

# Lezione

TECNICA DELLE COSTRUZIONI

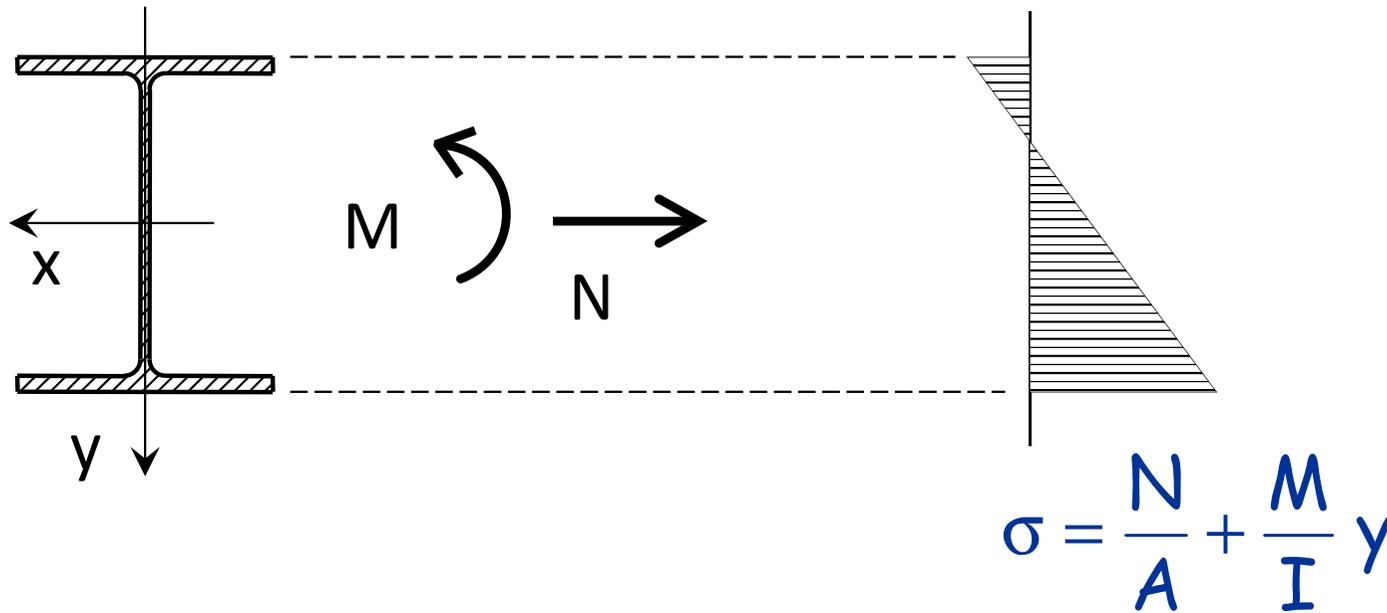
*Prof. Pier Paolo Rossi*

*Università degli Studi di Catania*

Flessione composta

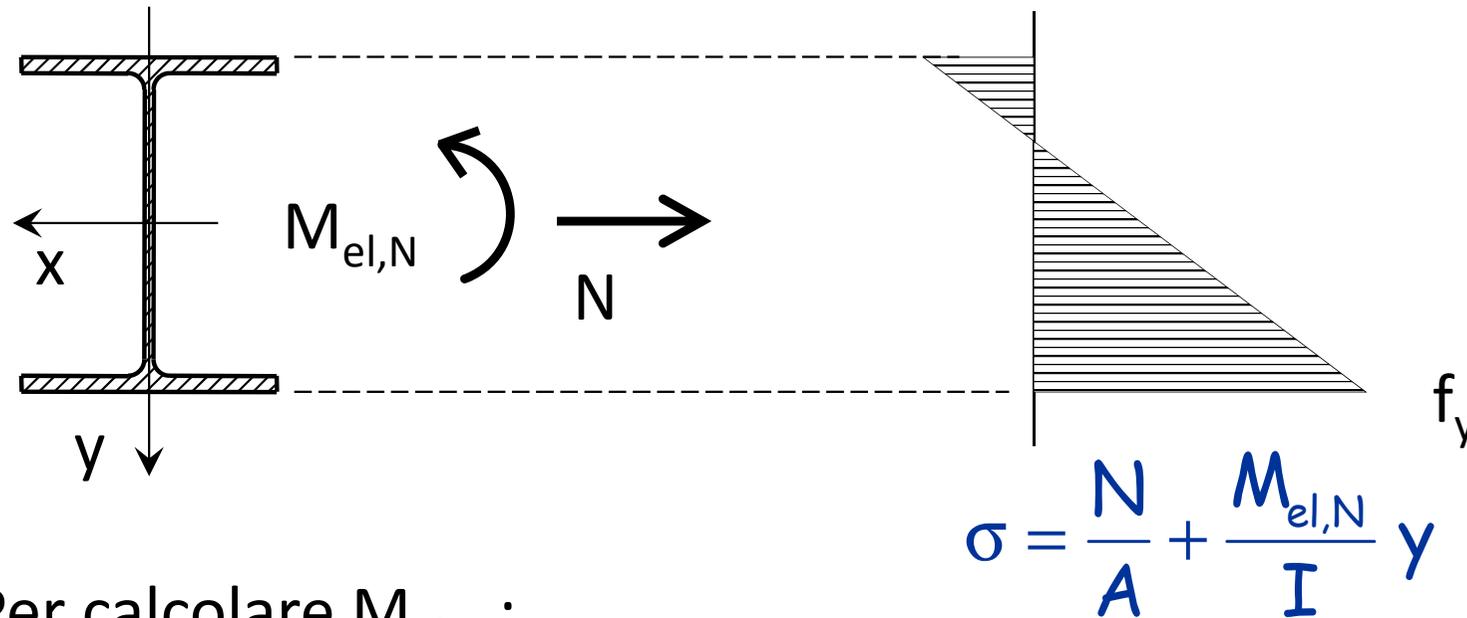
# Risposta della sezione

Campo elastico



# Risposta della sezione

Al limite elastico



Per calcolare  $M_{el,N}$ :

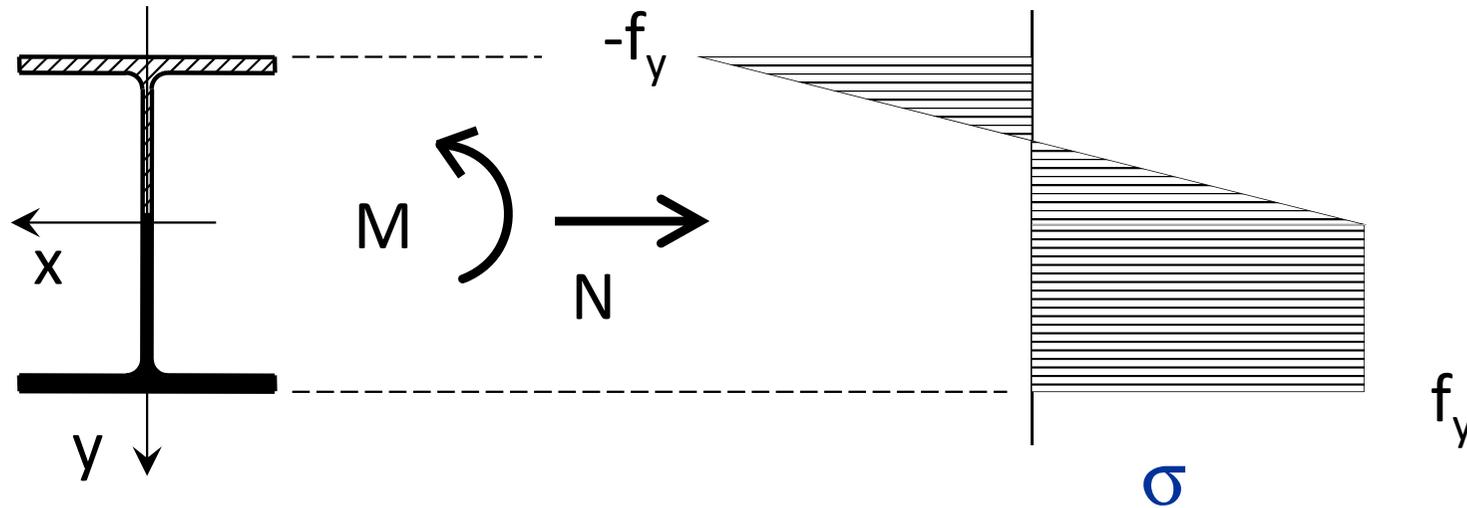
$$f_y = \frac{N}{A} + \frac{M_{el,N}}{W_{el}}$$



$$M_{el,N} = \left( f_y - \frac{N}{A} \right) W_{el}$$

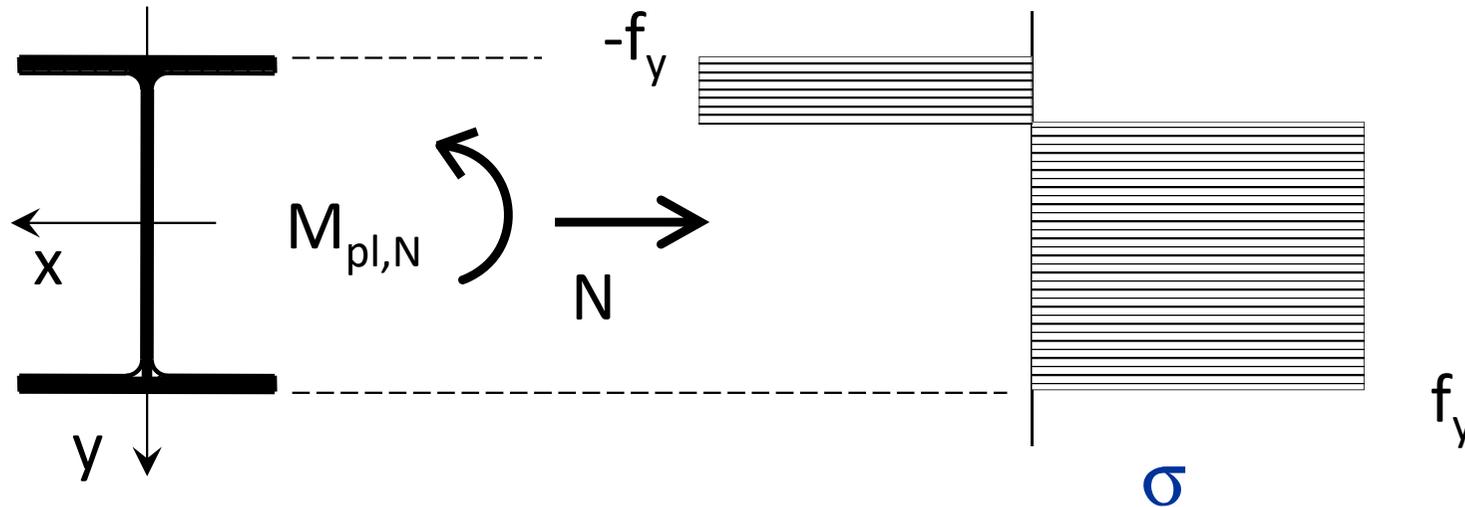
# Risposta della sezione

## Campo inelastico



Incrementando il momento flettente le deformazioni plastiche si propagano fino alla completa plasticizzazione della sezione

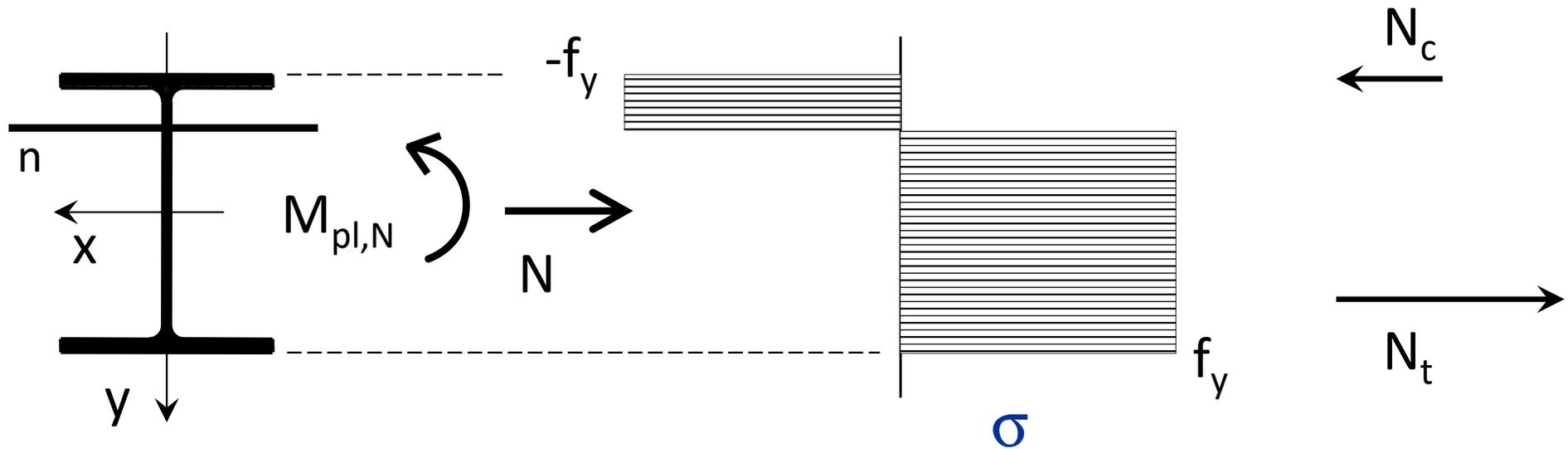
# Comportamento ultimo



Per calcolare  $M_{pl,N}$  :

1. Bisogna prima determinare la posizione dell'asse neutro, dall'equilibrio alla traslazione;
2. Si determina poi  $M_{pl,N}$  imponendo l'equilibrio alla rotazione rispetto all'asse baricentrico.

# Comportamento ultimo



## 1. Determinazione dell'asse neutro

$$N_c + N_t = N$$

(equilibrio alla traslazione)

