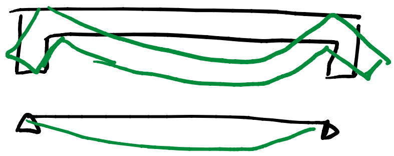
TORSIONE : PREMESSA

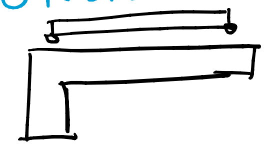
giovedì 11 giugno 2020 13:58

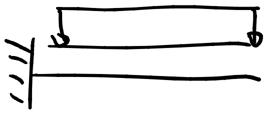
TORSIONE DA CONGRUENZA

LA TRAVE SI TORCE MA SOLO PER CONGRUENZA



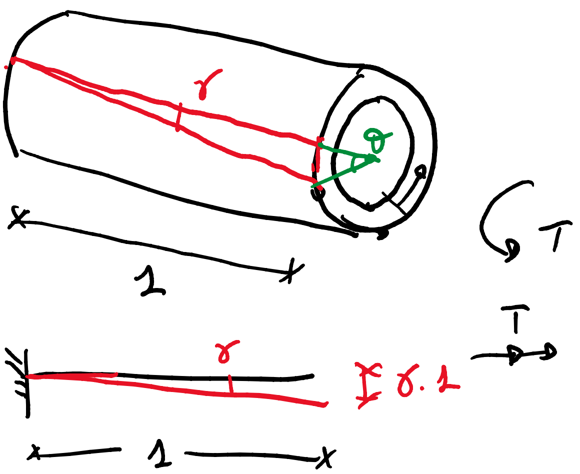
TORSIONE DA EQUILIBRIO





SE NON CONSIDERASSI TORSIONE AVREI UNO SCHEMA LABILE

TORSIONE : MATERIALE OMOGENEO E ISOTROPO SEZIONE ANULARE



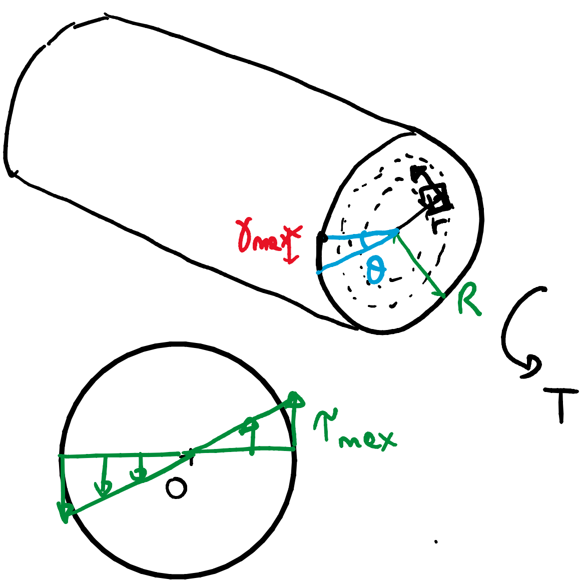
τ = COSTANTI

$\gamma = \dfrac{\tau}{G}$ SCORRIMENTO

θ = ANGOLO ROTAZIONE UNITARIA

$$\theta = \frac{\gamma}{r}$$

SEZIONE CIRCOLARE PIENA



$$\theta = \frac{\gamma_{max}}{R}$$

$$\gamma = \theta \cdot r = \frac{\gamma_{max}}{R} \cdot r$$

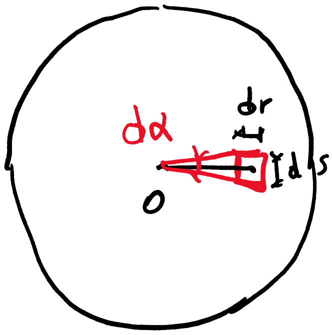
$$\tau = \gamma \cdot G = \gamma_{max} G \frac{r}{R}$$

$$\tau = \tau_{max} \cdot \frac{r}{R}$$

$$dT = \tau \, dA \cdot r \rightarrow T = \int \tau \cdot r \, dA = \int \tau_{max} \cdot \frac{r^2}{R} \, dA$$

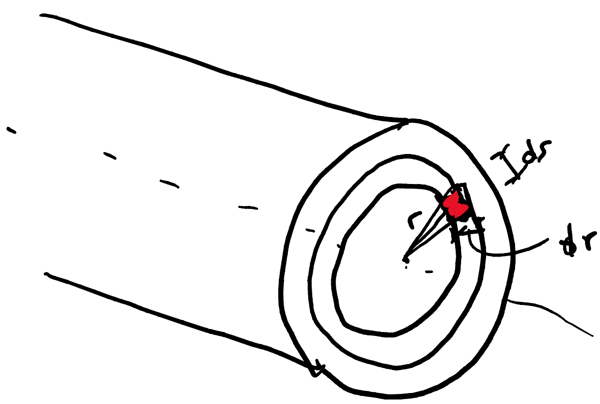
$$T = \frac{\tau_{max}}{R} \underbrace{\int r^2 \, dA}_{I_p}$$

