VERIFICA A FLESSIONE

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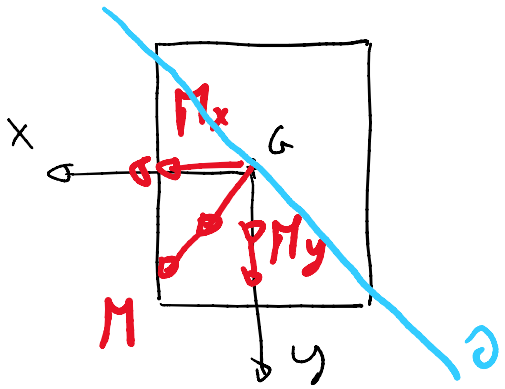
SEZ. CON MATERIALE OMOGENEO E ISOTROPO



$$\varepsilon = \varepsilon_G + \chi_x X + \chi_y Y$$

$$\varepsilon_G = \frac{N}{EA}$$

$$\chi_x = -\frac{M_y}{EI_y} \quad ; \quad \chi_y = \frac{M_x}{EI_x}$$

$N = 0 \Longrightarrow \varepsilon_G = 0 \iff$ ASSE NEUTRO BARICENTRICO

FLESSIONE RETTA $M = M_x$

SEZ. IN C.A

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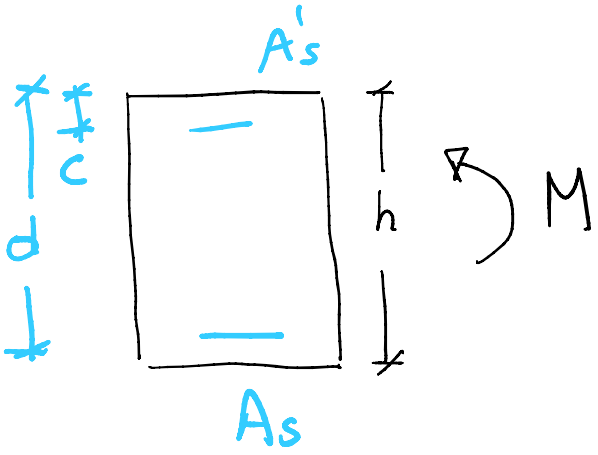
DEFINIZIONI



A_s ARMATURA TESA

A'_s ARMATURA COMPRESSA

d = ALTEZZA UTILE
(DISTANZA A_s DAL BORDO COMPRESSO

C = COPRIFERRO

$d = h - c$

I STADIO

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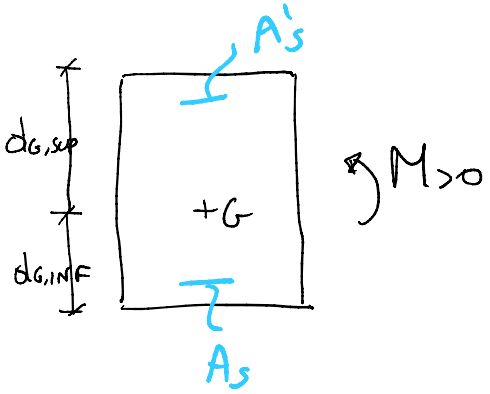
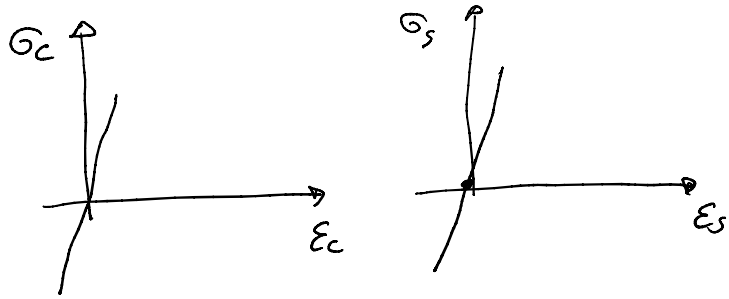
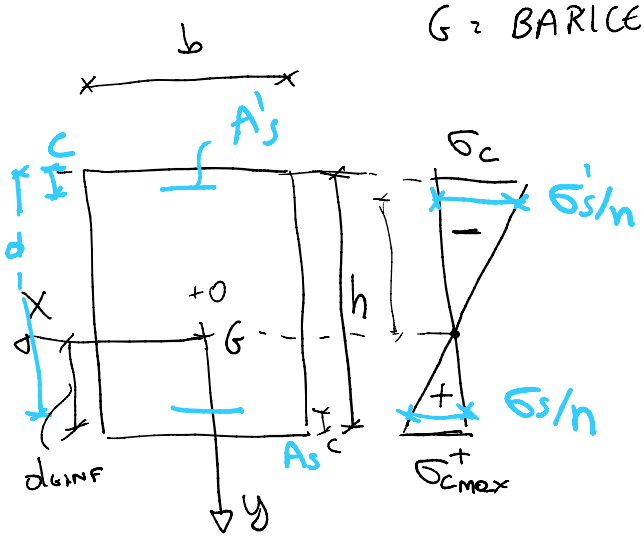