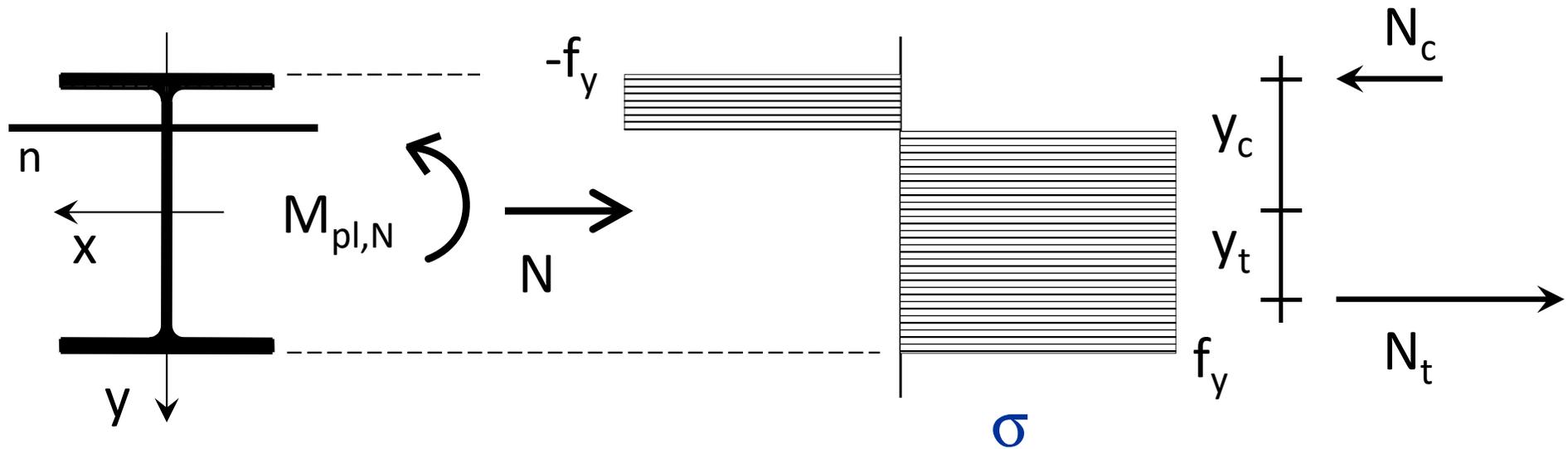
$$N_c = -f_y A_c$$

$$N_t = f_y A_t$$



Asse neutro

# Comportamento ultimo



## 2. Calcolo di $M_{pl,N}$

$$M_{pl,N} = N_t y_t + N_c y_c = f_y (A_t y_t - A_c y_c)$$

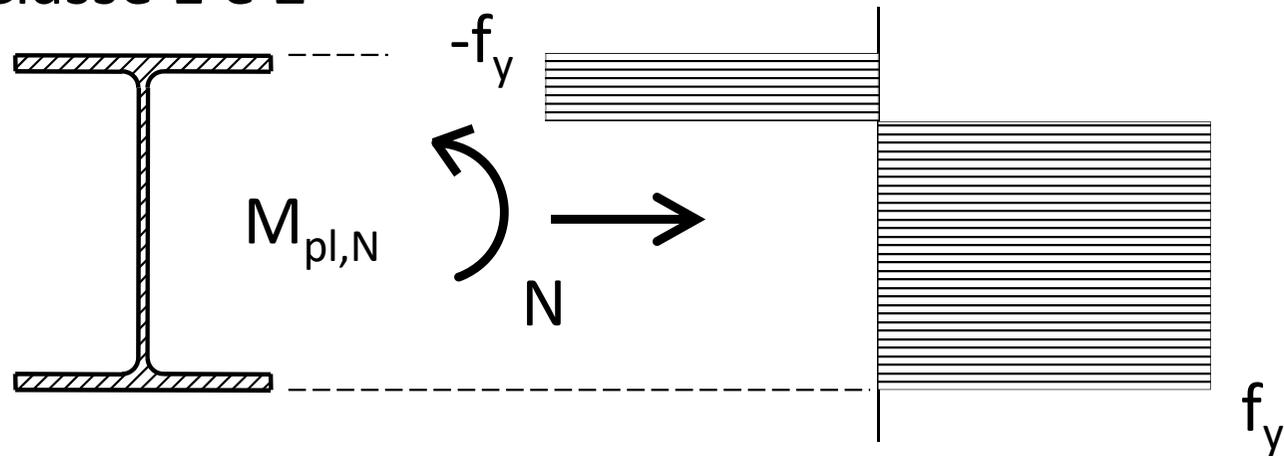
$$N_c = -f_y A_c$$

$$N_t = f_y A_t$$

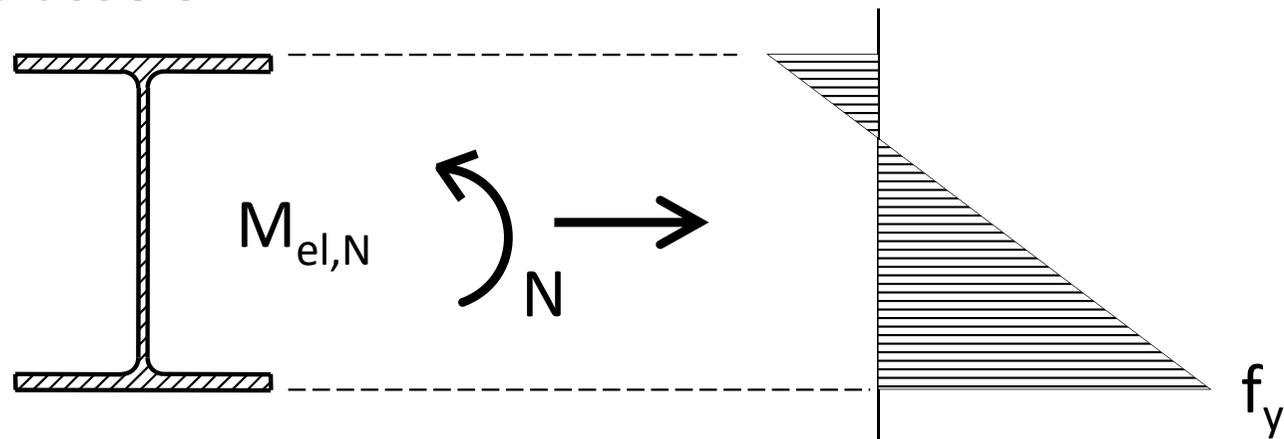
# Verifica di resistenza

## Stato limite ultimo

Classe 1 e 2



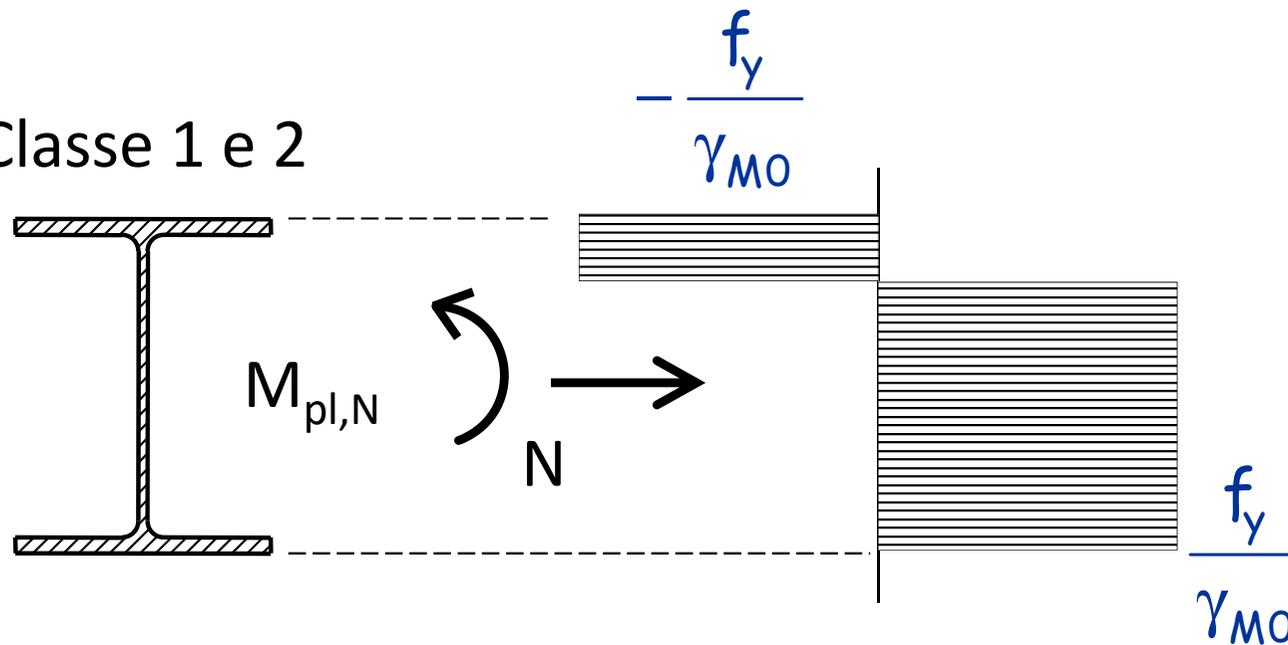
Classe 3



# Verifica di resistenza

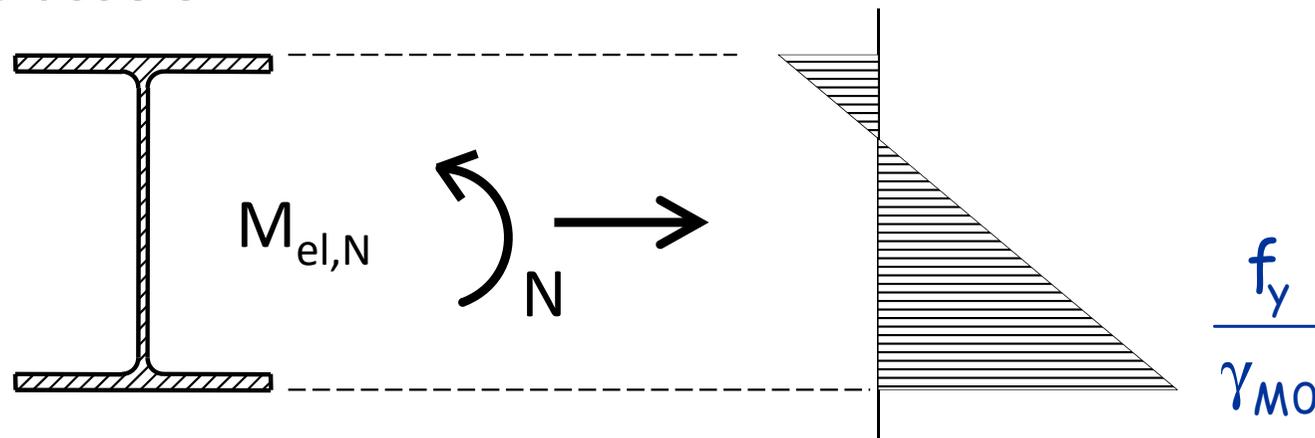
Stato limite ultimo

Classe 1 e 2



$$M_{Ed} \leq M_{pl,N,Rd}$$

Classe 3



$$M_{Ed} \leq M_{el,N,Rd}$$

# Domini di resistenza

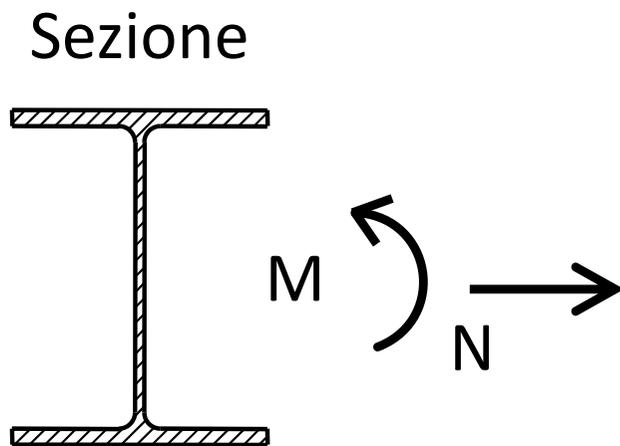
Stato limite ultimo

Dominio di resistenza, o  
curva di interazione

=

insieme delle coppie M-N  
per cui si ottiene lo stato limite  
ultimo della sezione

Per ricavare una coppia M-N del dominio



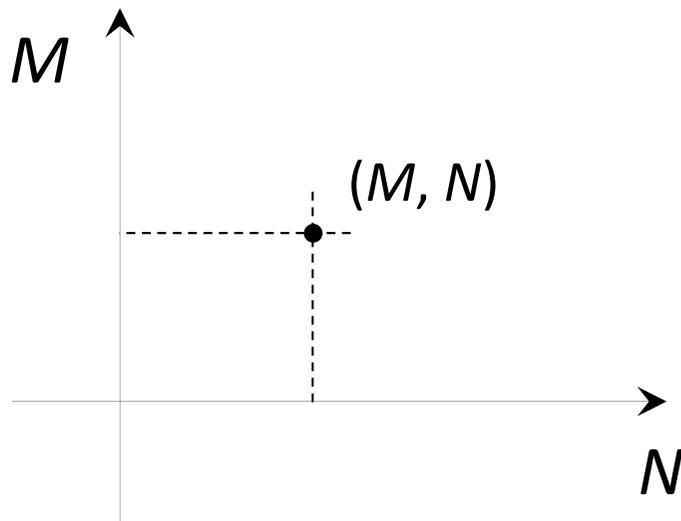
1. Si assegna una posizione dell'asse neutro
2. Si determina N
3. Si determina M ( $M_{pl,N}$  o  $M_{el,N}$ )

# Domini di resistenza

Stato limite ultimo

Dominio di resistenza, o curva di interazione = insieme delle coppie M-N per cui si ottiene lo stato limite ultimo della sezione

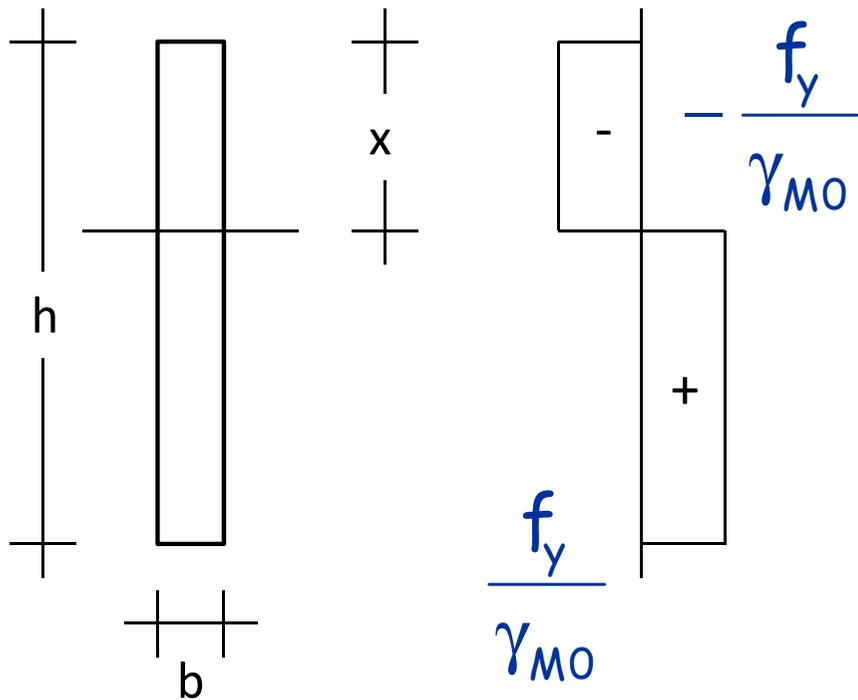
Per ricavare una coppia M-N del dominio



1. Si assegna l'asse neutro
2. Si determina N
3. Si determina M ( $M_{pl,N}$  o  $M_{el,N}$ )
4. Si riporta la coppia M – N nel diagramma

# Esempio di dominio

Sezione rettangolare (solo ideale)



$$N = b (h - 2x) \frac{f_y}{\gamma_{M0}}$$

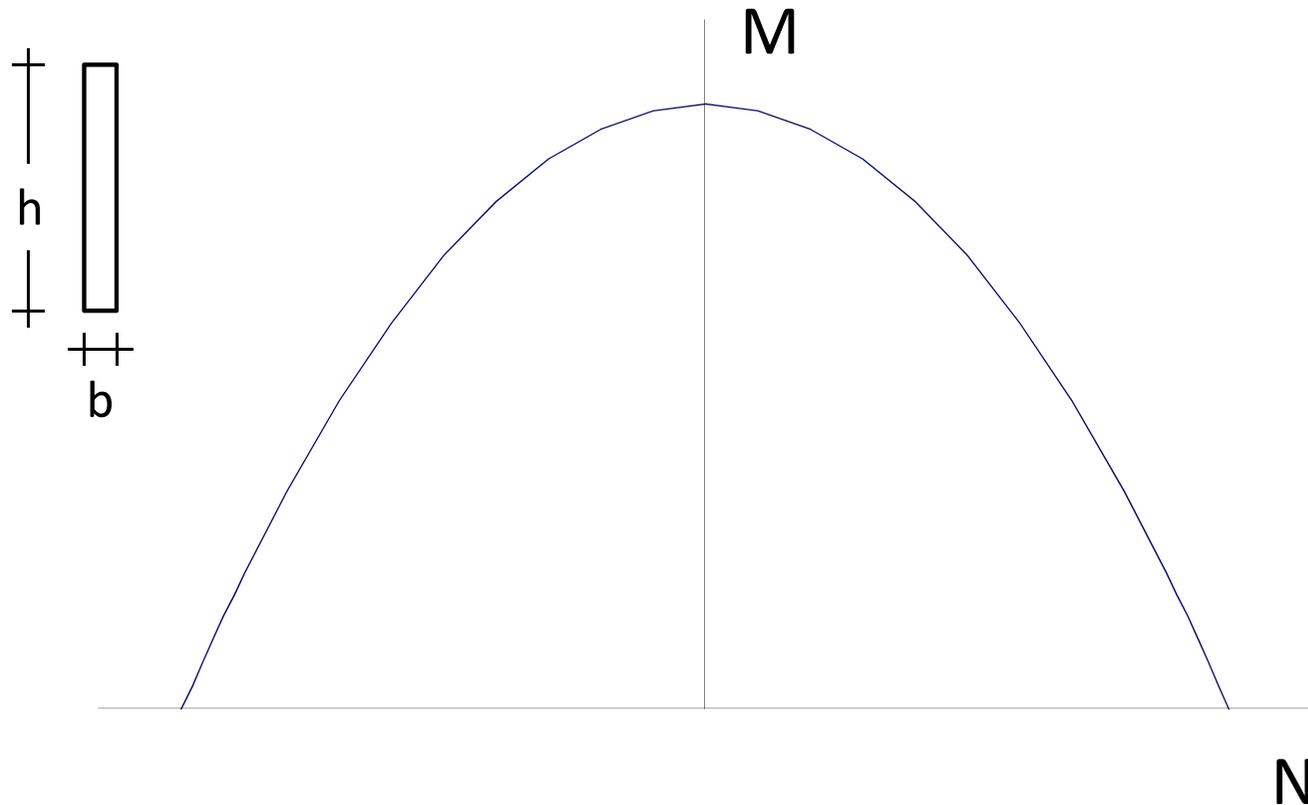
$$M = b x (h - x) \frac{f_y}{\gamma_{M0}}$$



$$M = \left[ \frac{bh^2}{4} \frac{f_y^2}{\gamma_{M0}^2} - \frac{1}{4b} N^2 \right] \frac{\gamma_{M0}}{f_y}$$