MOMENTO D'INERZIA POLARE



$$ds = r \cdot d\alpha \Rightarrow dA = ds \cdot dr$$
$$= r \, d\alpha \, dr$$

$$T = \frac{\tau_{max}}{R} \int_0^{2\pi} \int_0^{R} r^3 \, d\alpha \, dr = \frac{\tau_{max}}{R} \cdot 2\pi \frac{R^4}{4}$$



SEZIONE RETTANGOLARE

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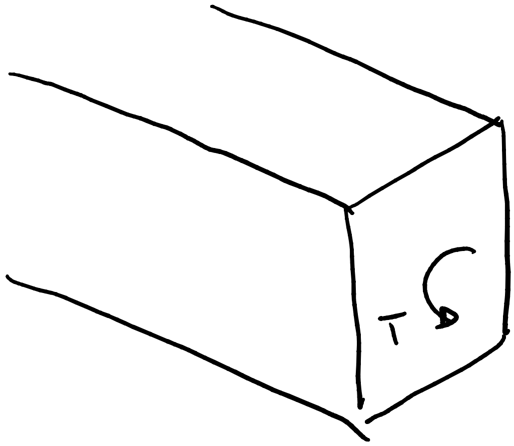
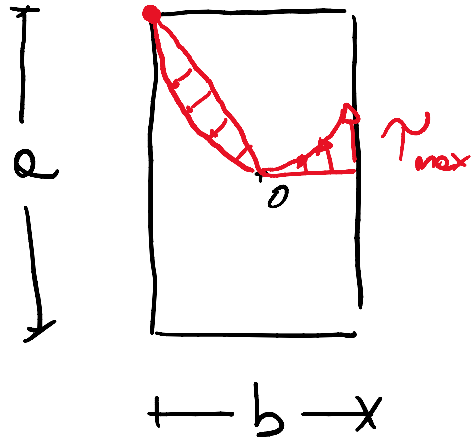




$$\tau_{max} = \psi \cdot \frac{T}{a b^2}$$

DEFORMAZIONI FUORI PIANO $\Rightarrow$ INGOBBIMENTO

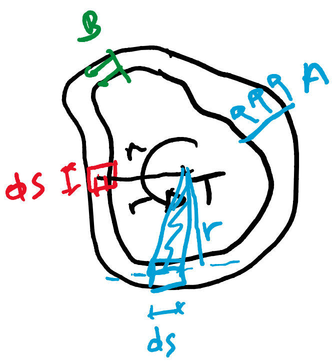
$$\psi = 3 + \frac{2,6}{0,45 + \frac{a}{b}}$$

SEZ. QUADRATA $a = b \Rightarrow \psi = 3 + \frac{2,6}{1,45} = 4,8$

SEZ MOLTO ALLUNGATA $a >> b \Rightarrow \psi \Rightarrow 3$

FORMULA DI BREDT

giovedì 11 giugno 2020 14:24



t_A SPESSORE IN A

t_B SPESSORE IN B

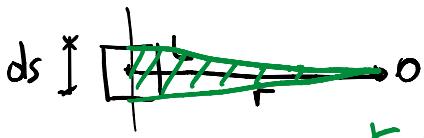
$$\tau_A \cdot t_A \cdot \cancel{1} = \tau_B \cdot t_B \cdot \cancel{1}$$

$$\rightarrow \tau \cdot t = \text{costante}$$

$dT = \tau dA \cdot r$ $\qquad dA = t \cdot ds$ $\qquad \Rightarrow dT = \tau \cdot t\, r\, ds$

$$T = \int \tau \cdot t \cdot r\, ds = \tau \cdot t \int r\, ds = \tau t \int 2\, dA_k$$

$$= \tau \cdot t \cdot 2 A_k$$



$r \cdot ds = 2\, dA_k$

A_k = AREA SEZIONE RACCHIUSA DALLA LINEA MEDIA

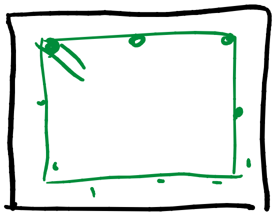
$$\rightarrow \quad \tau_{max} = \frac{T}{t_{min}\, 2 A_k}$$

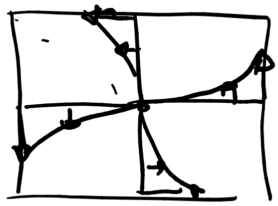
SEZIONE IN C.A.

giovedì 11 giugno 2020 14:31

PER VALUTARE LE τ PRODOTTE DA T IN SEZIONI IN C.A.