SEZ. OMOGENEIZZATA



G = BARICENTRO SEZ. OMOGEN.



$$\sigma_c = \frac{M_x}{I_x} y$$

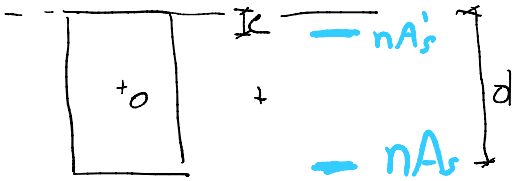
$$\sigma_s = n\sigma_c$$

$$n = E_s/E_c$$

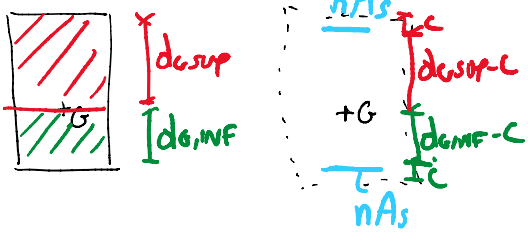
$$d_{G,sup} = \frac{S_{sup}}{A}$$

$$A = bh + nA'_s + nA_s$$

$$S_{sup} = bh\cdot\frac{h}{2} + nA'_s c + nA_s d$$



$$I_x = \frac{b d_{G,sup}^3}{3} + \frac{b d_{G,INF}^3}{3} + nA'_s (d_{G,sup} - c)^2 + nA_s (d_{G,INF} - c)^2$$



$$\sigma^{\oplus}_{cmax} = \frac{M}{I_x}\cdot d_{G,INF}$$

$$\sigma_{s,} = n\frac{M}{I_x}(d_{G,INF} - c)\,; \qquad \sigma'_s = -n\frac{M}{I_x}(d_{G\,sup} - c)$$

ESEMPIO

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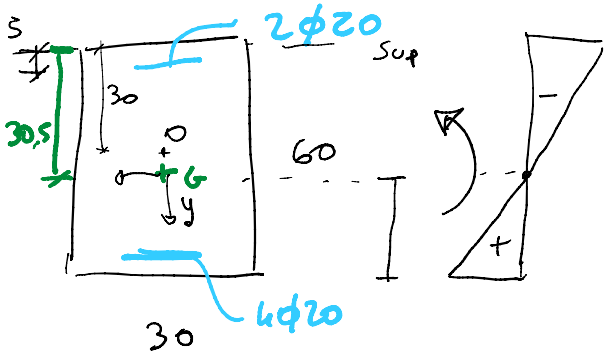


$M = 50$ KNm

Bus0C

C25/30

$c = 5$ cm

$d = 60 - 5 = 55$ cm

$n = \frac{E_s}{E_c} = 6{,}35$

$A_s = 4 \times \frac{\pi \cdot 2^2}{4} = 12{,}56\ cm^2$

($\frac{\pi \cdot 2^2}{4} = 3{,}14\ cm^2$)

$A'_s = 2\phi 20 \rightarrow 6{,}28\ cm^2$

1. DETERMINO G

$S_{sup} = 30 \times 60 \times \frac{60}{2} + 6{,}35 \times 6{,}28 \times 5 + 6{,}35 \times 12{,}56 \times 55$

$= 58586\ cm^3$

$A = 30 \times 60 + 6{,}35\,(6{,}28 + 12{,}56) = 1919{,}6\ cm^2$

$d_{G,sup} = \frac{58586}{1919{,}6} = 30{,}52$ cm $\qquad d_{G\,INF} = 60 - 30{,}52 = 29{,}48$ cm

$$I_x = \frac{30 \times 30{,}52^3}{3} + \frac{30}{3} \times 29{,}48^3 + 6{,}35 \times \left[6{,}28\,(30{,}5 - 5)^2 + 12{,}56\,(29{,}48 - 5)^2 \right]$$

$= 614213\ cm^4$

$$\sigma^{+}_{cmax} = \frac{50\ KNm}{614213\ cm^4} \times 29{,}48\ cm \times \frac{10^3 \times 10^3}{} \frac{N}{mm^2} = 2{,}4\ MPa$$

LA SEZIONE È FESSURATA ?