# Esempio di dominio

Sezione rettangolare (solo ideale)

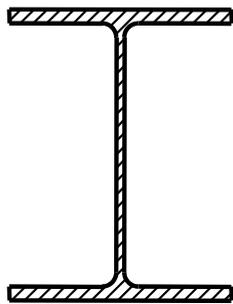


Il dominio ha un andamento parabolico

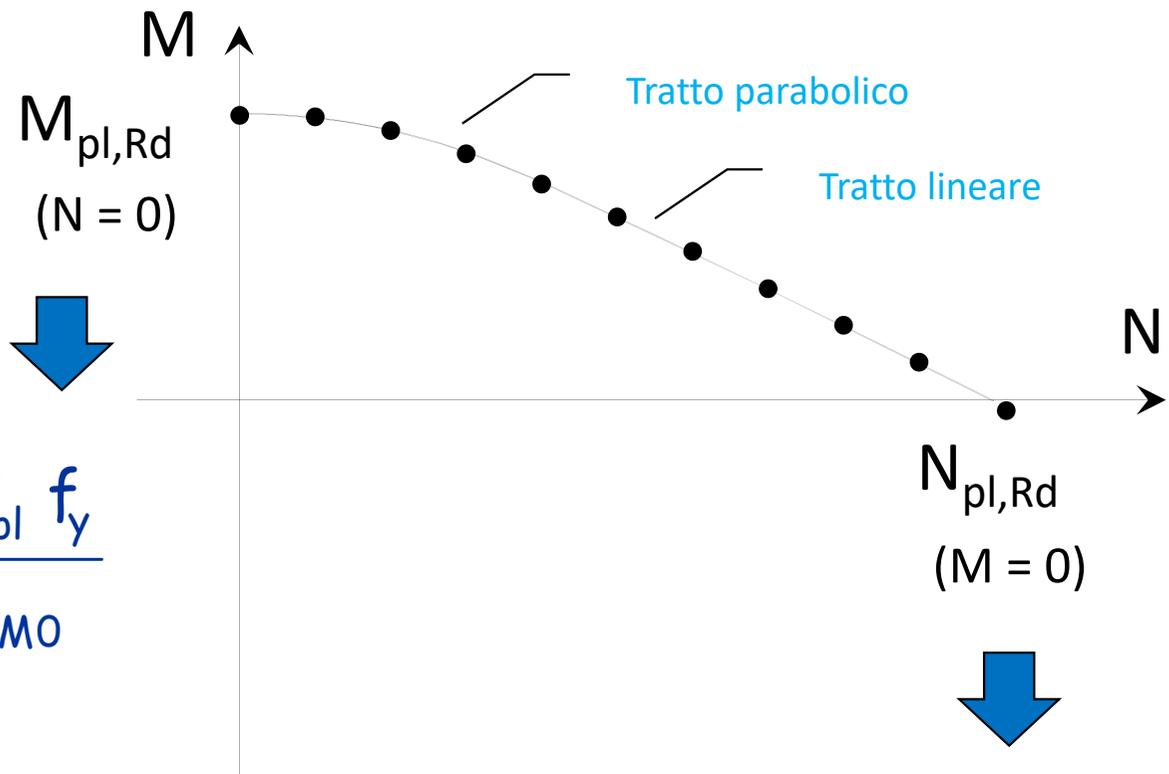
Questo vale in sostanza per un doppio T caricato nel piano debole

# Domini di resistenza

## Sezioni di classe 1 e 2



$$M_{pl,Rd} = \frac{W_{pl} f_y}{\gamma_{M0}}$$



$$N_{pl,Rd} = \frac{A f_y}{\gamma_{M0}}$$

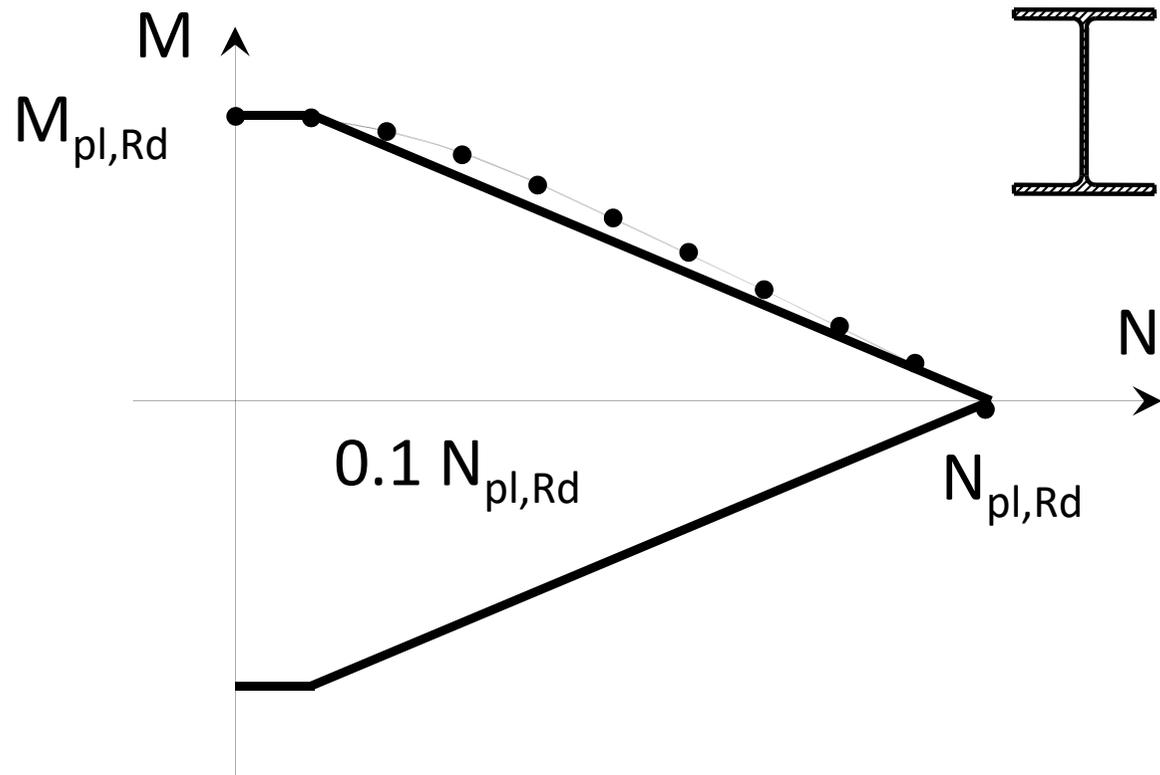
# Domini di resistenza

## Sezioni di classe 1 e 2

Caratteristiche plastiche

$$M_{pl,Rd} = W_{pl} f_y / \gamma_{M0}$$

$$N_{pl,Rd} = A f_y / \gamma_{M0}$$



$$M_{pl,N,Rd} = M_{pl,Rd}$$

$$N \leq 0.1 N_{pl,Rd}$$

$$M_{pl,N,Rd} = 1.11 M_{pl,Rd} \left(1 - N/N_{pl,Rd}\right)$$

$$N > 0.1 N_{pl,Rd}$$

# Domini di resistenza

## Sezioni di classe 1 e 2

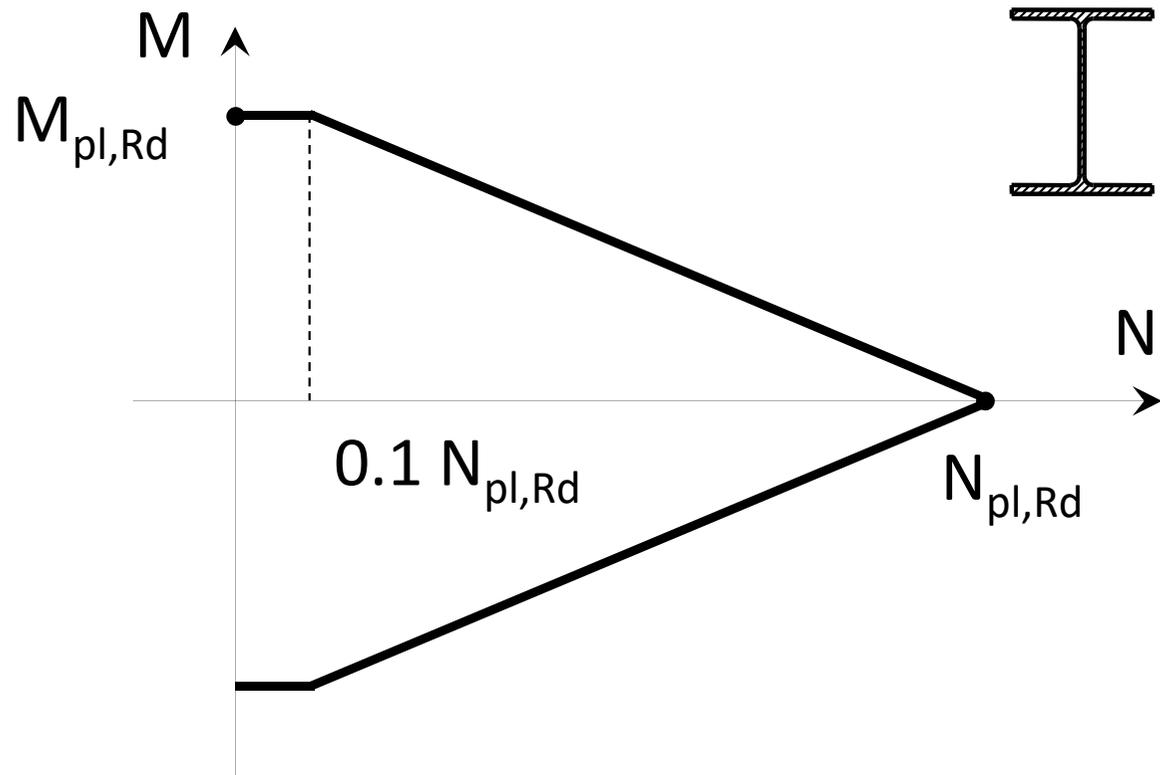
Caratteristiche plastiche

$$M_{pl,Rd} = W_{pl} f_y / \gamma_{M0}$$

$$N_{pl,Rd} = A f_y / \gamma_{M0}$$

Approssimazione  
“tradizionale”, cautelativa

Eurocodice 3, 1993



$$M_{pl,N,Rd} = M_{pl,Rd}$$

$$N \leq 0.1 N_{pl,Rd}$$

$$M_{pl,N,Rd} = 1.11 M_{pl,Rd} \left( 1 - N / N_{pl,Rd} \right)$$

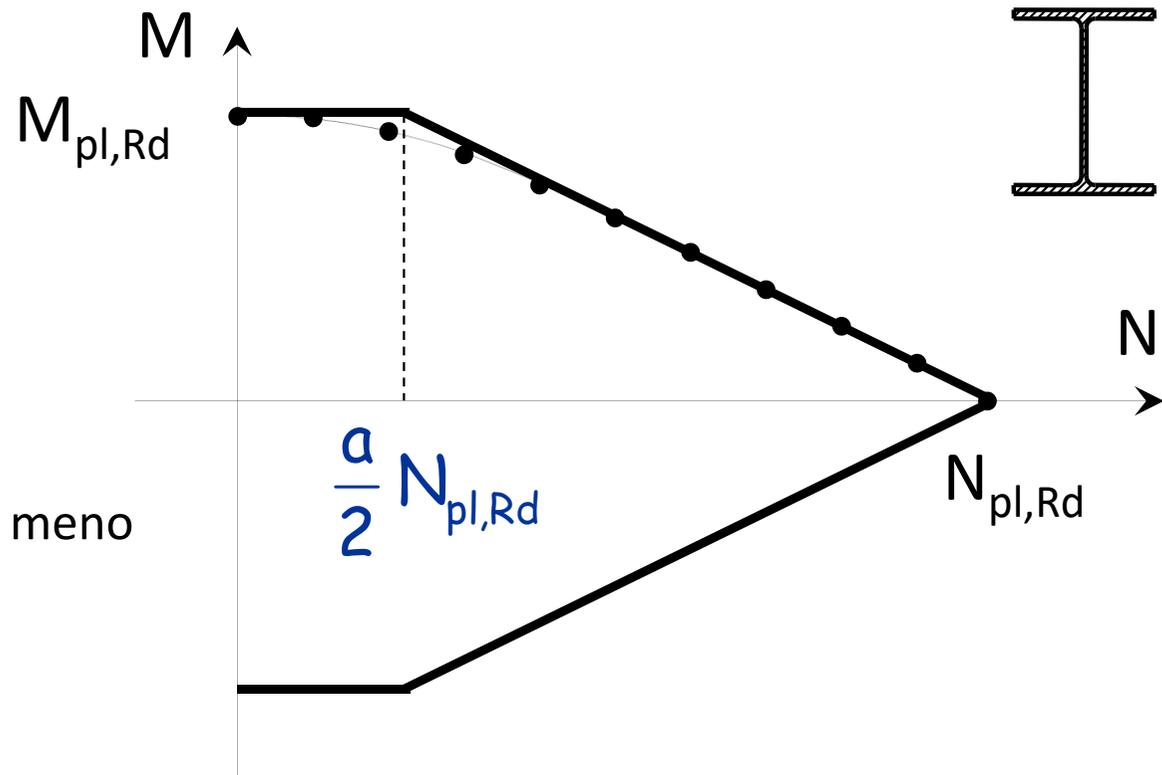
$$N > 0.1 N_{pl,Rd}$$

# Domini di resistenza

Approccio più preciso, ma meno cautelativo

$$a = \frac{A - 2 b t_f}{A} \leq 0.5$$

(differenze più forti per gli IPE, meno forti per gli HE)



$$M_{pl,N,Rd} = M_{pl,Rd}$$

$$N \leq N_{pl,Rd} \cdot a/2$$

$$M_{pl,N,Rd} = M_{pl,Rd} \left( \frac{1 - N/N_{pl,Rd}}{1 - 0.5a} \right)$$

$$N > N_{pl,Rd} \cdot a/2$$

# Domini di resistenza

## Sezioni di classe 3

Nota la relazione

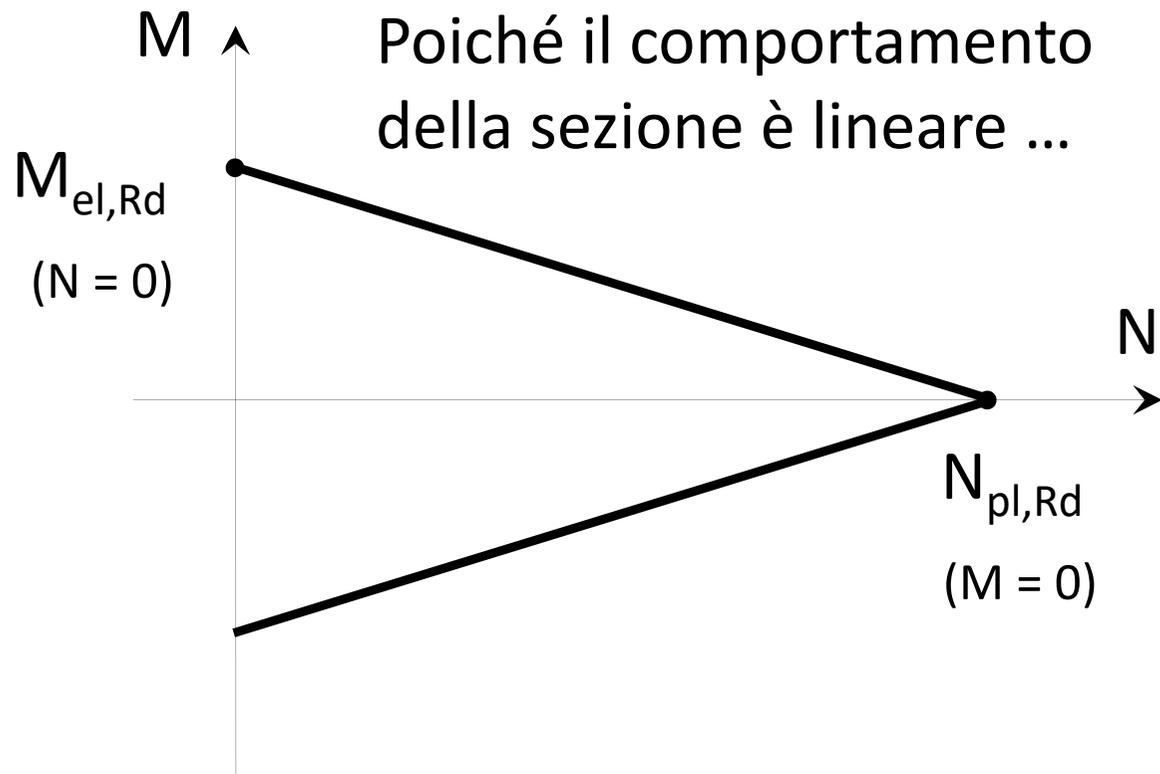
$$\frac{f_y}{\gamma_{M0}} = \frac{N}{A} + \frac{M_{el,N,Rd}}{W_{el}}$$

e sostituendo:

$$W_{el} = \frac{M_{el,Rd}}{f_y / \gamma_{M0}} \quad \text{e} \quad \frac{f_y}{\gamma_{M0}} = \frac{N_{pl,Rd}}{A}$$