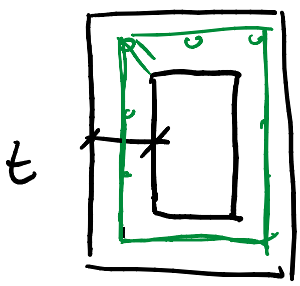
SCUOLA NAPOLETANA



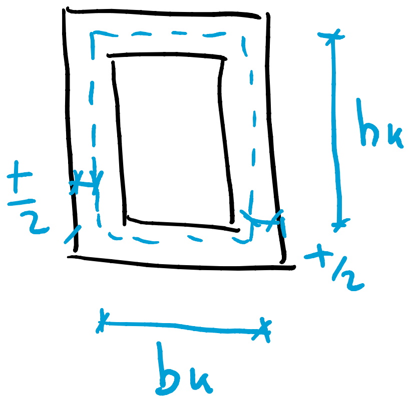


ALTRA POSSIBILITÀ (COERENTE CON COMPORTAMENTO DOPO LA FESSURAZIONE)

SI TRASFORMA LA SEZ IN C.A IN UNA SEZIONE TUBOLARE EQUIVALENTE DI SPESSORE



$$t = \max\left\{2c;\ \frac{A}{u}\right\}$$



$$b_k = b - t$$
$$h_k = h - t$$

$\Rightarrow$ APPLICO BREDT

QUADRO FESSURATIVO

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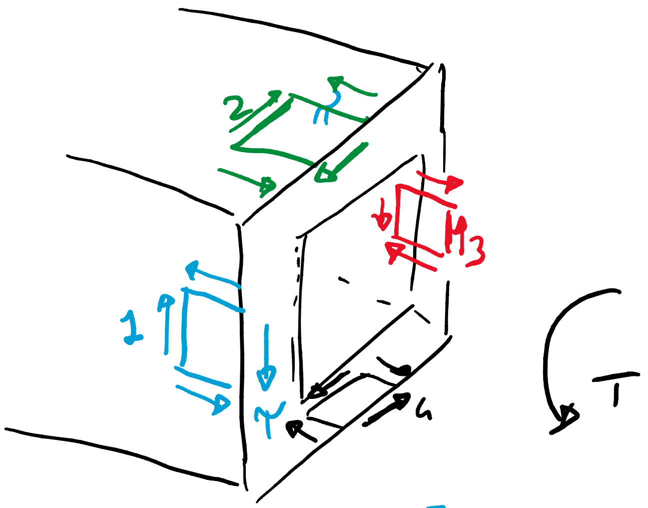
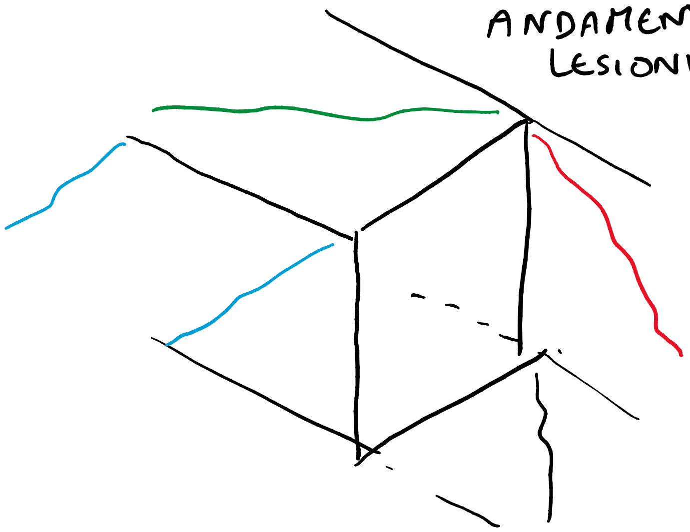
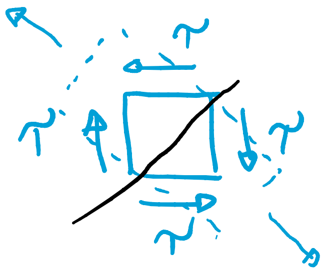


ANDAMENTO LESIONI

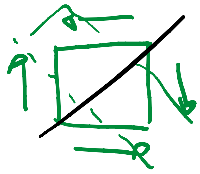


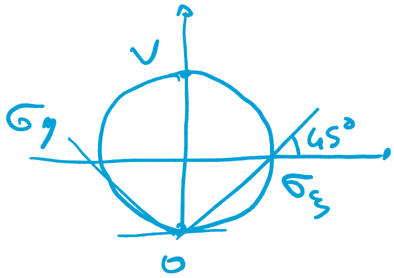
ELEMENTINO 1.



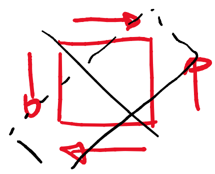
LESIONE A SPIRALE

ELEMENTINO 2

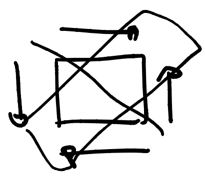




ELEMENTINO 3



ELEMENTINO 4



MODELLI PER ELEMENTO ARMATO A TORSIONE

giovedì 11 giugno 2020 14:42

- CAMPO LINEARE
  - CAMPI DI TENSIONE (σ_c INCLINATE A 45°)
  - TRALICCIO SPAZIALE DI RAUSCH

- CAMPO NON LINEARE
  - CAMPI DI TENSIONE — σ_c INCLINATE DI θ
  - TRALICCIO SPAZIALE AD INCLINAZIONE VARIABILE DEL PUNTONE