CONFRONTO CON $f_{cfk} = 1{,}2 f_{ctk} = 2{,}15\ MPa$

$f_{ctk} = 0{,}7 \times 0{,}3 \sqrt[3]{f_{ck}^2} = 0{,}7 \times 0{,}3 \sqrt[3]{25^2} = 1{,}8\ MPa$

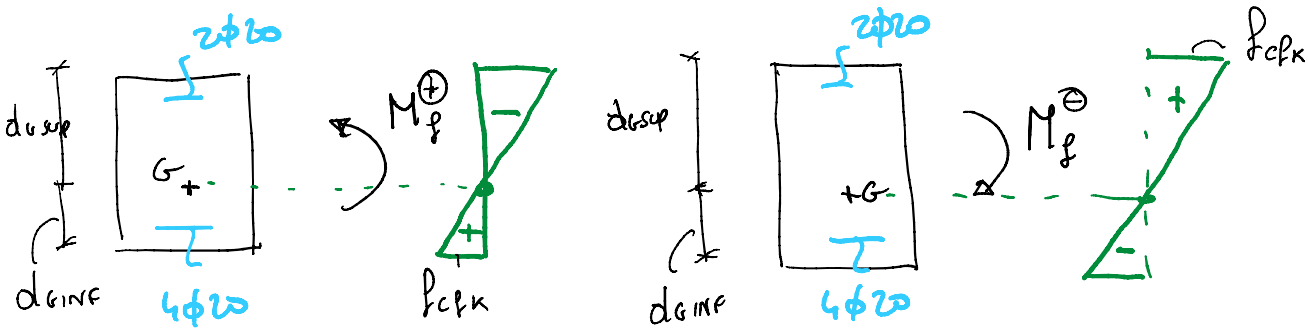
$\sigma^{+}_{cmax} > f_{cfk} \Rightarrow$ SEZ. È FESSURATA

MOMENTO DI FESSURAZIONE

$M > 0$

$\sigma^{+}_{cmax} = \frac{M}{I} d_{GINF}$

PONGO $\sigma^{+}_{cmax} = f_{cfk} \Rightarrow M_{fess} = f_{cfk} \cdot \frac{I}{d_{GINF}}$



$M^{\ominus}_{fess} = f_{cfk} \cdot \frac{I}{d_{GISUP}}$

NEL CASO DELL'ESEMPIO

$50\ kNm : 2{,}4\ MPa = M_f : 2{,}15\ MPa \Rightarrow$

$M_f = 44{,}58\ kNm$

$\sigma_s = \underset{\parallel}{n} \times \frac{44{,}58\ kNm}{614213\ cm^4} \times (29{,}48 - 5) \times 10^3 = 11{,}34\ MPa$

$n = 6{,}35$

II STADIO

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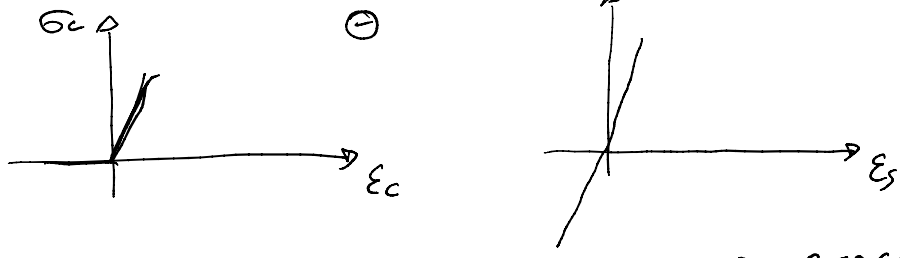
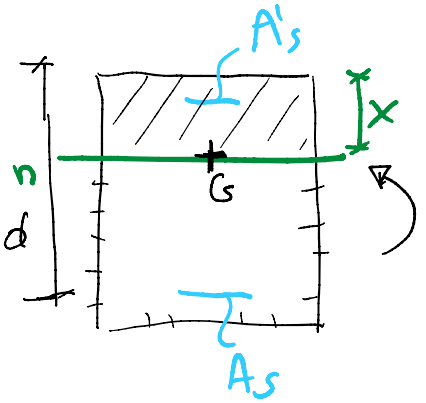
LEGAMI COSTITUTIVI