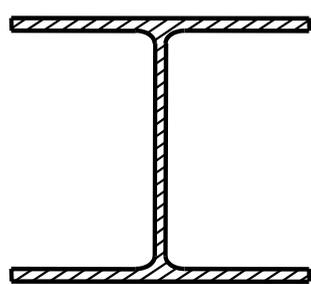
si ottiene .....

$$M_{el,N,Rd} = M_{el,Rd} \left( 1 - N / N_{pl,Rd} \right)$$



# Verifica di resistenza

## Esempio



$$M_{Ed} = 210 \text{ kNm}$$



$$N_{Ed} = 1500 \text{ kN}$$

Sezione HEB300 (S235)

A 149 cm<sup>2</sup>

W<sub>pl</sub> 1868 cm<sup>3</sup>

### 1. Classe della sezione

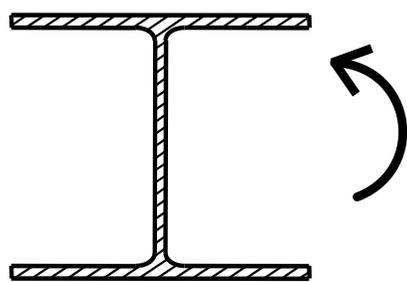
Anima:  $\frac{c_w}{t_w} = \frac{208}{11} = 19 \leq 72 \varepsilon = 72$

Flangia:  $\frac{c}{t_f} = \frac{150}{19} = 7.9 \leq 9 \varepsilon = 10$

La sezione appartiene alla classe 1.

# Verifica di resistenza

## Esempio



$$M_{Ed} = 210 \text{ kNm}$$



$$N_{Ed} = 1500 \text{ kN}$$

Sezione HEB300 (S235)

A 149 cm<sup>2</sup>

$W_{pl}$  1868 cm<sup>3</sup>

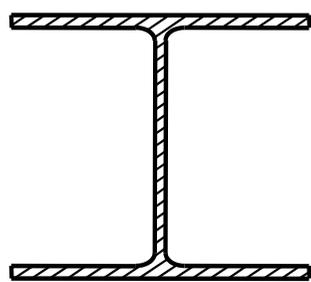
## 2. Determinazione di $N_{pl,Rd}$ ed $M_{pl,Rd}$

$$N_{pl,Rd} = \frac{f_y A}{\gamma_{M0}} = \frac{235 \times 149}{1.05 \times 10} = 3334.8 \text{ kN}$$

$$M_{pl,Rd} = \frac{f_y W_{pl}}{\gamma_{M0}} = \frac{235 \times 1868}{1.05 \times 10^3} = 418.1 \text{ kNm}$$

# Verifica di resistenza

## Esempio



$$M_{Ed} = 210 \text{ kNm}$$



$$N_{Ed} = 1500 \text{ kN}$$

Sezione HEB300 (S235)

A 149 cm<sup>2</sup>

W<sub>pl</sub> 1868 cm<sup>3</sup>

b=300 mm

t=19 mm

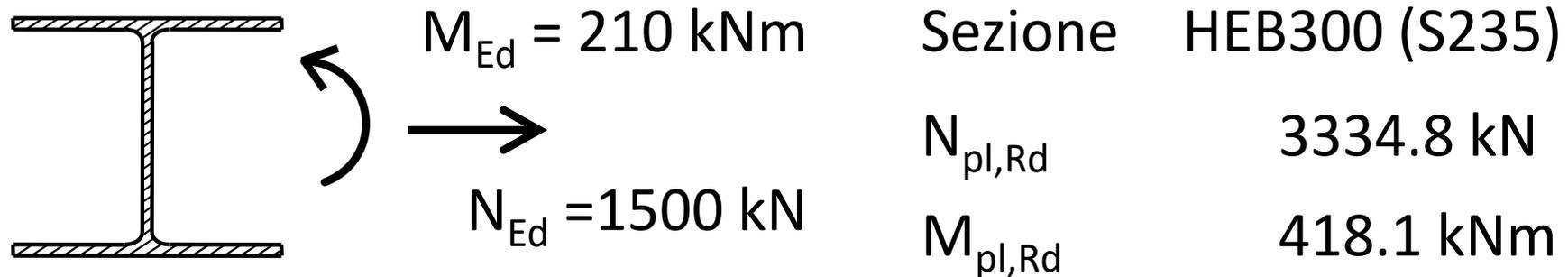
### 3. Determinazione di $\alpha$

$$\alpha = \frac{A - 2 b t_f}{A} = \frac{14900 - 2 \times 300 \times 19}{14900} = 0.235$$

$$\frac{\alpha}{2} N_{pl,Rd} = \frac{0.235}{2} 3334.8 = 391.8 \text{ kN}$$

# Verifica di resistenza

## Esempio



### 4. Determinazione di $M_{pl,N,Rd}$ e verifica

$$N_{Ed} = 1500 \text{ kN} \geq N_{pl,Rd} \alpha/2 = 391.8 \text{ kN}$$

$$M_{pl,N,Rd} = M_{pl,Rd} \left( 1 - \frac{N_{Ed}}{N_{pl,Rd}} \right) \frac{1}{1 - 0.5 \times 0.235} = 260.7 \text{ kNm} > M_{Ed} = 210.0 \text{ kNm}$$

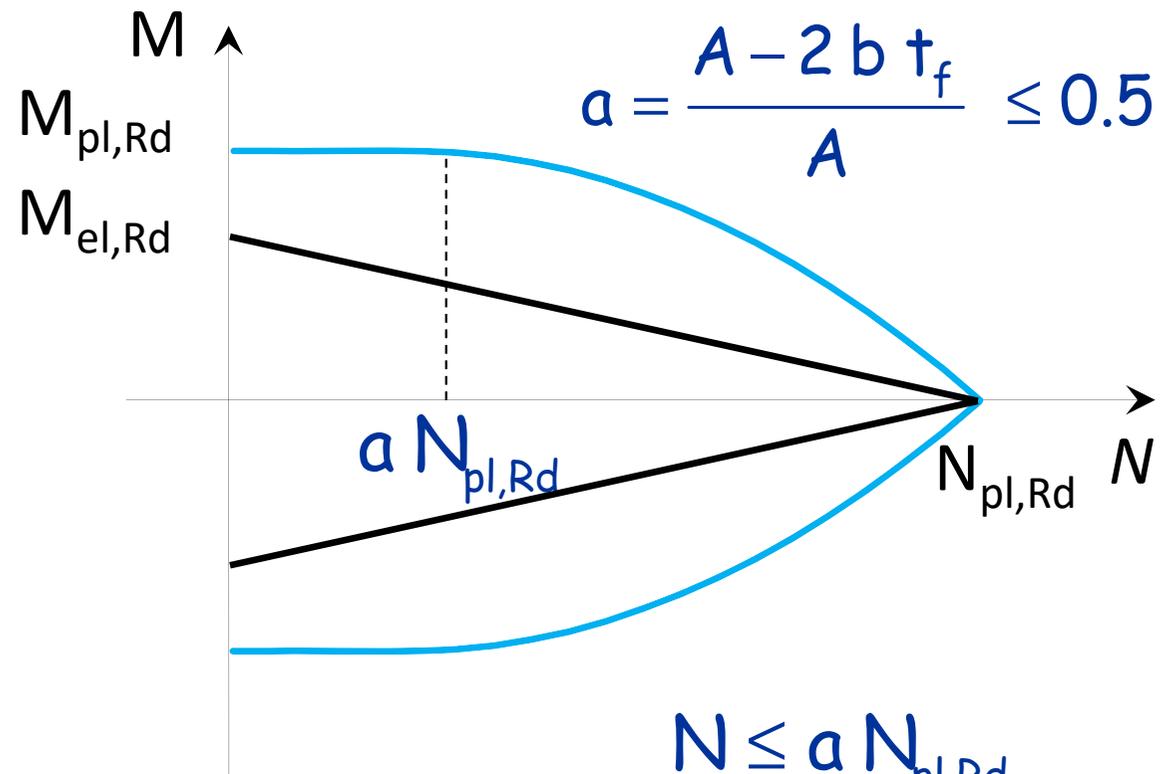
La sezione è verificata

# Domini di resistenza

Momento agente attorno all'asse debole

Il dominio plastico dipende dalla forma della sezione

Il dominio al limite elastico NO



$$M_{pl,N,Rd} = M_{pl,Rd}$$

$$M_{pl,N,Rd} = M_{pl,Rd} \left[ 1 - \left( \frac{N/N_{pl,Rd} - a}{1 - a} \right)^2 \right]$$

$$N \leq a N_{pl,Rd}$$

$$N > a N_{pl,Rd}$$

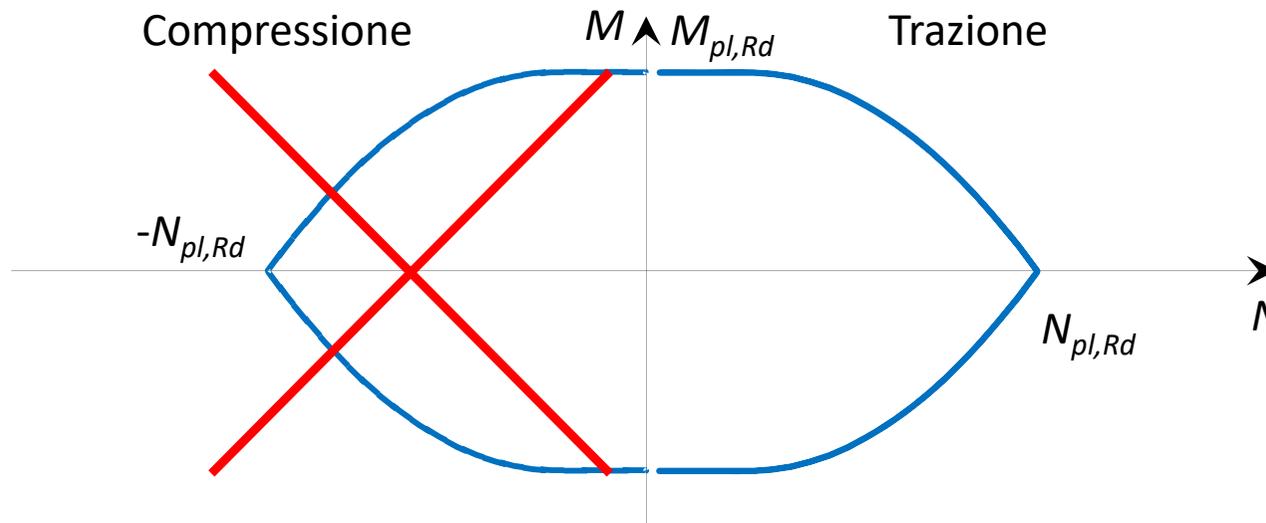
# Pressoflessione

# Domini di resistenza

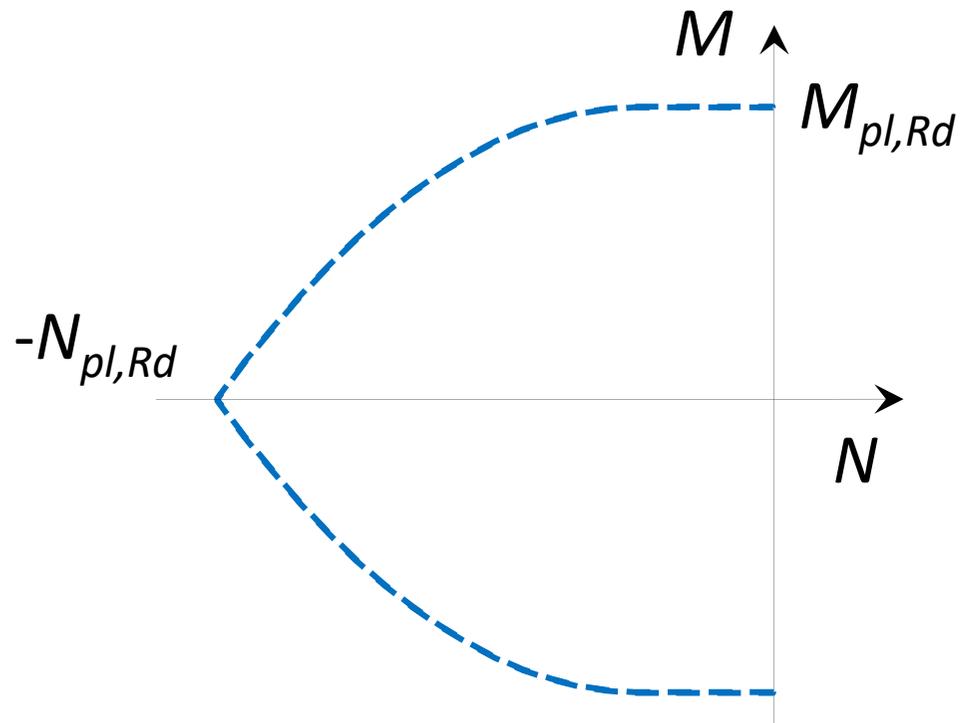
## Stato limite ultimo

Si possono ottenere semplicemente ribaltando il dominio M-N costruito nel caso di tenso-flessione?