MODELLO CAMPI DI TENSIONE

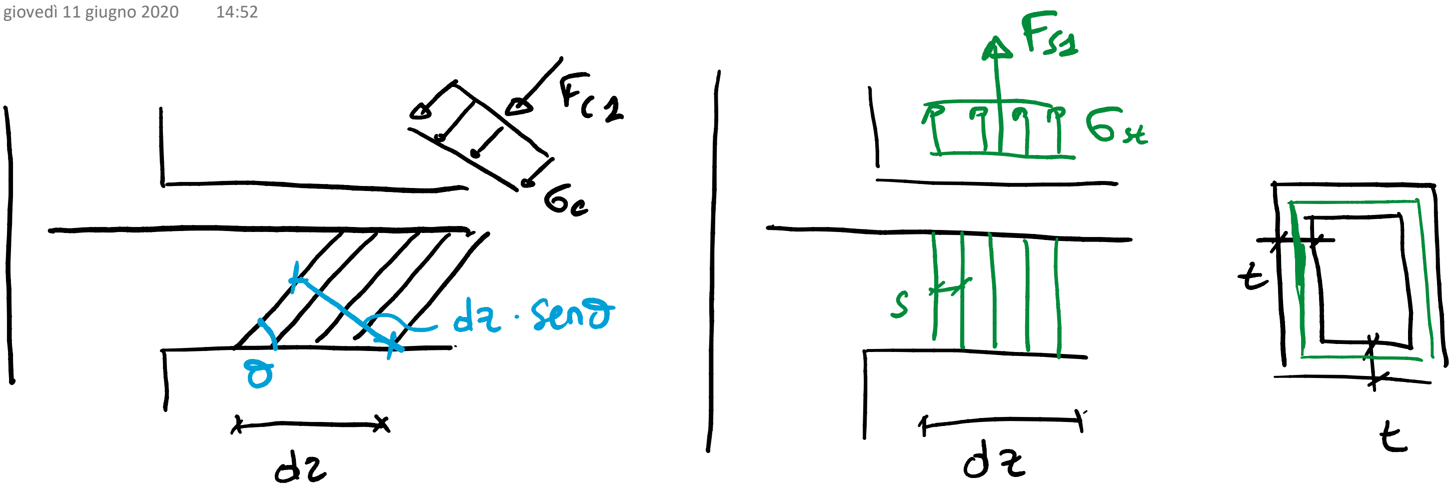
giovedì 11 giugno 2020 14:50

- σ_c LUNGO LE DIREZ. PRINCIPALI DI COMPRESSIONE INCLINATE DI θ
- σ_{st} STAFFE, VERTICALI
- σ_{se} ARMATURA DI PARETE (NECESSARIA)
- T MOMENTO TORCENTE AGENTE

APPLICHIAMO ALLA SEZIONE TUBOLARE

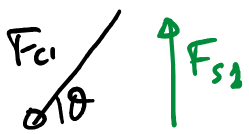
SEZIONE ORIZZONTALE

giovedì 11 giugno 2020 14:52



$$F_{c1} = \sigma_c \cdot dz \operatorname{sen}\theta \cdot t \qquad F_{s1} = \sigma_{st} A_{st} \cdot \underbrace{\frac{dz}{s}}_{\text{NUMERO DI STAFFE}}$$

EQUILIBRIO ALLA TRASLAZIONE VERTICALE



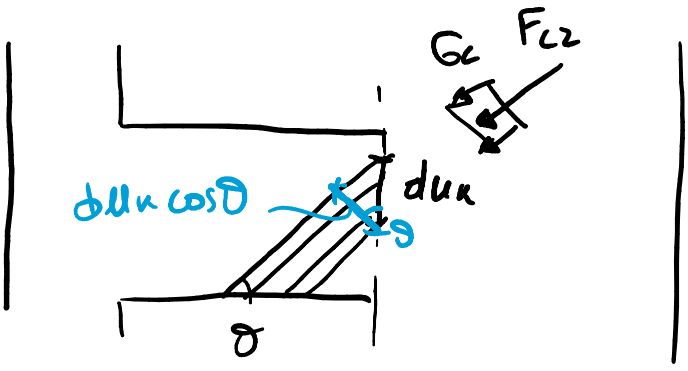
$$F_{c1,v} = F_{c1} \operatorname{sen}\theta = F_{s1} \rightarrow$$

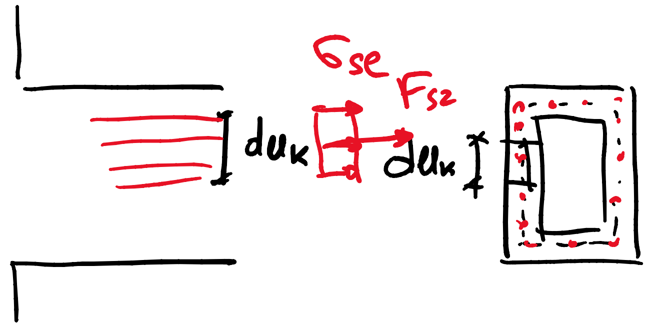
$$\sigma_c \cdot \cancel{dz} \operatorname{sen}^2\theta \cdot t = \sigma_{st} A_{st} \cdot \frac{\cancel{dz}}{s}$$

$$\boxed{\sigma_c = \sigma_{st} \frac{A_{st}}{s\,t} \frac{1}{\operatorname{sen}^2\theta}} \qquad \boxed{1}$$