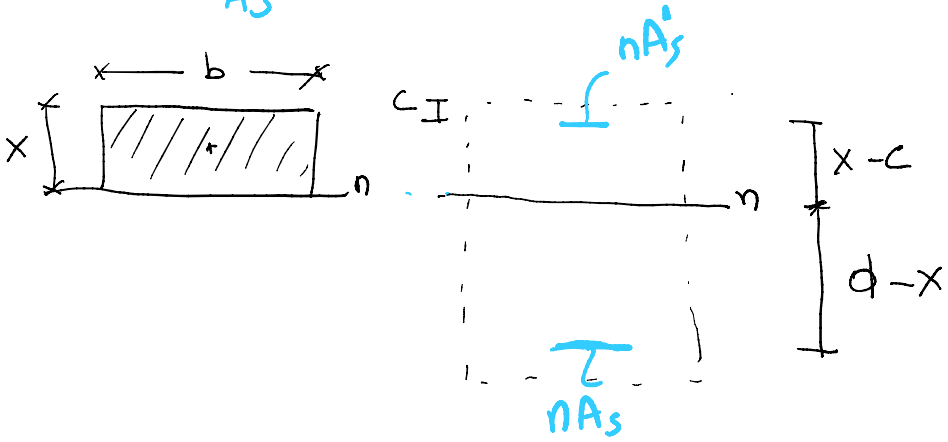


SEZIONE DI RIFERIMENTO : SEZ. REAGENTE OMOGEN.



x DISTANZA ASSE NEUTRO DAL BORDO COMPRESSO

$S_n = 0 \iff$ ASSE NEUTRO BARICENTRICO



$$S_n = -\frac{bx^2}{2} - nA'_s(x-c) + nA_s(d-x) = 0$$

$$\frac{bx^2}{2} + nA'_s x - nA'_s c - nA_s d + nA_s x = 0$$

$$x^2 + \frac{2n}{b}(A'_s + A_s)x - \frac{2}{b}n(A'_s c + A_s d) = 0$$

ax^2 $+bx$ $+c$

$$x = -\frac{n}{b}(A'_s + A_s) \pm \sqrt{\frac{n^2}{b^2}(A'_s + A_s)^2 + \frac{2n}{b}(A'_s c + A_s d)}$$

UNICA SOLUZIONE CON SIGNIFICATO FISICO $x>0$

$$x = -\frac{n}{b}(A'_s+A_s) + \sqrt{\frac{n^2}{b^2}(A'_s+A_s)^2\left[1+\frac{b^2}{n^2(A'_s+A_s)^2}\frac{2n}{b}(A'_s c+A_s d)\right]}$$

$$x = d\frac{n}{bd}(A'_s+A_s)\left[-1+\sqrt{1+\frac{2bd(A'_s c+A_s d)}{dn(A'_s+A_s)(A'_s+A_s)}}\right]$$

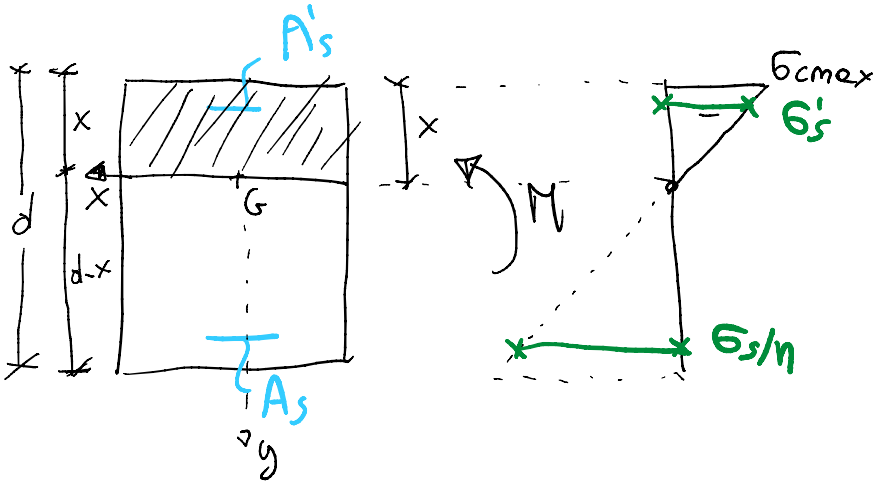
ψ

ψ = PERC. ELASTICA DI ARMATURA $= \dfrac{n(A'_s+A_s)}{bd}$

$$x = \psi d\left[-1+\sqrt{1+\frac{2}{\psi d}\cdot\frac{A'_s c+A_s d}{(A'_s+A_s)}}\right]$$

