarrow F_{c1} \, \mathrm{sen}\theta = F_{s1}$$

$$\sigma_c \, dz \, \mathrm{sen}^2\theta \cdot t = \sigma_{st} \, A_{st} \, \frac{dz}{s}$$

$$\boxed{\sigma_c = \sigma_{st} \, \frac{A_{st}}{s \, t} \, \frac{1}{\mathrm{sen}^2\theta}} \quad (1)$$

SEZIONE VERTICALE



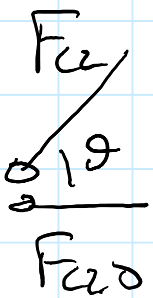


$$F_{c2} = \sigma_c \cdot d\mu_k\, t \cos\vartheta \qquad F_{s2} = \sigma_{se} A_{se} \frac{d\mu_k}{\mu_k}$$

A_{se} = ARMATURA DI PARETE TOTALE

EQ. TRASLAZIONE ORIZZONTALE $\quad F_{c2,0} = F_{s2}$

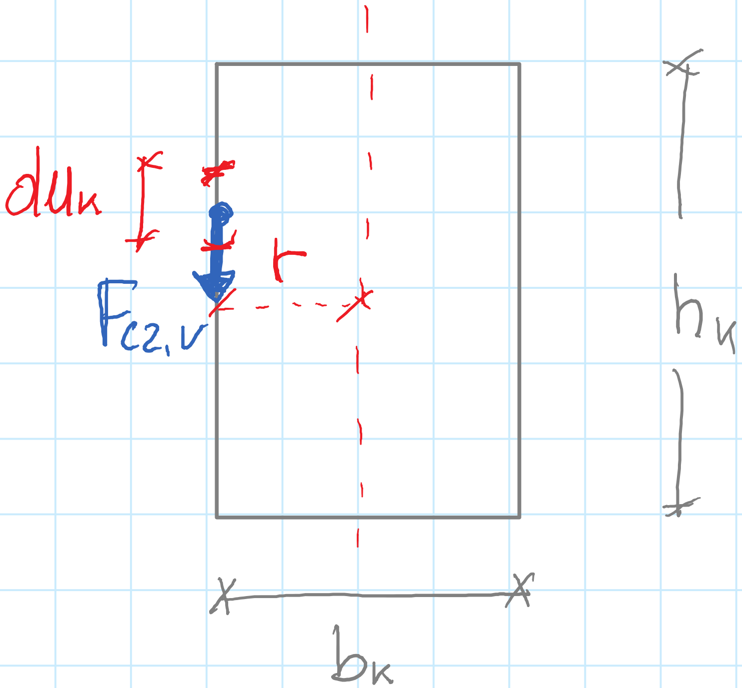
$$F_{c2} \cdot \cos\vartheta = F_{s2} \longrightarrow$$



$$\sigma_c \cdot d\mu_k\, t \cos^2\vartheta = \sigma_{se} A_{se} \frac{d\mu_k}{\mu_k}$$

$$\boxed{\sigma_c = \sigma_{se} \cdot \frac{A_{se}}{\mu_k\, t} \frac{1}{\cos^2\vartheta}} \qquad [2]$$

NEL PIANO DELLA SEZIONE



$$U_k = 2 b_k + 2 h_k$$

$$F_{c2v} = F_{c2} \cdot \operatorname{sen}\vartheta$$

$$dT = \sigma_c \, dU_k t \cos\vartheta \cdot \operatorname{sen}\vartheta \cdot r$$

$$dU_k \cdot r = 2\, dA_k$$

$$dT = \sigma_c t \frac{\cos\vartheta}{\operatorname{sen}\vartheta} \operatorname{sen}^2\vartheta \cdot 2\, dA_k$$

$$\operatorname{sen}^2\vartheta = \frac{1}{1 + \operatorname{cotg}^2\vartheta} \rightarrow dT = \sigma_c t \frac{\operatorname{cotg}\vartheta}{1 + \operatorname{cotg}^2\vartheta} 2\, dA_k$$

$$T = \int dT = \int \sigma_c \, t \, \frac{\cot\theta}{1+\cot^2\theta} \, 2 \, dA_k \Longrightarrow$$