SEZIONE VERTICALE

giovedì 11 giugno 2020 14:58

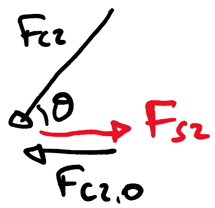




$$F_{cz} = \sigma_c \cdot du_k \cos\vartheta \cdot t \qquad F_{sz} = \sigma_{se} A_{sp} \cdot \frac{du_k}{u_k}$$

A_{sp} ARMATURA DI PARETE TOTALE

EQUILIBRIO ALLA TRASLAZIONE ORIZZONTALE



$$F_{cz,o} = F_{sz}$$

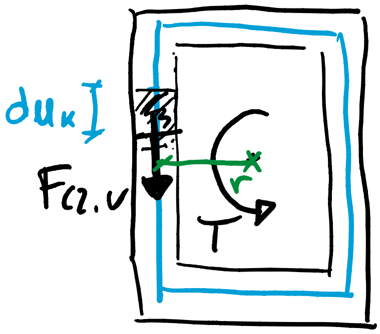
$$F_{cz} \cos\vartheta = F_{sz} \quad \rightarrow$$

$$\sigma_c \cdot \cancel{du_k} \cos^2\vartheta \cdot t = \sigma_{se} \cdot A_{sp} \cdot \frac{\cancel{du_k}}{u_k} \quad \rightarrow$$

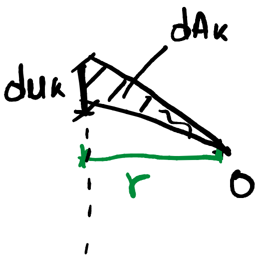
$$\boxed{\sigma_c = \sigma_{se} \cdot \frac{A_{sp}}{u_k t} \frac{1}{\cos^2\vartheta}} \qquad \boxed{2}$$

NEL PIANO DELLA SEZIONE TRASVERSALE

giovedì 11 giugno 2020 15:04



$$dT = F_{c2,v} \cdot r$$
$$= F_{c2} \cdot \sin\vartheta \cdot r$$
$$= \sigma_c \, du_k \cos\vartheta \cdot t \cdot \sin\vartheta \cdot r$$



$$r \cdot du_k = 2 dA_k \quad \Rightarrow$$

$$dT = \sigma_c \cdot \cos\vartheta \sin\vartheta \cdot t \, 2 \, dA_k$$

$$T = \int \sigma_c \cos\vartheta \sin\vartheta \, t \, 2 \, dA_k \quad \Rightarrow$$

$$T = \sigma_c \frac{\cos\vartheta}{\sin\vartheta} \cdot \sin^2\vartheta \, 2 A_k t$$

$$\Rightarrow T = \sigma_c \cdot \cot\vartheta \cdot \sin^2\vartheta \cdot 2 A_k t \quad \leftarrow$$

$$\sin^2\vartheta = \frac{1}{1+\cot^2\vartheta} \quad \Rightarrow \quad T = \sigma_c \cdot 2 A_k t \cdot \frac{\cot\vartheta}{1+\cot^2\vartheta}$$

CRISI NEL CLS: SE $\sigma_c = \nu f_{cd} \rightarrow$

$$T_{Rcd} = \nu f_{cd} \cdot 2 A_k t \, \frac{\cot\vartheta}{1+\cot^2\vartheta}$$

DAL LEGAME TRA σ_c E σ_{st}

giovedì 11 giugno 2020 15:13

$$\begin{cases} \sigma_c = \sigma_{st} \cdot \dfrac{A_{st}}{st} \cdot \dfrac{1}{sen^2\vartheta} \\ T = \sigma_c \, 2A_k t \cdot \cos\vartheta \cdot sen\vartheta \end{cases} \Longrightarrow$$

$$T = \sigma_{st} \cdot \frac{A_{st}}{s \cancel{t}} \cdot 2A_k \cancel{t} \, \frac{\cos\vartheta \cdot \cancel{sen\vartheta}}{sen^{\cancel{2}}\vartheta} \Longrightarrow$$

$$T = \sigma_{st} \cdot \frac{A_{st}}{s} \cdot 2A_k \cot g\vartheta$$

CRISI NELLE STAFFE SE $\sigma_{st} = f_{yd}$

$$T_{Rsd} = \frac{A_{st}}{s} \cdot 2A_k f_{yd} \cot g\vartheta$$

DAL LEGAME TRA σ_c E σ_{se}

giovedì 11 giugno 2020 15:17

$$\begin{cases} \sigma_c = \sigma_{se} \cdot \dfrac{A_{sp}}{u_k t} \dfrac{1}{\cos^2\vartheta} \\ T = \sigma_c \cdot 2 A_k t \cos\vartheta \cdot \sin\vartheta \end{cases} \Longrightarrow$$