$d_{G,S}$ = DISTANZA BARICENTRO ARMATURE DAL BORDO COMPRESSO $= \dfrac{A'_s c+A_s d}{A'_s+A_s}$

$$x = \psi d\left[-1+\sqrt{1+\frac{2d_{GS}}{\psi d}}\right]$$



$$\sigma_{cmax} = -\frac{M}{I_x}x$$

$$\sigma_s = n\frac{M}{I_x}(d-x)$$

$$\sigma'_s = -n\frac{M}{I_x}(x-c)$$

$$I_x = \frac{bx^3}{3} + n A'_s(x-c)^2 + n A_s(d-x)^2$$

ALTRO APPROCCIO

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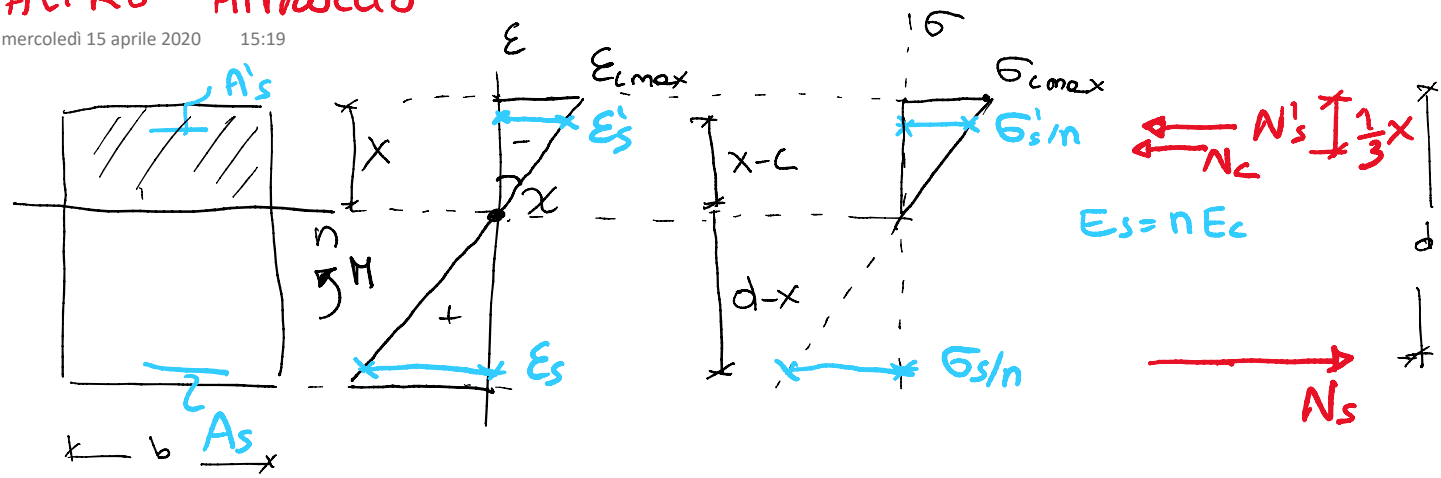

$$\varepsilon_{cmax} = -\chi x \qquad \sigma_{cmax} = \varepsilon_{cmax} E_c = -E_c \chi \cdot x$$

$$\varepsilon'_s = -\chi (x-c) \qquad \sigma'_s = E_s \cdot \varepsilon'_s = -n E_c \chi (x-c)$$

$$\varepsilon_s = \chi (d-x) \qquad \sigma_s = E_s \varepsilon_s = n E_c \chi (d-x)$$

FLESSIONE SEMPLICE $N_{TOT} = 0$ EQ. TRASLAZIONE

$$N_{TOT} = N_c + N'_s + N_s = 0$$

$$N_c = b x \frac{\sigma_{cmax}}{2} = -E_c \chi \frac{b x^2}{2}$$

$$N'_s = A'_s \cdot \sigma'_s = -A'_s \cdot n E_c \chi (x-c)$$

$$N_s = A_s \cdot \sigma_s = A_s \cdot n E_c \chi (d-x)$$

$$-E_c \chi \frac{b x^2}{2} - n A'_s E_c \chi (x-c) + n A_s E_c \chi (d-x) = 0$$

$$\boxed{-\frac{b x^2}{2} - n A'_s (x-c) + n A_s (d-x) = 0} \Rightarrow x$$

EQ. ROTAZIONE

$$N'_s \cdot c + N_c \frac{x}{3} + N_s d = M_{Ed} \Rightarrow E_c \chi$$

ESEMPIO

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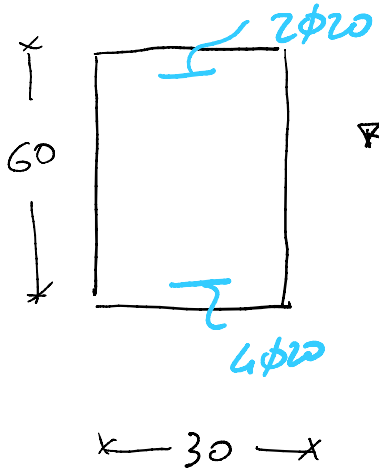