$$T = \sigma_c \, t \, \frac{\cot\theta}{1+\cot^2\theta} \cdot 2 A_k$$

ROTTURA PER COMPRESSIONE DEL CLS SE

$$\sigma_c = \nu f_{cd} \longrightarrow$$

$$T_{Rd\,max} = \nu f_{cd} \, t \, 2 A_k \, \frac{\cot\theta}{1+\cot^2\theta}$$

RESISTENZA DELLE STAFFE

PARTO DA $T = \sigma_c \, t \cos\theta \, \text{sen}\theta \cdot 2A_k$

RICAVO σ_c DALLA 1 →

$$T = \sigma_{st} \frac{A_{st}}{s\,\cancel{t}} \frac{1}{\text{sen}^2\theta} \cancel{t} \cos\theta \, \cancel{\text{sen}\theta} \cdot 2A_k$$

$$T = \sigma_{st} \frac{A_{st}}{s} \cot g\theta \cdot 2A_k$$

CRISI STAFFE SE $\sigma_{st} = f_{yd}$

$$T_{Rd,st} = \frac{A_{st}}{s} f_{yd} \, 2A_k \cot g\theta$$

CRISI ARMATURA LONGITUDINALE

PARTO DA $T = \sigma_c \, t \cos\vartheta \, \text{sen}\,\vartheta \cdot 2A_k$

RICAVO σ_c DALLA (2) $\longrightarrow$

$$T = \sigma_{se} \frac{A_{se}}{u_k \, \cancel{t}} \frac{1}{\cos^2\vartheta} \cancel{t} \cancel{\cos\vartheta} \, \text{sen}\,\vartheta \cdot 2A_k$$

CRISI ARMATURA LONGITUDINALE SE $\sigma_{se} = f_{yd}$

$$T_{Rd,se} = \frac{A_{se}}{u_k} \cdot f_{yd} \cdot 2A_k \, \frac{1}{\cot\vartheta}$$