$$T = \sigma_{se} \cdot \frac{A_{sp}}{u_k \cancel{t}} \frac{1}{\cos^{\cancel{2}}\vartheta} \cdot 2 A_k \cancel{t} \cancel{\cos\vartheta} \cdot \sin\vartheta \Longrightarrow$$

$$T = \sigma_{se} \frac{A_{sp}}{u_k} \cdot \frac{2 A_k}{\cot g\vartheta}$$

CRISI ARMATURA LONGITUDINALE SE $\sigma_{se} = f_{yd}$

$$T_{Rd} = \frac{A_{se}}{u_k} \cdot 2 A_k f_{yd} \frac{1}{\cot g\vartheta}$$

MOMENTO TORCENTE RESISTENTE

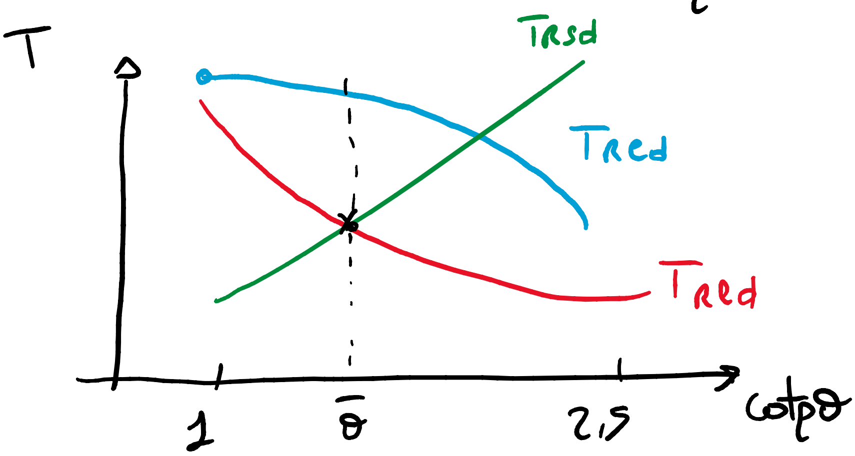
giovedì 11 giugno 2020 15:20

$$T_{Rd} = \min \left\{ T_{Rcd},\ T_{Rsd},\ T_{Rld} \right\} \qquad 1 \le \cot\theta \le 2{,}5$$

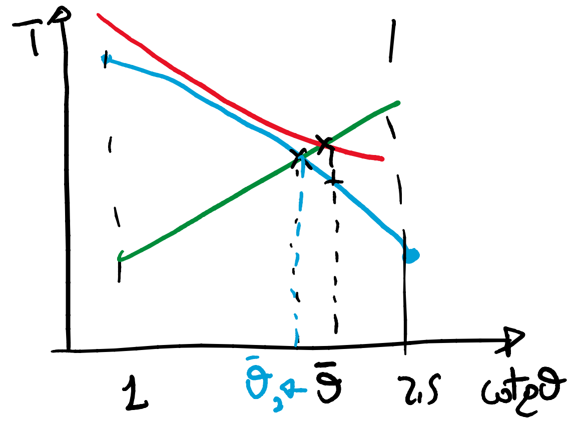
$$T_{Rcd} = 2 A_k t \cdot \nu f_{cd} \cdot \frac{\cot\theta}{1+\cot^2\theta}$$

$$T_{Rsd} = \frac{A_{st}}{s} \cdot 2 A_k f_{yd} \cot\theta$$

$$T_{Rld} = \frac{A_{sl}}{u_k} \cdot 2 A_k \frac{f_{yd}}{\cot\theta}$$



CASO 1



CASO 2

PROCEDIMENTO DI VERIFICA

CERCO $\cot\bar\theta$: $T_{Rsd} = T_{Rld}$ → CALCOLO $T_{Rcd}(\cot\bar\theta)$

SE $T_{Rcd}(\cot\bar\theta) > T_{Rsd}$ (CASO 1) → $T_{Rd} = T_{Rsd}$

SE $T_{Rcd}(\cot\bar\theta) < T_{Rsd}$ (CASO 2) → CERCO $\bar\theta_1$: $T_{Rsd} = T_{Rcd}$

PROGETTO

DIMENSIONAMENTO SEZIONE PER TORSIONE

$$t = \max\left\{\frac{A}{u}, 2c\right\} \Rightarrow \text{FISSO } t = 2c \rightarrow$$

?

$$\text{da } T_{Rcd} = \nu f_{cd}\, 2A_k t \cdot \frac{\cot\theta}{1+\cot^2\theta}$$