B450C C = 5cm

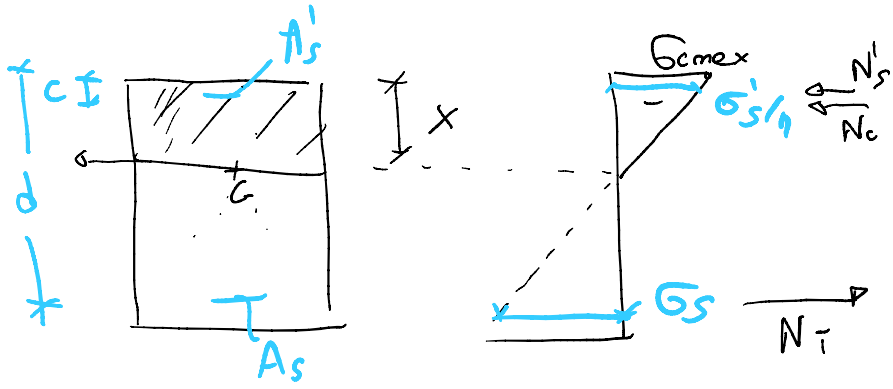
C25/30 d = 55cm

CARICHI DI LUNGA DURATA ⇒ $n = 15$

$\bar{\sigma}_{cmax} = ?$

$\bar{\sigma}_s = ?$

1. TROVARE x



$A_s = 12.56\ cm^2$

$A'_s = 6.28\ cm^2$

$$x = \psi d \left[-1 + \sqrt{1 + \frac{2 d_{G,s}}{\psi d}} \right]$$

$$\psi = \frac{n(A'_s + A_s)}{bd} = \frac{15 \times (6.28\ cm^2 + 12.56\ cm^2)}{30 \times 55\ cm^2}$$

$$= 0,17$$

$$d_{G,s} = \frac{A_s \cdot d + A'_s c}{A_s + A'_s} = \frac{12,56 \times 55 + 6,28 \times 5}{12,56 + 6,28} = 38,33\ cm$$

$$x = 0,17 \times \underset{\uparrow}{55}_{cm} \left[-1 + \sqrt{1 + \frac{2 \times 38,33}{0,17 \times 55}} \right] = 19,1\ cm$$

2. CALCOLO $\sigma_{c\,max}$

$$I_x = \frac{30 \times 19{,}1^3}{3} + 15 \times 6{,}28 \left(19{,}1 - 5\right)^2 + 15 \times 12{,}56 \left(55 - 19{,}1\right)^2$$

$$= 331218\ cm^4$$

$$\sigma_{c\,max} = -\frac{160\ kNm}{331218\ cm^4} \times 19{,}1\,cm \times 10^3 = -9{,}23\ MPa$$

CONFRONTARE
- $-0{,}45\,f_{ck}$ $11{,}25\ MPa$ (COMB. QUASI PERM.)
- $-0{,}60\,f_{ck}$ (COMB. RARA)

$$\sigma_s = 15 \frac{160\ kNm}{331218\ cm^4} \left(55 - 19{,}1\right) \times 10^3 = 260\ MPa$$

CONFRONTARE CON $0{,}8\,f_{yk} = 0{,}8 \times 450\ MPa = 360\ MPa$
(COMB. RARA)

CASO CON $M<0$

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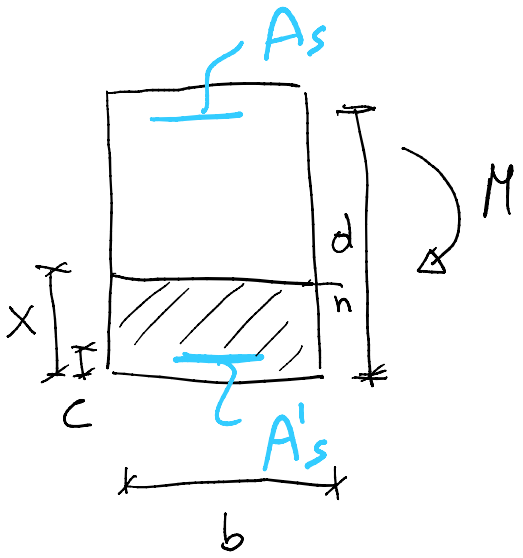