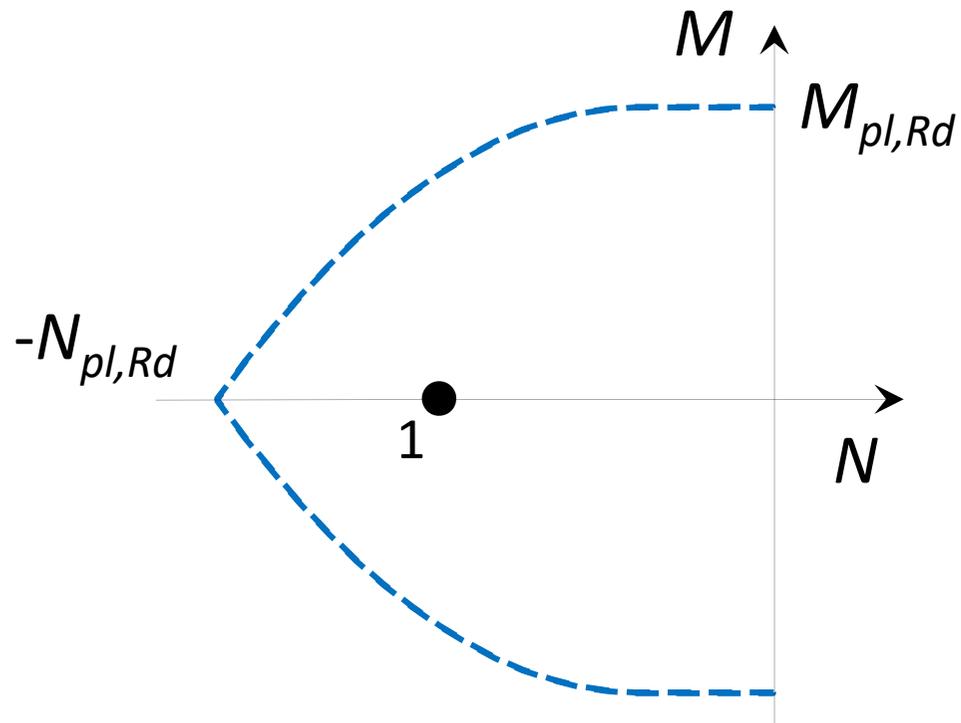
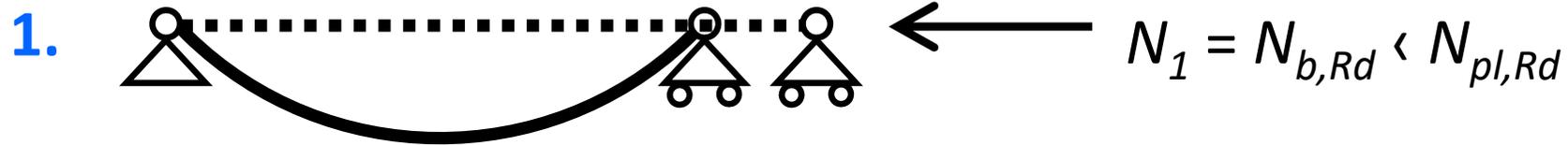
Va bene per la singola sezione,  
ma per l'asta bisogna tener conto dell'instabilità



# Costruzione dei domini di resistenza

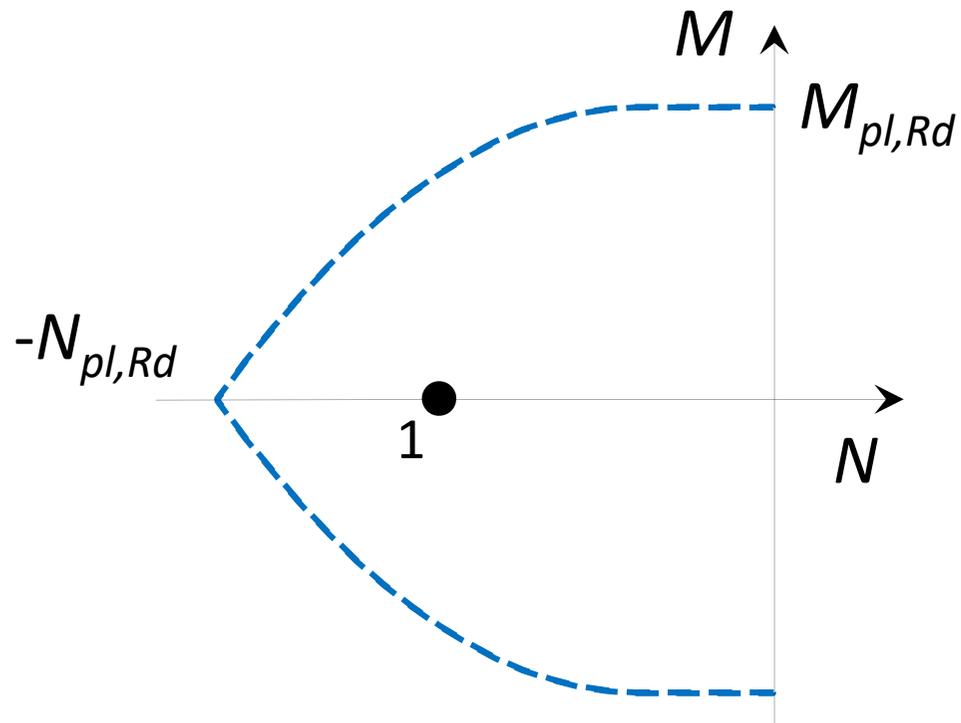


# Costruzione del domini di resistenza



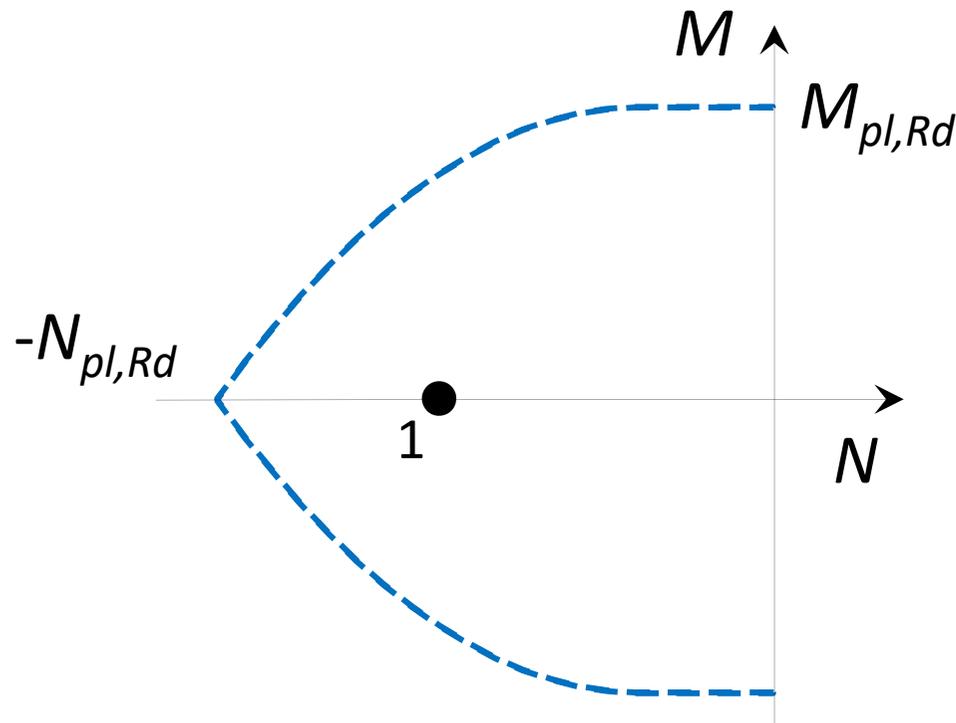
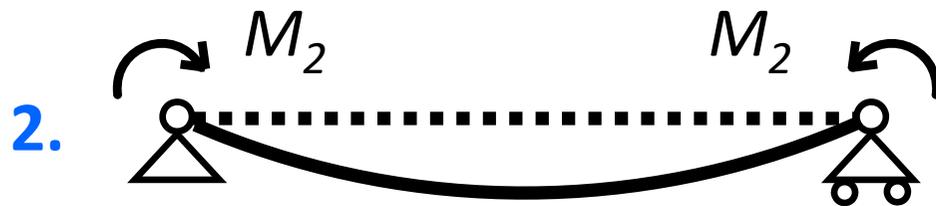
1.  $N_1 = N_{b,Rd}, M_1 = 0$

# Costruzione del domini di resistenza



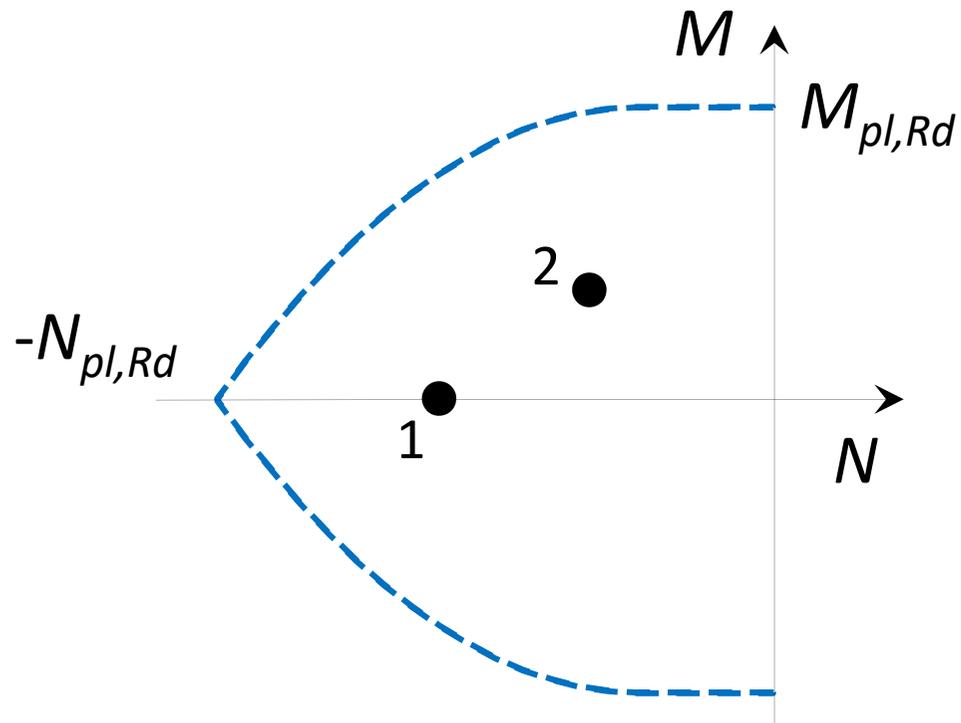
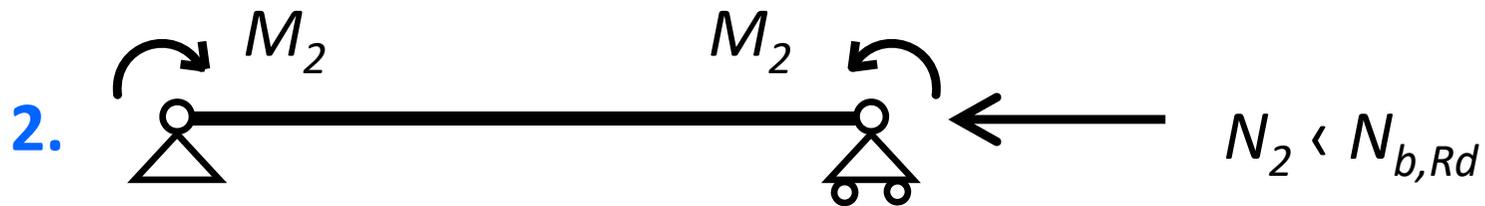
1.  $N_1 = N_{b,Rd}, M_1 = 0$

# Costruzione del domini di resistenza



1.  $N_1 = N_{b,Rd}, M_1 = 0$

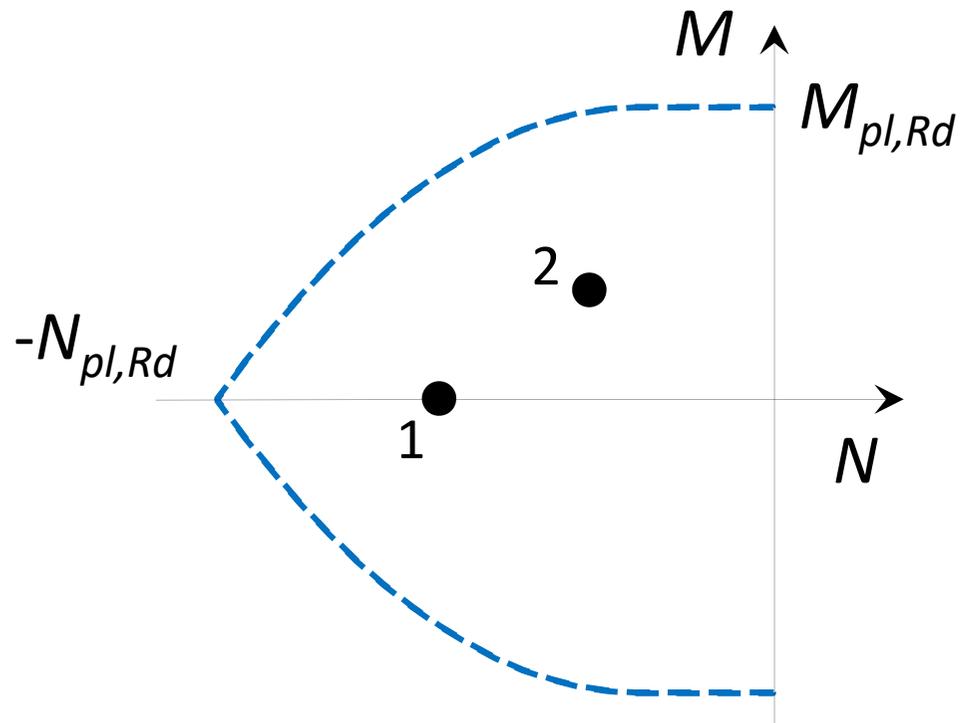
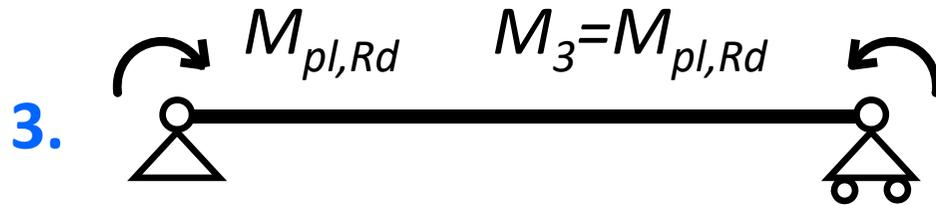
# Costruzione del domini di resistenza