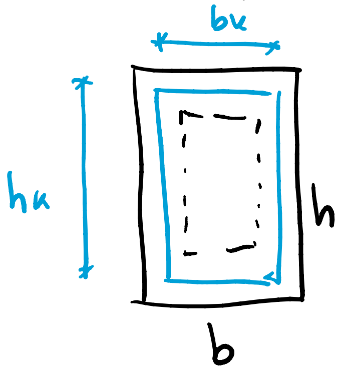
CONSIGLIO $\cot\theta = 2{,}5$ (CAUTELATIVO)

PONGO $T_{Rcd} = T_{Ed} \Rightarrow A_k = b_k \cdot h_k$

FISSO $b \rightarrow b_k = b - t \rightarrow h_k = \dfrac{A_k}{b_k}$

$\downarrow$

$h = h_k + t$



RICALCOLO $t = \max\left\{\frac{A}{u}, 2c\right\}$

$\Rightarrow$ AGGIORNO I VALORI DI b_k, h_k

PROGETTO ARMATURE LONGITUDINALI E STAFFE

FISSO $\cot\theta \simeq 2 \Rightarrow \dfrac{A_{st}}{s} = \dfrac{T_{Ed}}{2A_k f_{yd} \cot\theta}$

$$A_{sp} = \frac{T_{Ed}\, u_k}{2A_k f_{yd}} \cot\theta$$

NOTA: NON HO VANTAGGIO AD USARE VALORI DI $\cot\theta$ MOLTO ALTI PERCHÈ SE RIDUCO A_{st}/s AUMENTA A_{se}

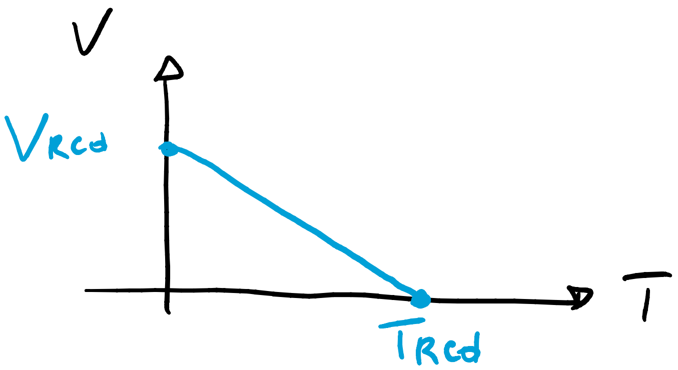
TORSIONE + TAGLIO + MOMENTO FLETTENTE

giovedì 11 giugno 2020 15:52

VERIFICA DELLA DIMENSIONE DELLA SEZIONE DI CLS

$$M_{Ed} \leq \frac{bd^2}{r'^2} \Rightarrow d$$

$$V_{Ed},\ T_{Ed} \Rightarrow \frac{V_{Ed}}{V_{Rcd}} + \frac{T_{Ed}}{T_{Rcd}} \leq 1$$



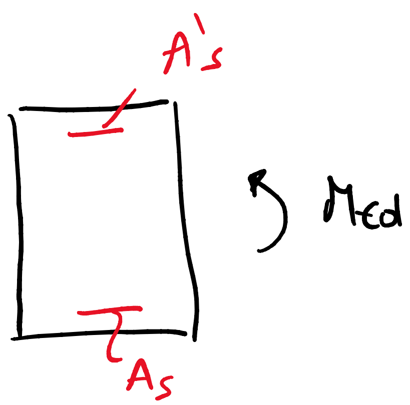
DOMINIO DI INTERAZIONE TAGLIO - TORSIONE

$$\frac{V_{Ed}}{b_w z \cdot \nu f_{cd} \cdot \underbrace{\frac{\cot\theta}{1+\cot^2\theta}}} + \frac{T_{Ed}}{2A_k t \cdot \nu f_{cd} \cdot \underbrace{\frac{\cot\theta}{1+\cot^2\theta}}} \leq 1$$

UTILIZZO LO STESSO $\cot\theta$ PER TAGLIO E TORSIONE

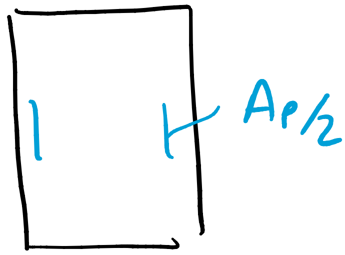
PROGETTO ARMATURE

$$M_{Ed} \Rightarrow \underset{(cm^2)}{A_s,\ A'_s}$$



$V_{Ed} \Rightarrow \dfrac{A_{st}}{s} \left[\dfrac{cm^2}{m}\right]$

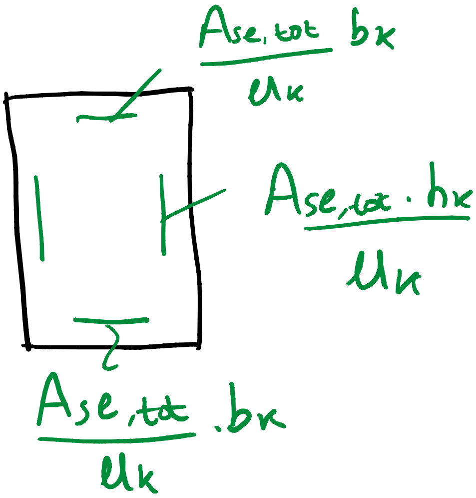
$A_p \; (cm^2)$



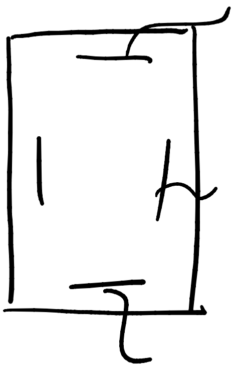
$T_{Ed} \Rightarrow \dfrac{A_{st}}{s} \left[\dfrac{cm^2}{m}\right]$