$$\sum N = 0$$

$$\frac{b x^2}{2} + n A'_s (x - c) - n A_s (d - x) = 0$$

$$d_{GS} = \frac{A'_s \cdot c + A_s \cdot d}{A_s + A'_s}$$

SEZIONI A T

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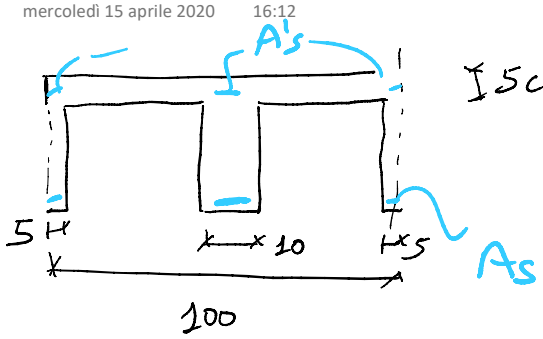
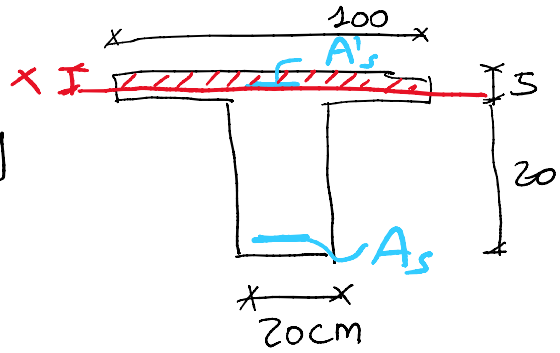
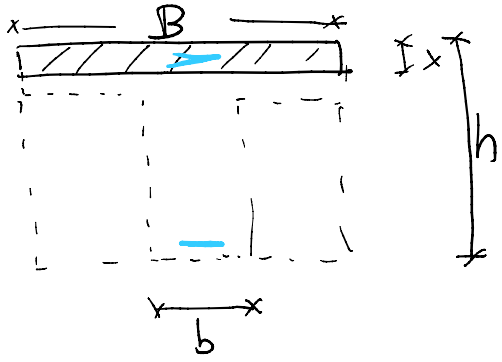


15cm
25
M



SE ASSE NEUTRO : $x \leq \partial$



$\Rightarrow$ SEZ. RETTANGOLARE

$B \times h$

POSSO RICAVARE x

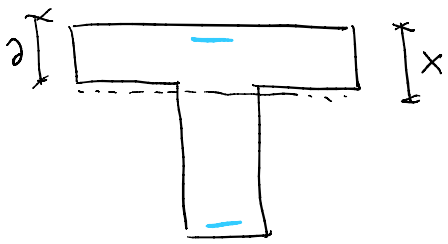
(VALIDA PER SEZ. RETTANGOLARE)

$$x = \psi d \left[-1 + \sqrt{1 + \frac{2 \phi_{cs}}{\psi d}} \right]$$

SE $x \leq \partial$ $\Rightarrow$ OK $\Rightarrow$ VERIFICO COME SEZ. RETTANGOLARE