1.  $N_1 = N_{b,Rd}, M_1 = 0$

2.  $N_2 < N_{b,Rd}, M_2 < M_{pl,Rd}$

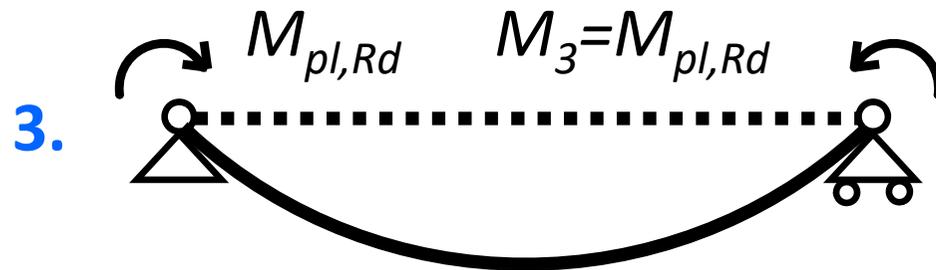
# Costruzione del domini di resistenza



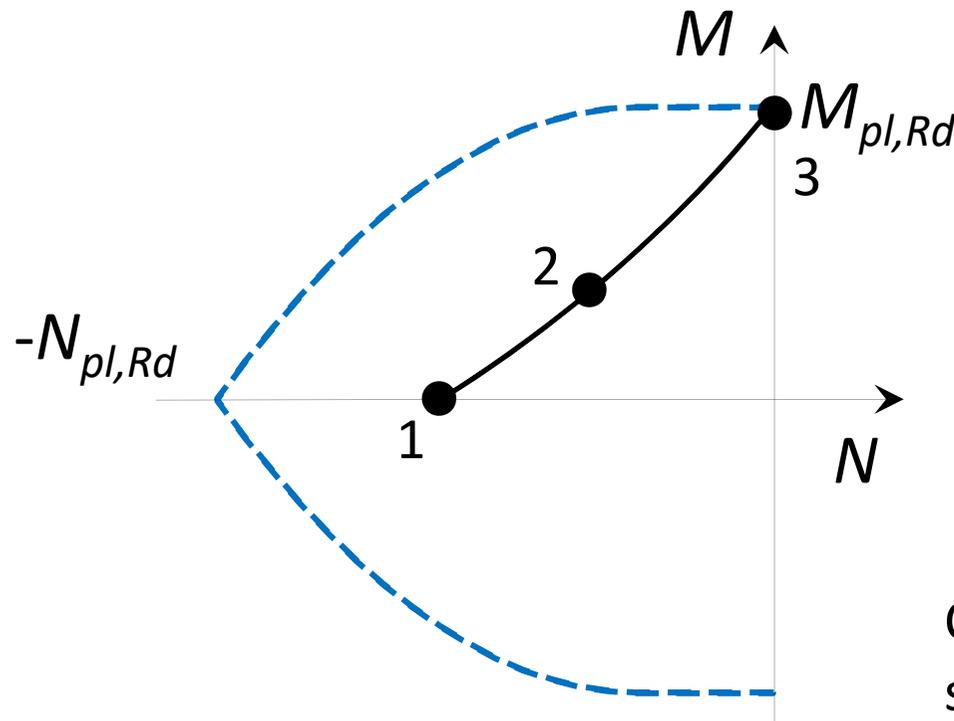
1.  $N_1 = N_{b,Rd}$ ,  $M_1 = 0$

2.  $N_2 < N_{b,Rd}$ ,  $M_2 < M_{pl,Rd}$

# Costruzione del domini di resistenza



L'asta si plasticizza e collassa in assenza di sforzo normale



**1.**  $N_1 = N_{b,Rd}, M_1 = 0$

**2.**  $N_2 < N_{b,Rd}, M_2 < M_{pl,Rd}$

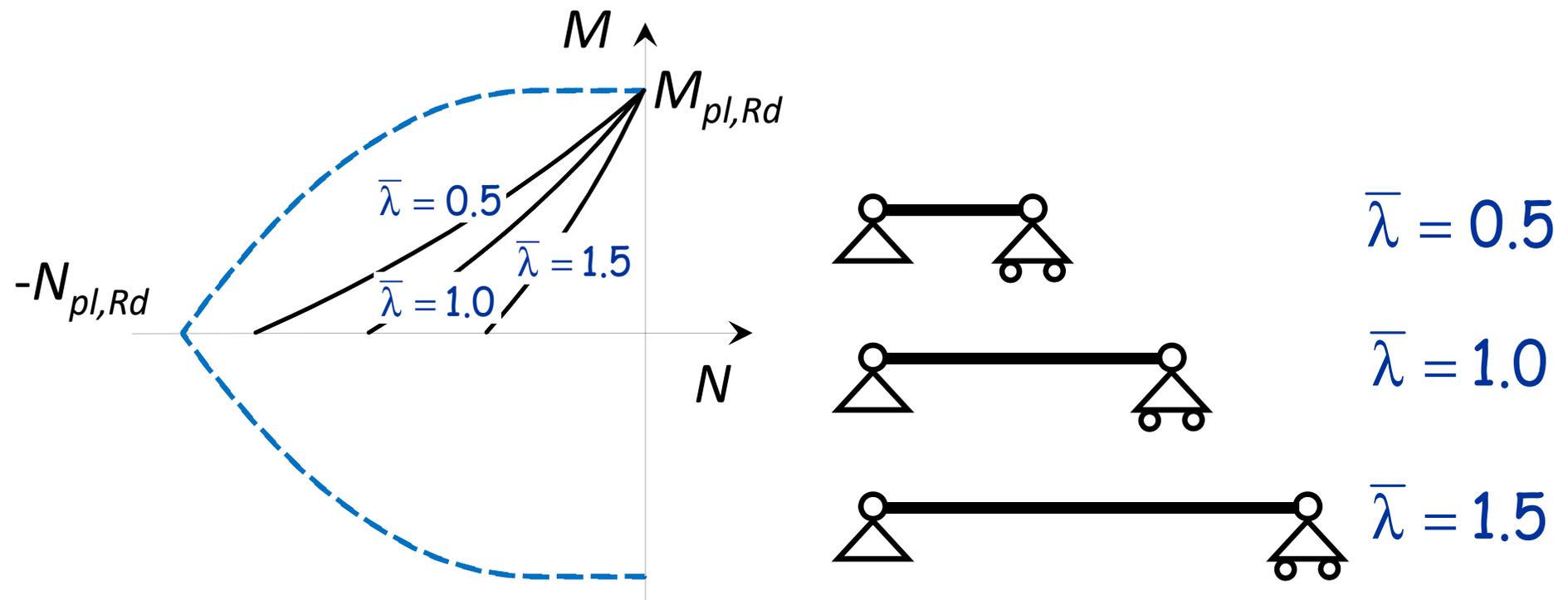
**3.**  $N_3 = 0, M_3 = M_{pl,Rd}$

Collegando i punti si ottiene il dominio

# Influenza della snellezza

Il dominio dipende dalla snellezza dell'asta:

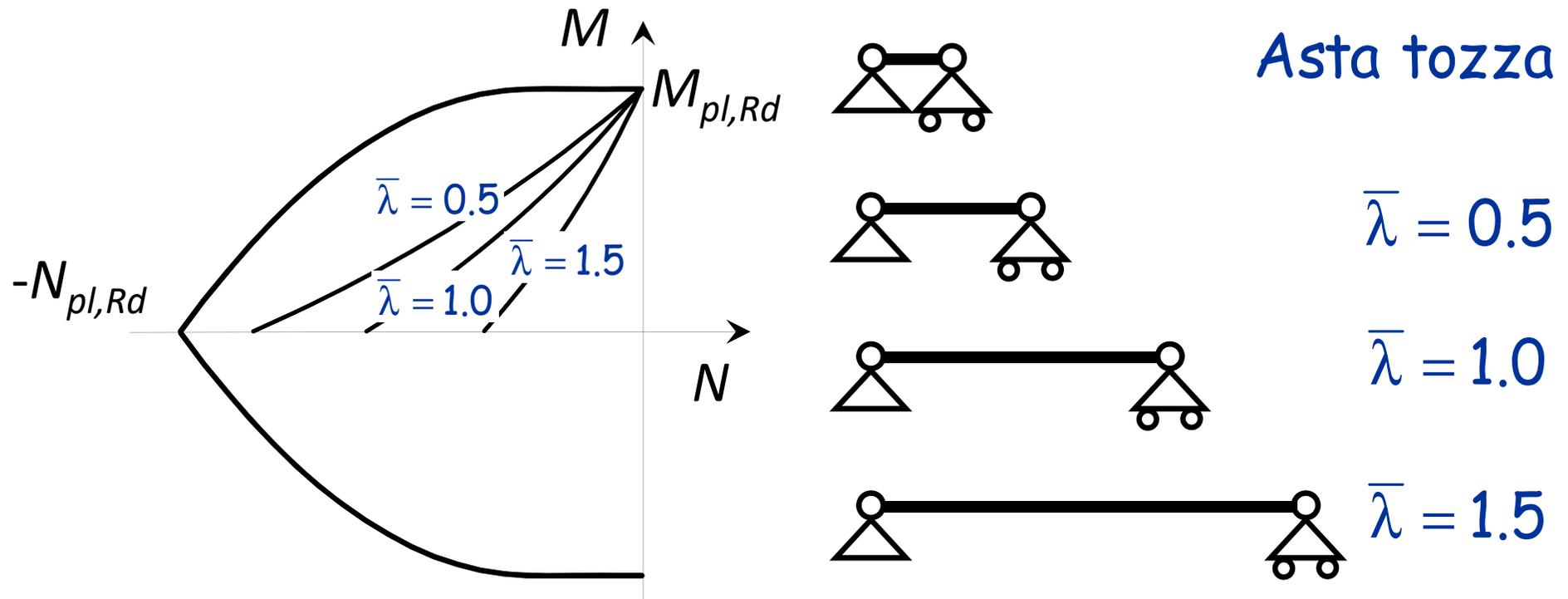
- L'ampiezza del dominio si riduce all'aumentare della snellezza;



# Influenza della snellezza

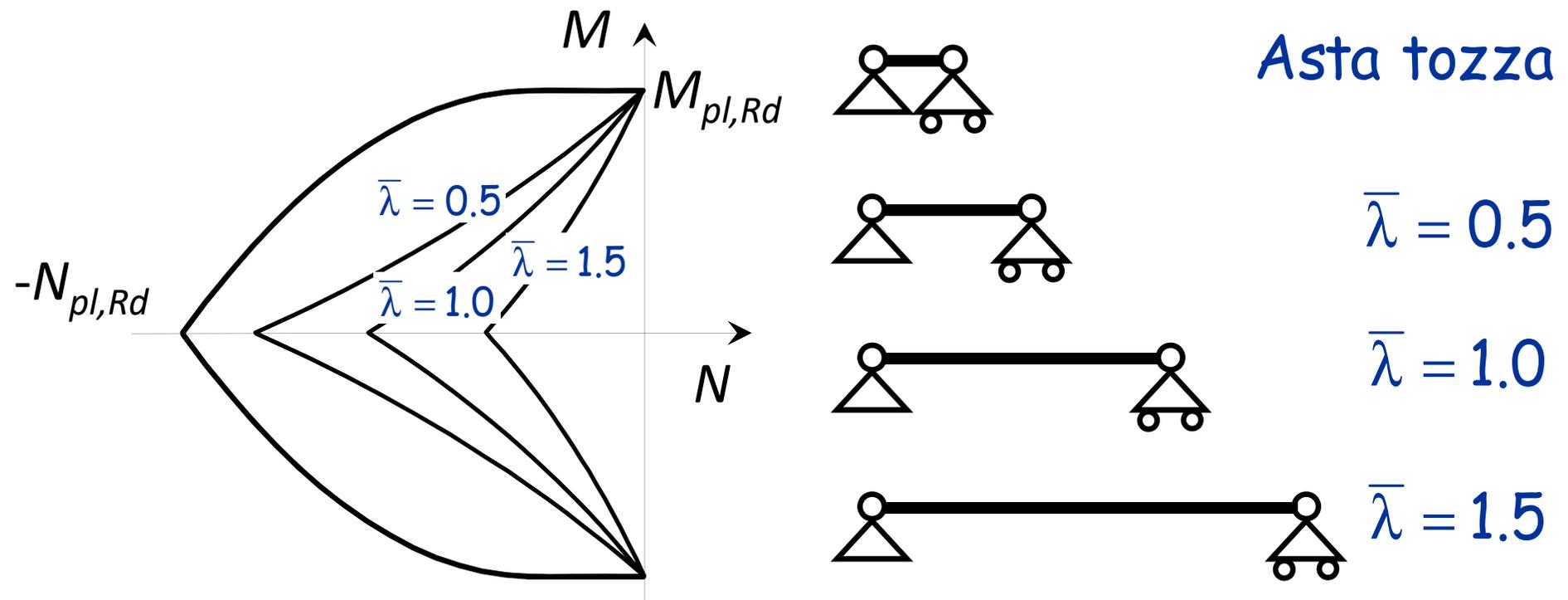
Il dominio dipende dalla snellezza dell'asta:

- Nel caso di aste tozze coincide con quello per presso-flessione della sezione

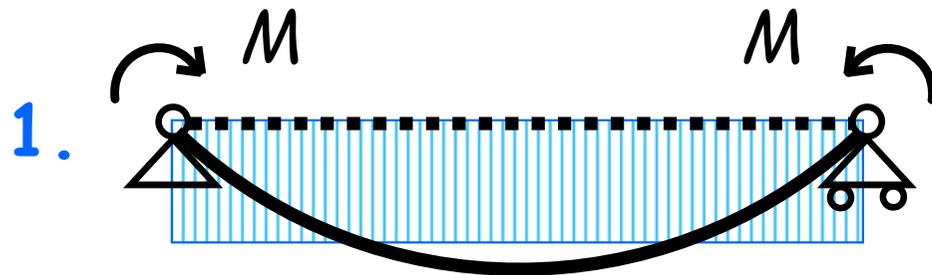


# Influenza della snellezza

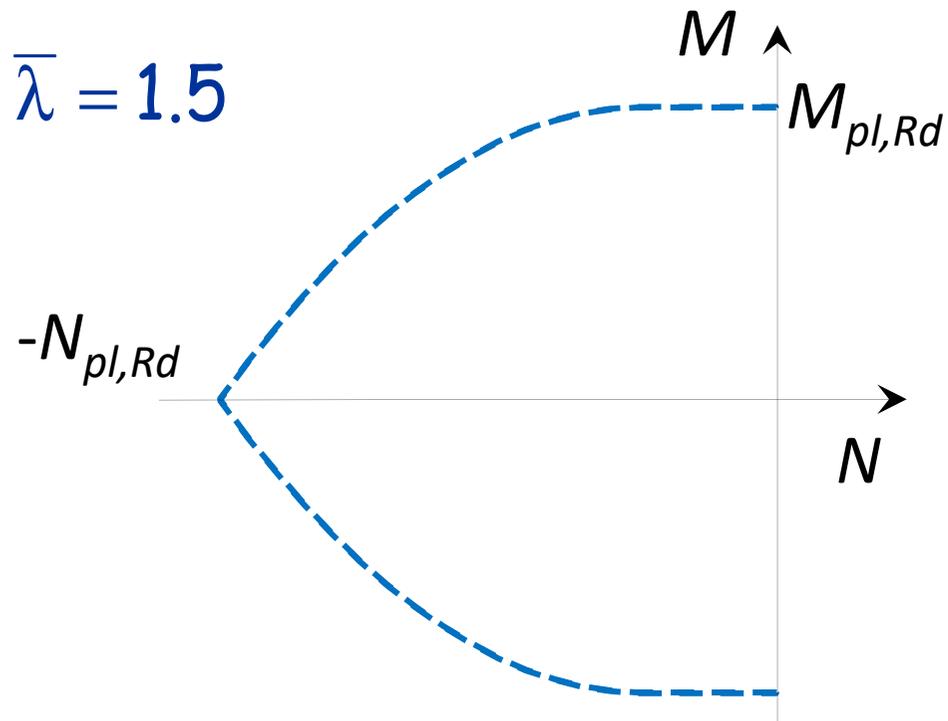
Il dominio è simmetrico  
per aste con sezione trasversale simmetrica



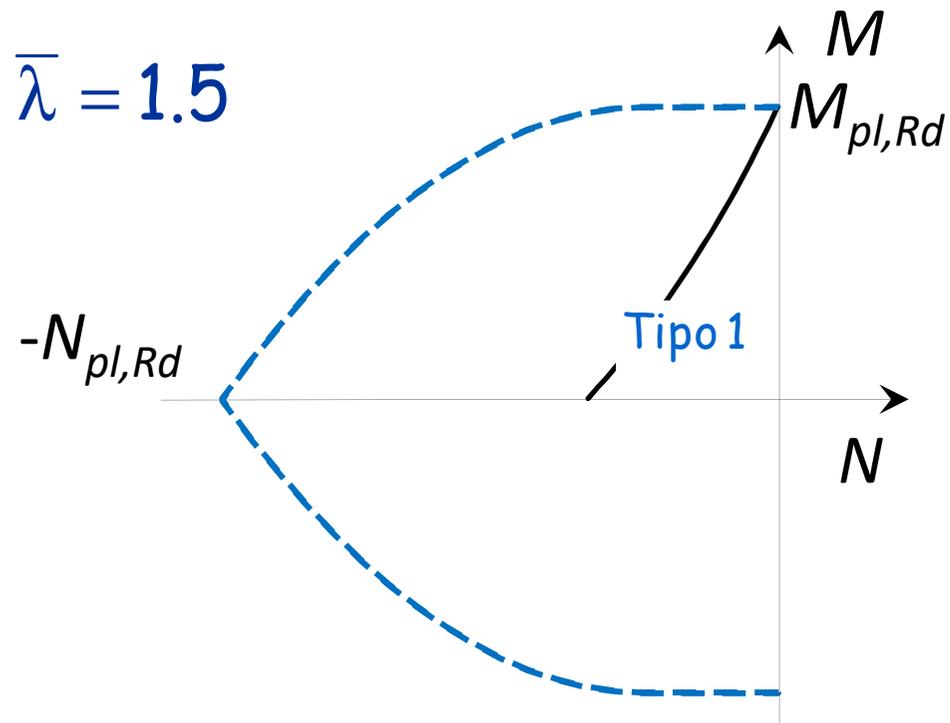
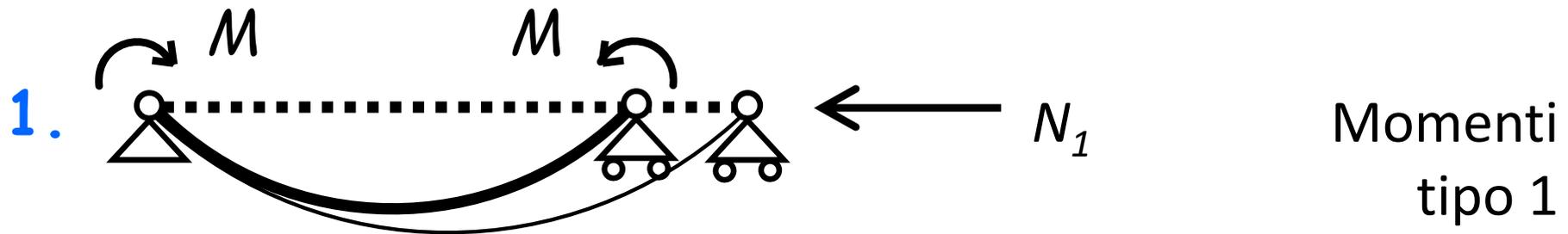
# Influenza del diagramma del momento