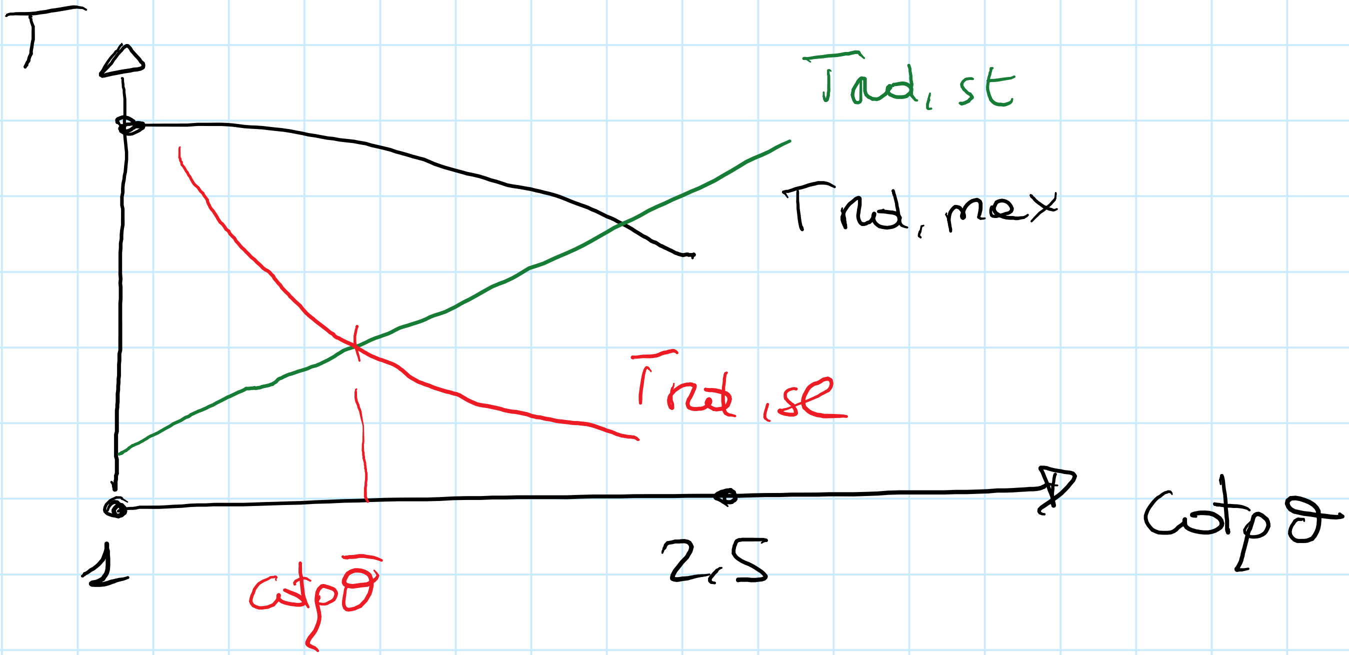
RESISTENZA DELLA SEZIONE

$$T_{Rd} = \min\{T_{Rd,max};\ T_{Rd,se};\ T_{Rd,st}\}$$

INDICAZIONI NTC18

$$1 \leq \cot\theta \leq 2{,}5$$

$$t = \max\left\{2c;\ \frac{A}{u}\right\}$$



VERIFICA

CERCO $\cot g\,\bar{\theta}$: $T_{Rd,sc} = T_{Rd,st}$

DEVO POI CAPIRE SE HO PROBLEMI NEL CLS

PROGETTO

PONGO $t = 2c$; $\cot g\,\theta = 2.5$ (CONSIGLIO)

$T_{Ed} = T_{Rd\,max} \Longrightarrow A_k = f(T_{Ed})$

FISSO $b \Longrightarrow b_k = b - t$

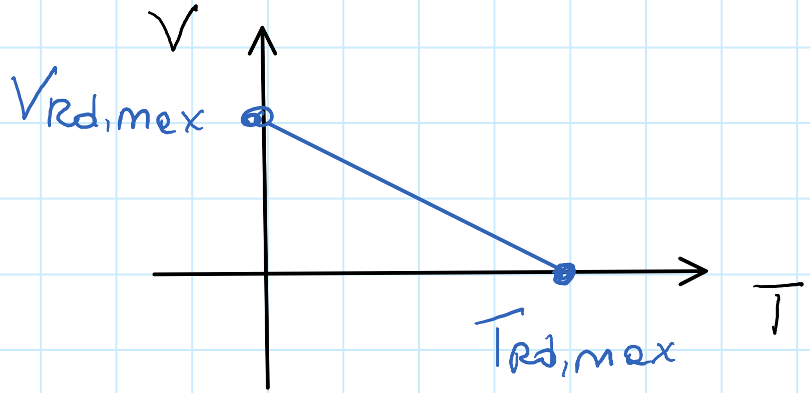
$$h_k = \frac{A_k}{b_k} \longrightarrow h = h_k + t$$