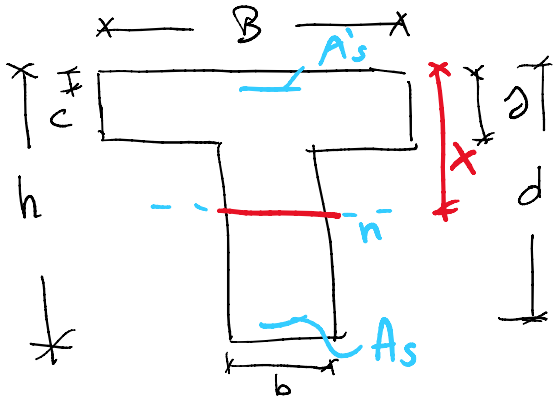
SE $x > \partial$

CASO 1

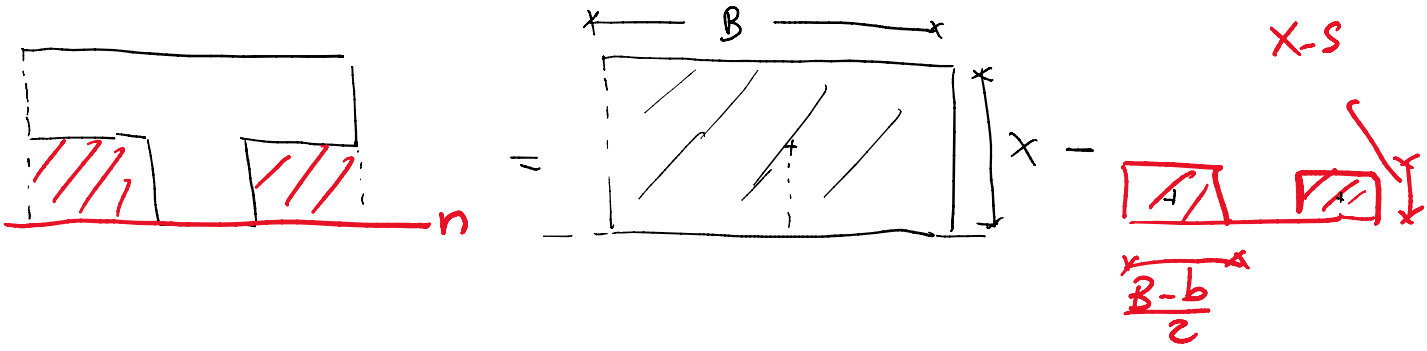


$\rightarrow$ IN MODO APPROSSIMATO CONSIDERO SEZ. RETTANGOLARE

CASO 2



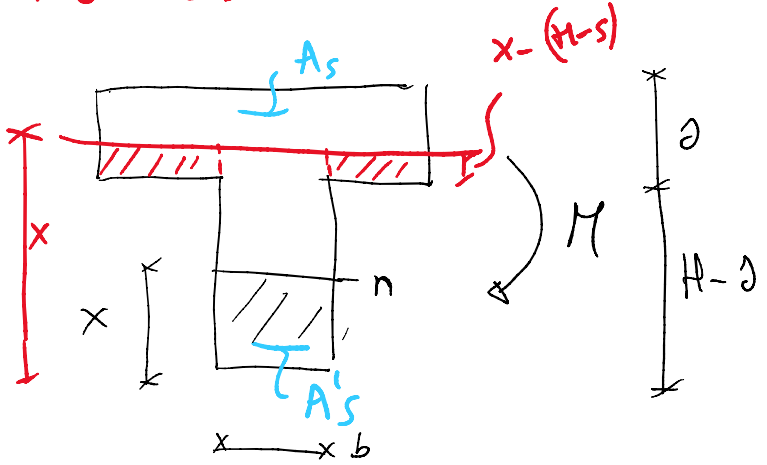
$S_N = 0$



$$S_N = -\frac{Bx^2}{2} + \frac{(B-b)(x-s)^2}{2} + nA_s(d-x) - nA'_s(x-c) = 0$$

↳ x

NEL CASO DI $M_{ed} < 0$



Se $x \leq H-d$ $\quad S_{n\,cls} = \frac{bx^2}{2}$

Se $x > H-s$ $\quad S_{n\,cls}: \frac{bx^2}{2} + \frac{(B-b)}{2} \cdot [x-(H-s)]^2$

+ ARMATURE $\quad S_{n's} = A'_s n (x-c)$

$S_{As} = -A_s \cdot n (d-x)$

ALTRE SEZ. ASSIMILABILI ALLA RETTANGOLARE

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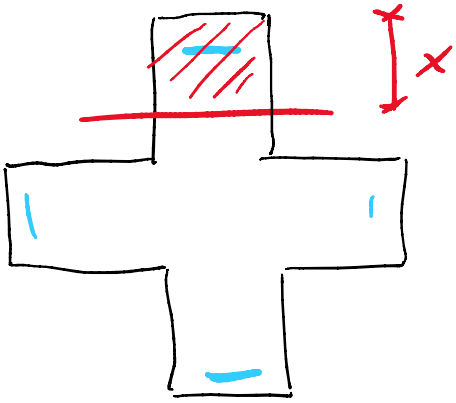

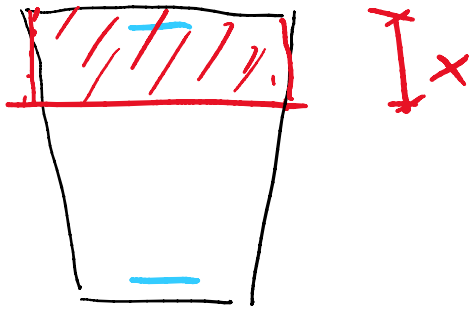

SOLUZIONI NUMERICHE

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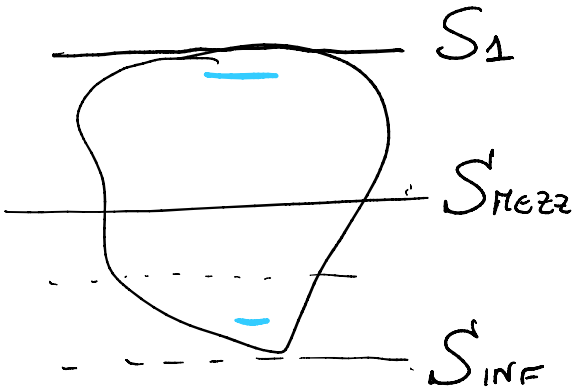

$S_1 > 0$

$S_{INF} < 0$

$S_{MEZZ} \begin{cases} > 0 \\ < 0 \end{cases}$

METODO DI BISEZIONE

VEDI FOGLIO EXCEL PER SEZIONE A T