$A_{se,tot} \; [cm^2]$

RIPARTITA SUI QUATTRO LATI IN PROPORZIONE ALLA LORO LUNGHEZZA



ARMATURA COMPLESSIVA NECESSARIA



$A'_s + \dfrac{A_{se}}{u_k} \cdot b_k \Rightarrow$ DEFINISCO BARRE

$\dfrac{A_p}{2} + \dfrac{A_{se}}{u_k} \cdot h_k \Rightarrow$

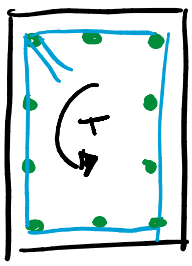
$A_s + \dfrac{A_{se}}{u_k} b_k \Rightarrow$

STAFFE $\dfrac{A^V_{st}}{s} + \dfrac{A^T_{st}}{s} \Rightarrow \dfrac{cm^2}{m}$

FISSO $A_{\phi 8} \Rightarrow s$

ESEMPIO VERIFICA

giovedì 11 giugno 2020 16:03



I C

60

40

$A_{se} = 10\,\phi\,14$

STAFFE $\phi 8/20$ $\Rightarrow$ $T_{Rd} = ?$

$c = 5\ cm$

C25/30

$$t = \max\left\{\frac{A}{u},\, 2c\right\} = \left\{\underbrace{\frac{40\times 60}{2\times 100}}_{12},\ 10\right\} = 12\ cm$$

$$b_k = 40 - t = 40 - 12 = 28\ cm$$

$$h_k = 60 - t = 60 - 12 = 48\ cm$$

$$u_k = 2\times(b_k + h_k) = 2\times(28+48) = 152\ cm$$

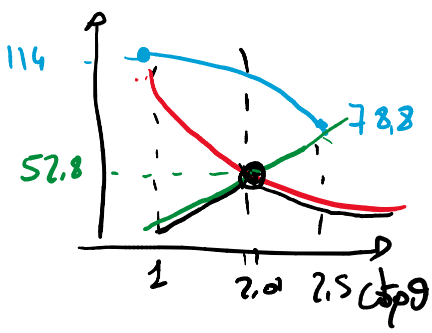
$$A_k = b_k \cdot h_k = 28\times 48\ cm^2 = 1344\ cm^2$$

$$T_{Rcd} = 0{,}5\times 14{,}17\,\frac{N}{mm^2}\times 2\times 1344\ cm^2\times 12\ cm \times \frac{\cot\theta}{1+\cot^2\theta}\cdot\frac{1}{10^3}$$

$$= 228{,}5\cdot\frac{\cot\theta}{1+\cot^2\theta}$$

se $\cot\theta = 1 \Rightarrow T_{Rcd} = 114{,}2\ KNm$

$\cot\theta = 2{,}5 \Rightarrow T_{Rcd} = 78{,}8\ KNm$



CERCO $\cot\theta$: $T_{Rsd} = T_{Rse}$

$$\frac{A_{st}}{s} \cdot 2A_k \cancel{f_{yd}} \cot\theta = \frac{A_{se}}{u_k} \cdot 2A_k \cancel{f_{yd}} \frac{1}{\cot\theta}$$

$$\cot^2\theta = \frac{A_{se}}{u_k} \cdot \frac{s}{A_{st}} \Rightarrow \cot\theta = \sqrt{\frac{A_{se}}{u_k} \frac{s}{A_{st}}}$$

$$A_{se} = 10\,\phi 14 = 10 \times 1{,}54\ cm^2 = 15{,}4\ cm^2$$

$$A_{st}\quad \phi 8 = 0{,}5\ cm^2$$

$$\cot\theta = \sqrt{\frac{15{,}4\ cm^2}{152\ \cancel{cm}} \cdot \frac{20\ \cancel{cm}}{0{,}5\ \cancel{cm^2}}} = 2{,}01$$

$$T_{Rsd} = \frac{0{,}5\ cm^2}{20\ cm} \times 2 \times 1344\ cm^2 \times 391{,}3\ \frac{N}{mm^2} \times 2{,}01\ \frac{1}{10^3}$$

$$= 52{,}85\ KNm$$

$$T_{Rd} = 52{,}85\ KNm$$ (PERCHÉ $T_{Rcd} > T_{Rsd}$)
PER $\cot\theta = 2.01$