TORSIONE - TAGLIO - FLESSIONE

VERIFICA E PROGETTO SEZIONE CLS

$$M_{Ed} \le M_{Rd}$$

$$\frac{V_{Ed}}{V_{Rd,max}} + \frac{T_{Ed}}{T_{Rd,max}} \le 1$$

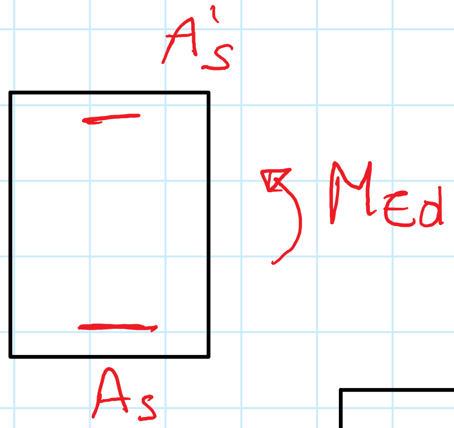


N.B. USARE STESSO VALORE DI $\cot\theta$ PER
$V_{Rd,max}$; $T_{Rd,max}$

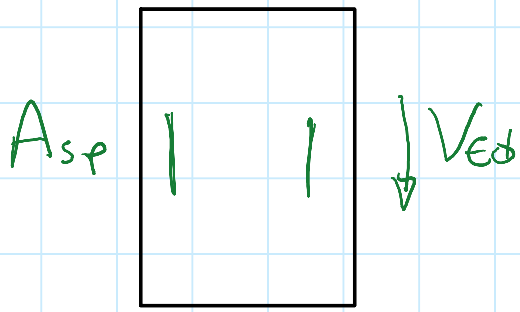
TORSIONE - TAGLIO - MOMENTO

PROGETTO ARMATURE

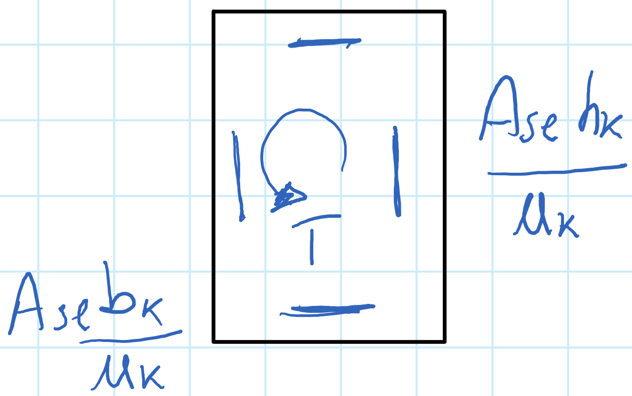
- DA M_{Ed} CALCOLO
  A_s, A'_s (cm²)



- DA V_{Ed} CALCOLO
  $n_b \frac{A_{st}}{s}$ (cm²/m) , A_{sp} (cm²)
  $n_b = 2$

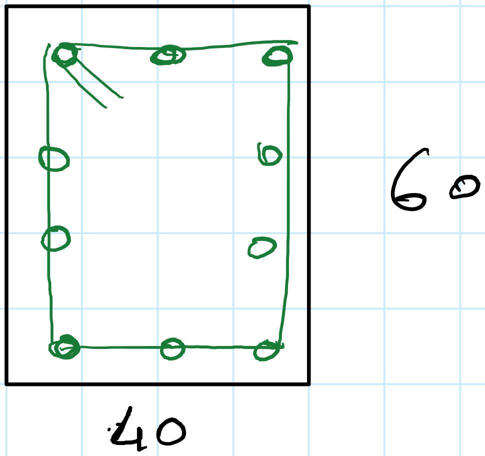


- DA T_{Ed} CALCOLO
  $\frac{A_{st}}{s}$ (cm²/m) A_{sl} (cm²)
  $n_b = 1$



- SOMMO ARMATURE E DEFINISCO BARRE

ESEMPIO



DATI:

$c = 5\,cm$

$A_{se} = 10\phi 14$

$\phi 8/20$

C 30/37

TROVARE

T_{Rd} ?

$$t = \max\left\{2c,\ \frac{A}{u}\right\} = \max\left\{10;\ \frac{40\times 60}{2\times(40+60)}\right\} = 12\ cm$$

$$b_k = 40 - 12 = 28\ cm$$

$$h_k = 60 - 12 = 48\ cm$$

$$\longrightarrow \begin{cases} A_k = 28\times 48 = 1344\ cm^2 \\ u_k = 2\times(28+48) = 152\ cm \end{cases}$$

RESISTENZA CLS

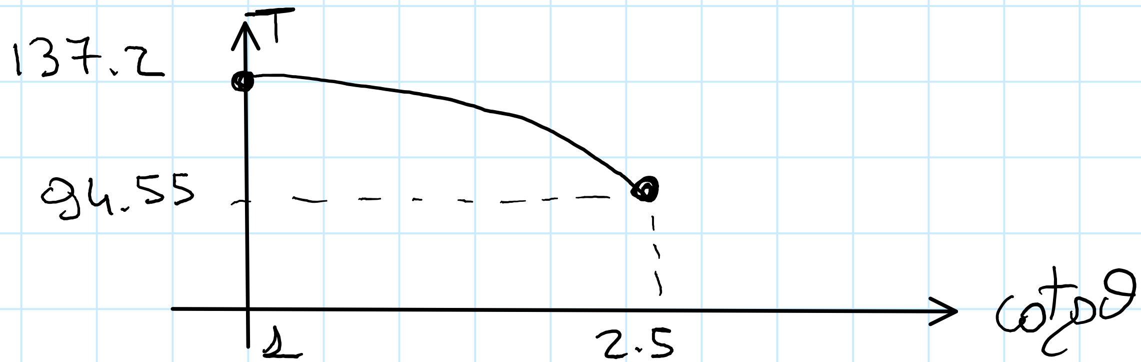
$$T_{Rd\,max} = \nu f_{cd}\, t\, 2A_k \frac{\cot g\theta}{1+\cot g\theta^2}$$

$$= 0,5 \times 17 \frac{N}{mm^2}\; 12\,cm \times 2 \times 1344\,cm^2 \frac{\cot g\theta}{1+\cot g^2\theta} \frac{1}{10^3}$$

$$= 274.2 \frac{\cot g\theta}{1+\cot g^2\theta}$$

SE $\cot g\theta = 1 \rightarrow T_{Rd\,max} = 137.2$ kNm

SE $\cot g\theta = 2,5 \rightarrow T_{Rd\,max} = 94.55$ kNm



RESISTENZA ARMATURA

CERCO $\cot\bar{\vartheta}$: $\bar{T}_{Rd,st} = \bar{T}_{Rd,se}$

$$\frac{A_{st}}{s} f_{yd}\, 2A_k \cot\vartheta = \frac{A_{se}}{u_k} f_{yd}\, 2A_k \frac{1}{\cot\vartheta}$$