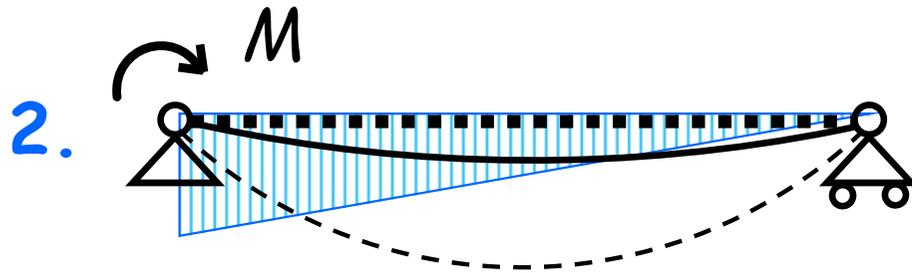
Distribuzione  
di momenti tipo 1



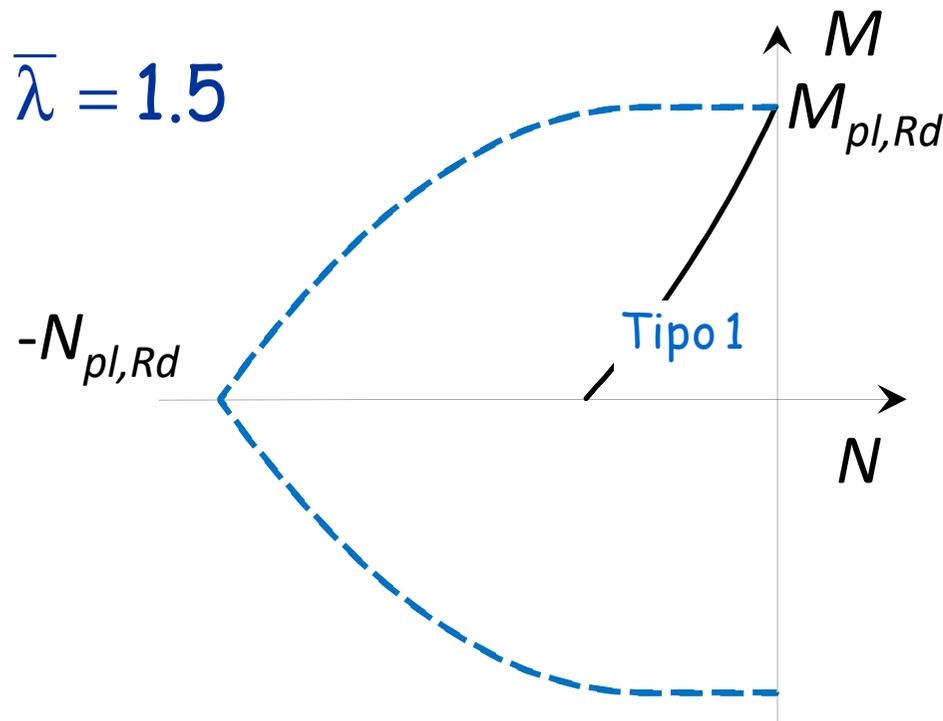
# Influenza del diagramma del momento



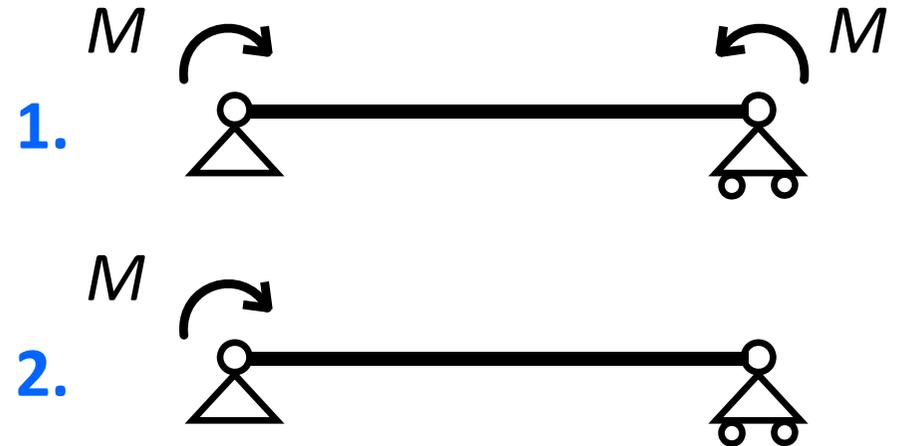
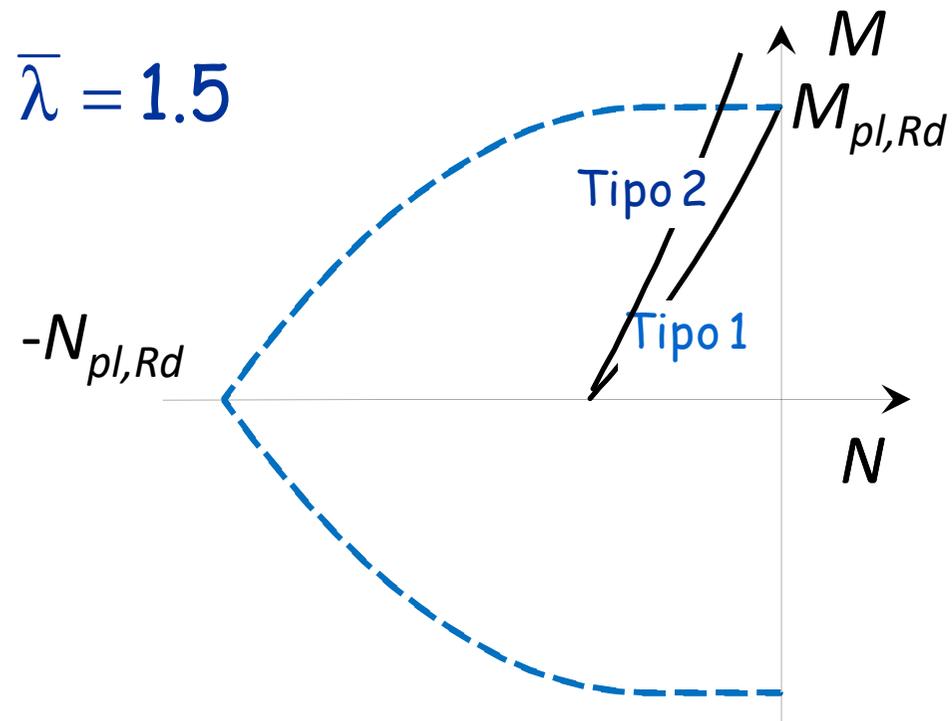
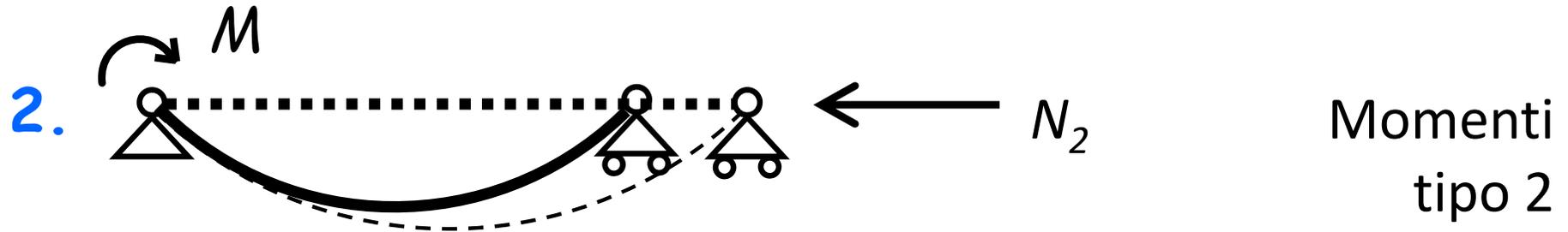
# Influenza del diagramma del momento



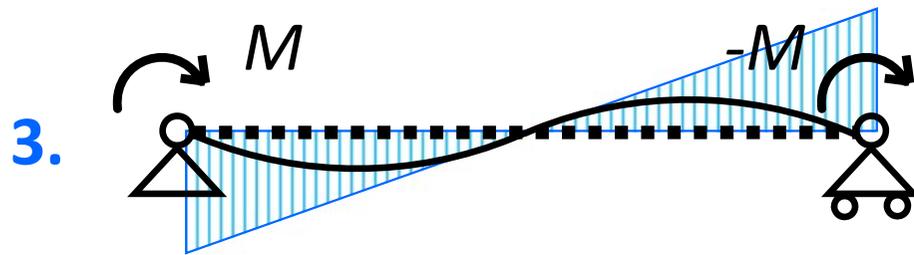
Distribuzione  
di momenti tipo 2



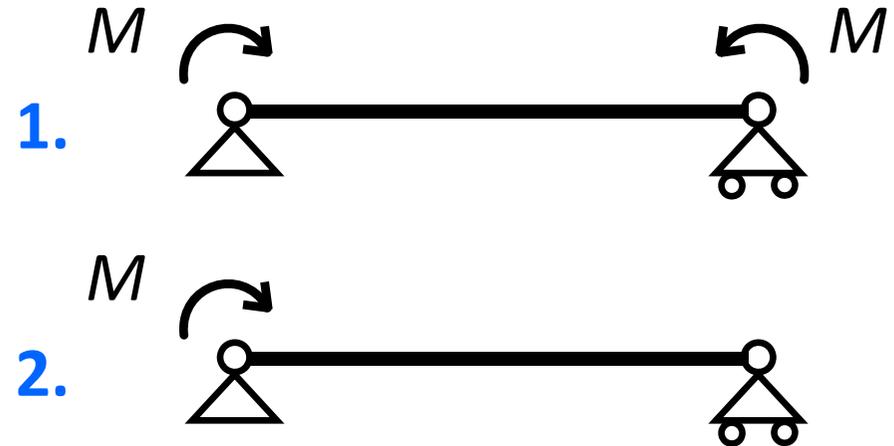
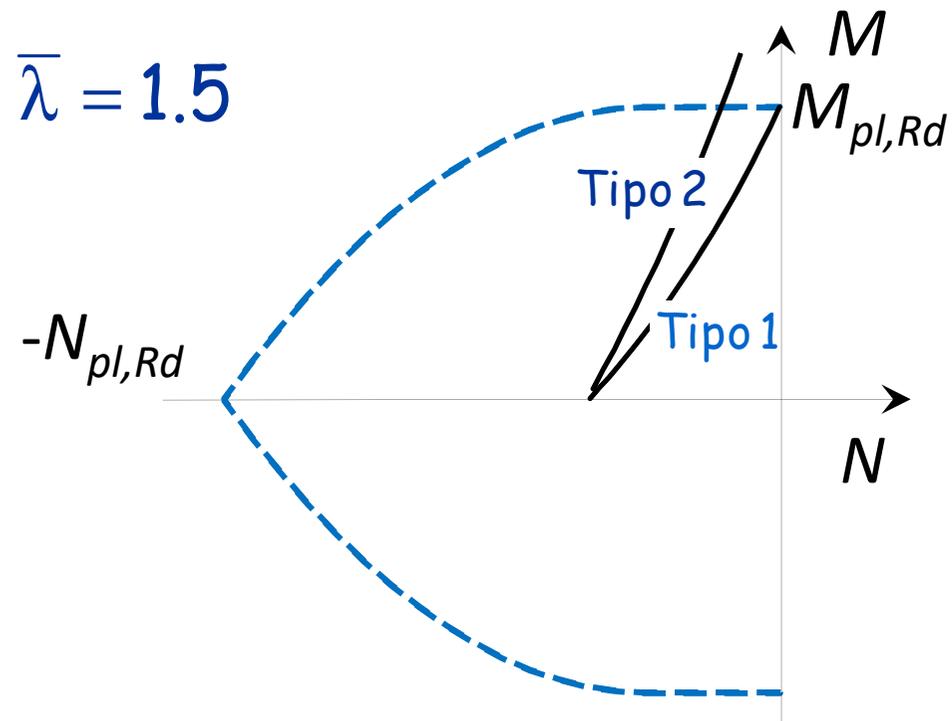
# Influenza del diagramma del momento



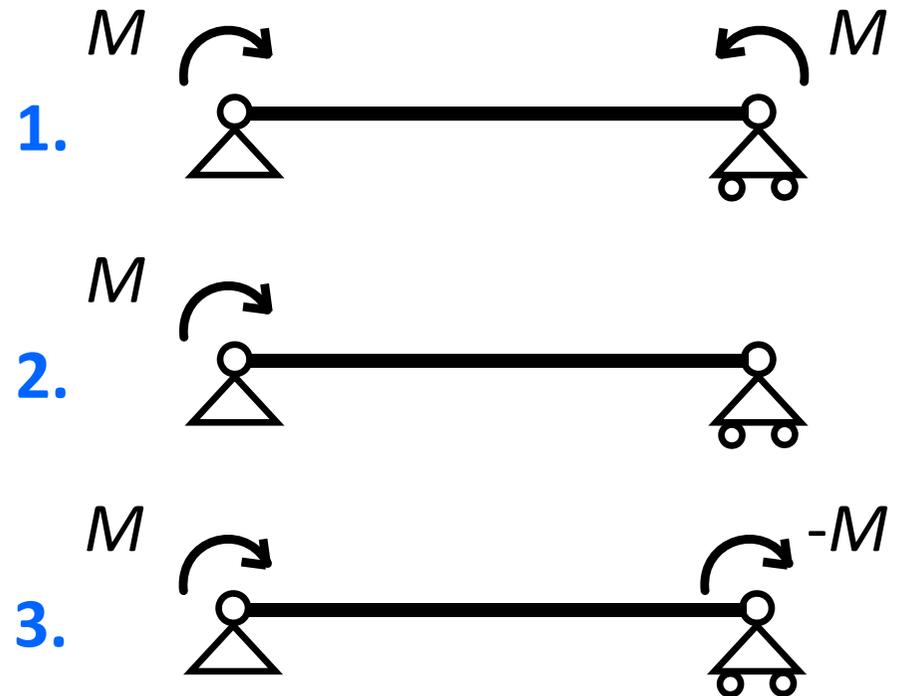
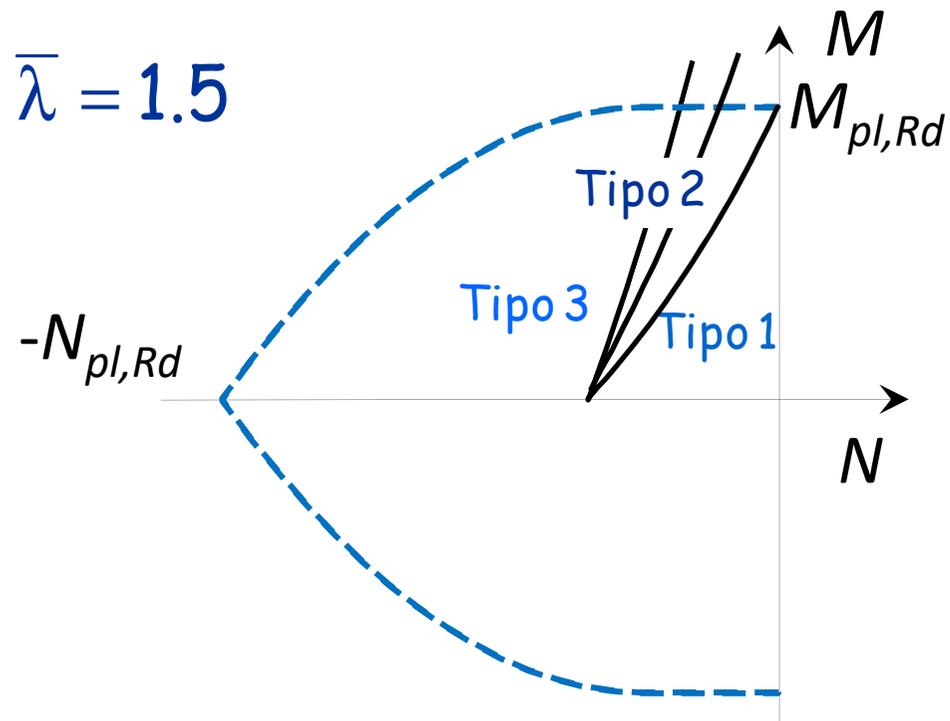
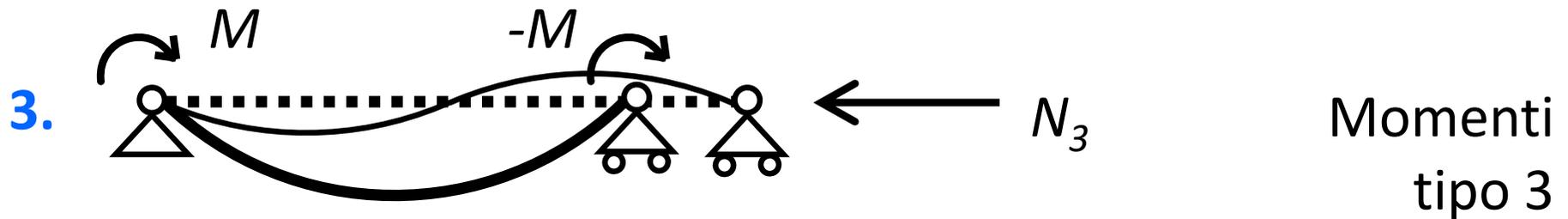
# Influenza del diagramma del momento