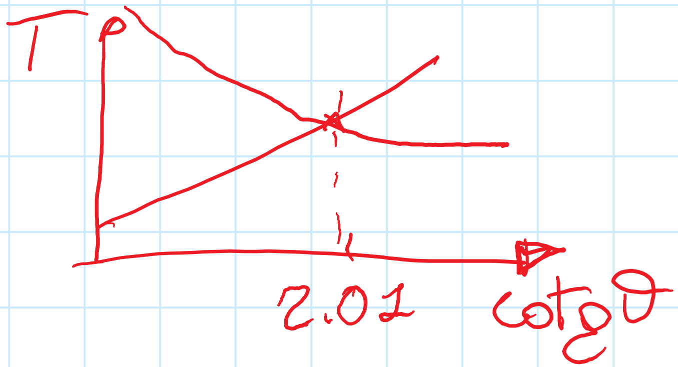
$$\cot\vartheta = \sqrt{\frac{A_{se}}{u_k} \cdot \frac{s}{A_{st}}} = \sqrt{\frac{15.4}{152} \times \frac{20}{0.5}} = 2.01$$

$A_{st} = A_{\phi 8} = 0{,}5\ cm^2$

$s = 20\ cm$

$A_{se} = 10 \times A_{\phi 14} = 10 \times 1{,}54 = 15{,}4\ cm^2$

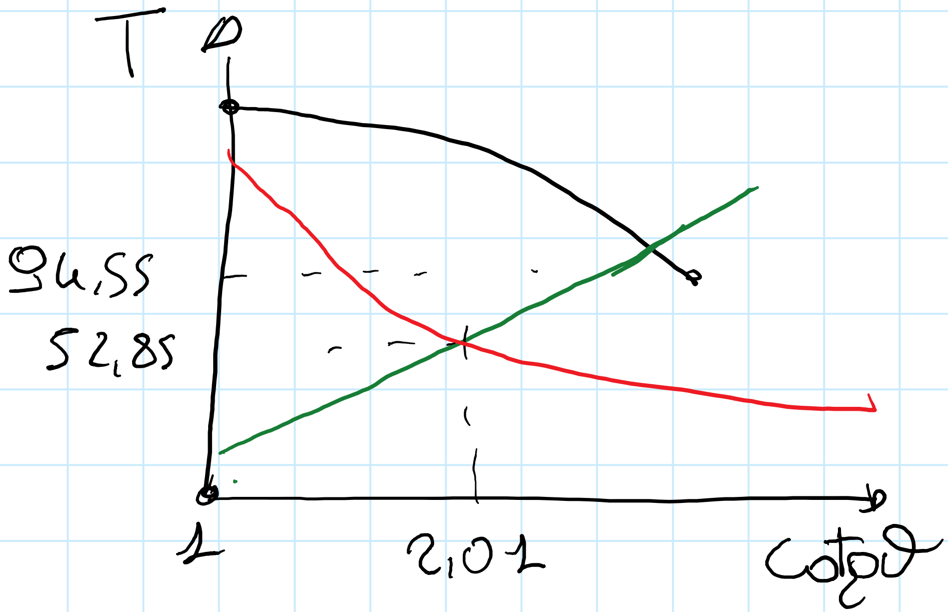
$u_k = 152\ cm$



RESISTENZA ARMATURA

$$T_{Rd,s} = \frac{1}{10^3} \frac{0{,}5\ cm^2}{20\ cm} \times 391{,}3 \frac{N}{mm^2} \times 2 \times 1364\ cm^2 \times 2{,}01$$

$$= 52{,}85\ KNm$$



IL VALORE DEL MOMENTO TORCENTE RESISTENTE È

$$T_{Rd} = 52{,}85\ KNm$$

(SE AVESSI TROVATO $T_{Rd,s} < T_{Rd,max}$ AVREI DOVUTO MODIFICARE $\cot\theta$)