Distribuzione  
di momenti tipo 3



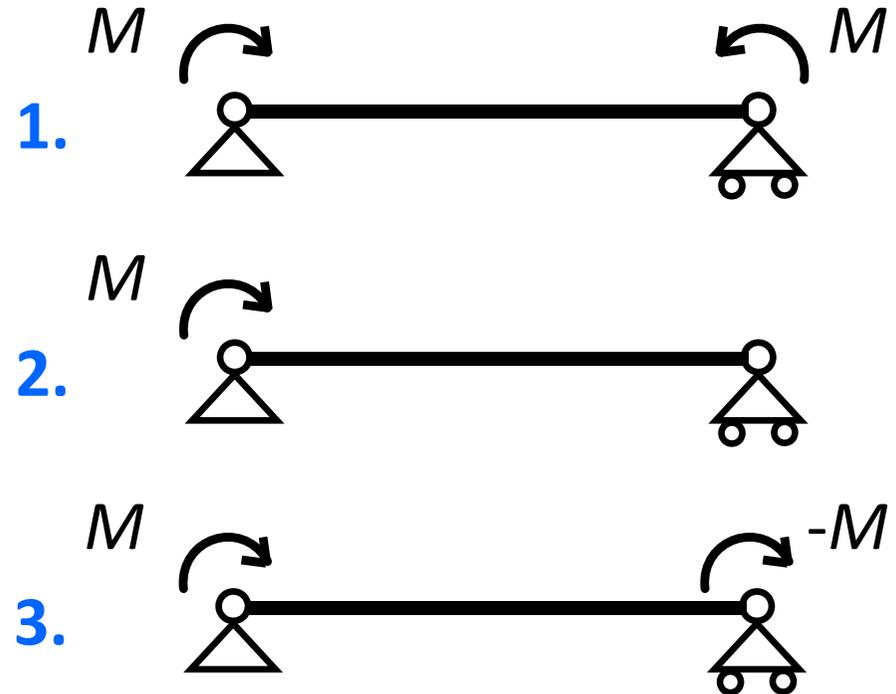
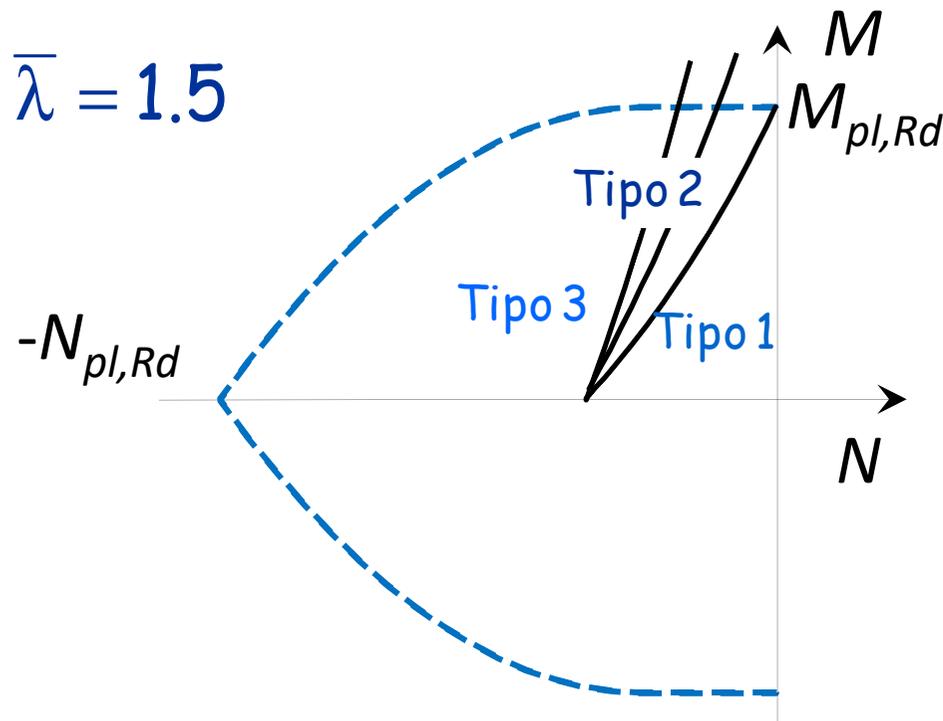
# Influenza del diagramma del momento



# Influenza del diagramma del momento

Il dominio dipende dal diagramma del momento flettente :

- L'ampiezza del dominio aumenta passando dal diagramma di tipo 1 a quello di tipo 3.



# Verifica di resistenza

## Stato limite ultimo

E' possibile usare un approccio semplificato.

Se non vi è rischio di instabilità flessione-torsionale, occorre controllare che:

$$\frac{N_{Ed} \gamma_{M1}}{\chi_{\min} f_{yk} A} + \frac{M_{x,eq,Ed} \gamma_{M1}}{f_{yk} W_x \left(1 - \frac{N_{Ed}}{N_{cr,x}}\right)} + \frac{M_{y,eq,Ed} \gamma_{M1}}{f_{yk} W_y \left(1 - \frac{N_{Ed}}{N_{cr,y}}\right)} \leq 1$$

o, in sostanza:

$$\frac{N_{Ed}}{N_{bRd}} + \frac{M_{x,eq,Ed}}{M_{x,Rd} \left(1 - \frac{N_{Ed}}{N_{cr,x}}\right)} + \frac{M_{y,eq,Ed}}{M_{y,Rd} \left(1 - \frac{N_{Ed}}{N_{cr,y}}\right)} \leq 1$$

$N_{cr}$  è il carico critico Euleriano

# Verifica di resistenza

## Stato limite ultimo

- Il momento equivalente  $M_{eq,Ed}$  tiene conto della variazione del momento nell'asta e può essere preso pari a

$$M_{eq,Ed} = 1.3 M_{m,Ed}$$

dove:

$M_{m,Ed}$  è il valore medio del momento nell'asta

ed essendo comunque  $0.75 M_{max,Ed} \leq M_{eq,Ed} \leq M_{max,Ed}$

# Verifica di resistenza

## Stato limite ultimo

- Per asta vincolata agli estremi con momento variabile linearmente si può assumere

$$M_{eq,Ed} = 0.6 M_a - 0.4 M_b$$

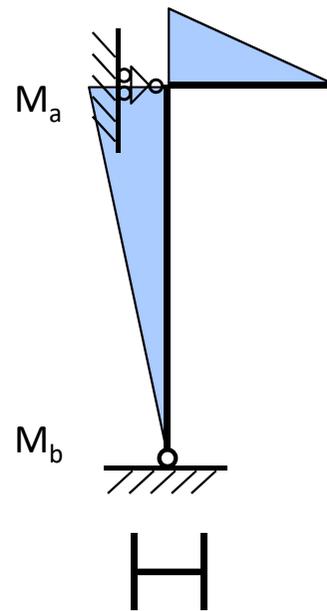
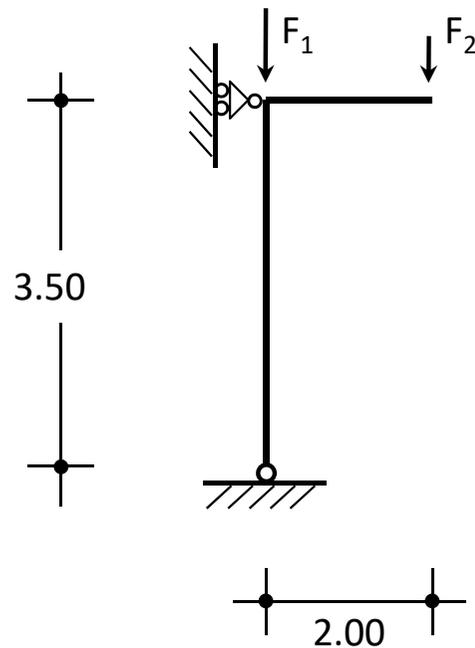
dove:

$M_a$  è il massimo tra i due ed il segno si riferisce al verso della coppia  $M$   
(se  $M_a = -M_b$  il diagramma di  $M$  è costante e  $M_{eq,Ed} = M_a$ )

ed essendo comunque  $M_{eq,Ed} \geq 0.4 M_a$

# Verifica di resistenza

## Esempio



$$F_1 = 800 \text{ kN}$$

$$F_2 = 60 \text{ kN}$$

$$M_a = 120 \text{ kNm}$$

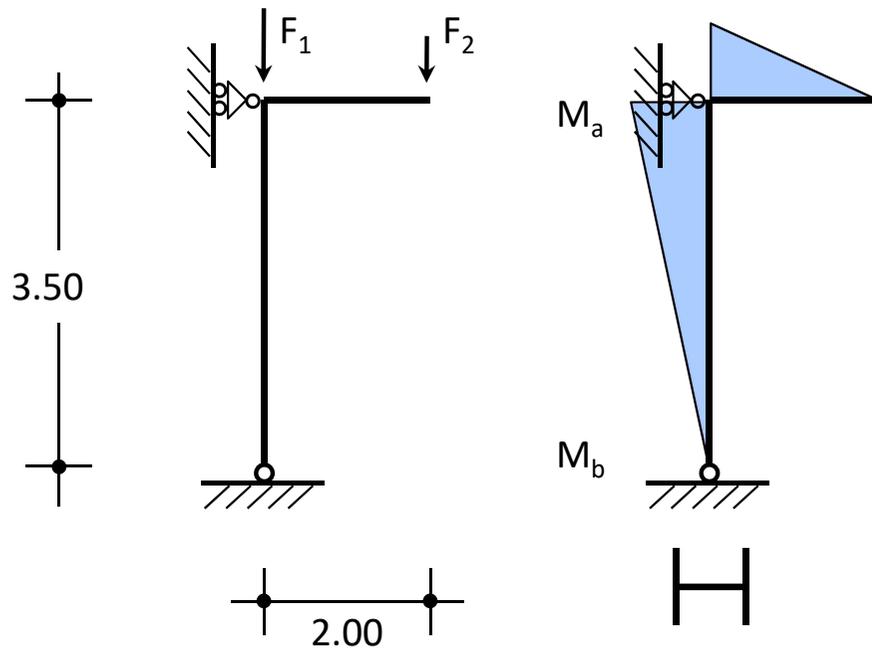
$$M_b = 0 \text{ kN}$$

$$N_{Ed} = -860 \text{ kN}$$

$$M_{eq,Ed} = 0.6 \times 120 - 0.4 \times 0 = 72 \text{ kNm}$$

# Verifica di resistenza

## Esempio



Sezione

HEB300 (S235)

A

149 cm<sup>2</sup>

W<sub>pl</sub>

1868 cm<sup>3</sup>

$$M_{pl,Rd} = 418.1 \text{ kNm}$$

$$N_{bRd} = 2826.8 \text{ kN}$$



instabilità intorno a y

$$l_0 = 3.50 \text{ m}$$

$$N_{cr} = \frac{\pi^2 E I_x}{l_0^2}$$

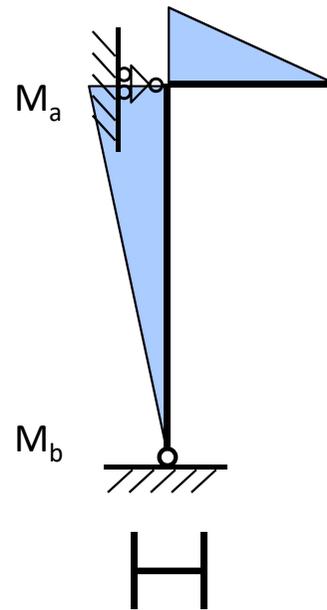
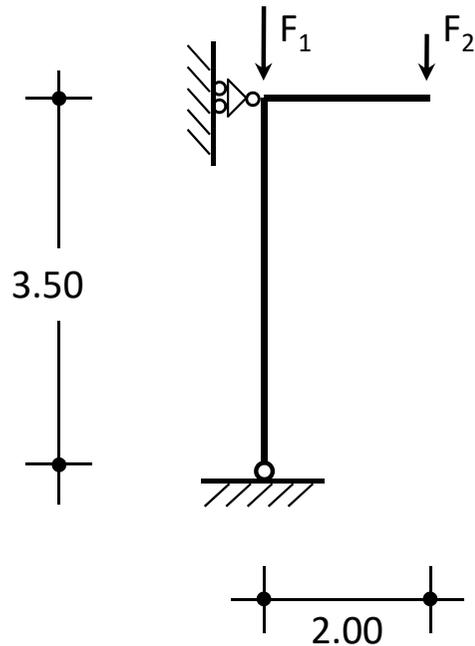


instabilità intorno a x

$$N_{cr} = 42579 \text{ kN}$$

# Verifica di resistenza

## Esempio



Sezione

HEB300 (S235)

A

149 cm<sup>2</sup>

W<sub>pl</sub>

1868 cm<sup>3</sup>

M<sub>pl,Rd</sub> = 418.1 kNm

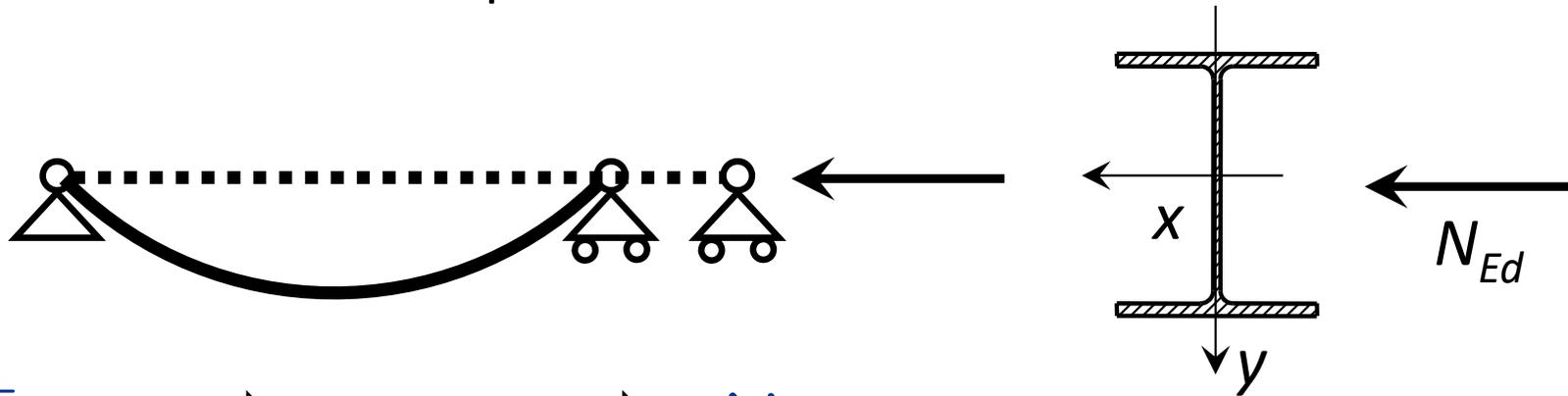
N<sub>bRd</sub> = 2826.8 kN

N<sub>bRd,x</sub> = 3231.2 kN

$$\frac{N_{Ed}}{N_{bRd}} + \frac{M_{x,eq,Ed}}{M_{x,Rd} \left(1 - \frac{N_{Ed}}{N_{cr,x}}\right)} = \frac{860}{2826.8} + \frac{72}{418.1 \left(1 - \frac{860}{42579}\right)} = 0.304 + 0.176 = 0.480 \leq 1$$

# Considerazioni

L'asta si instabilizza nel piano di minor resistenza



$$\bar{\lambda}_x \Rightarrow \chi_x \Rightarrow N_{b,Rd,x}$$

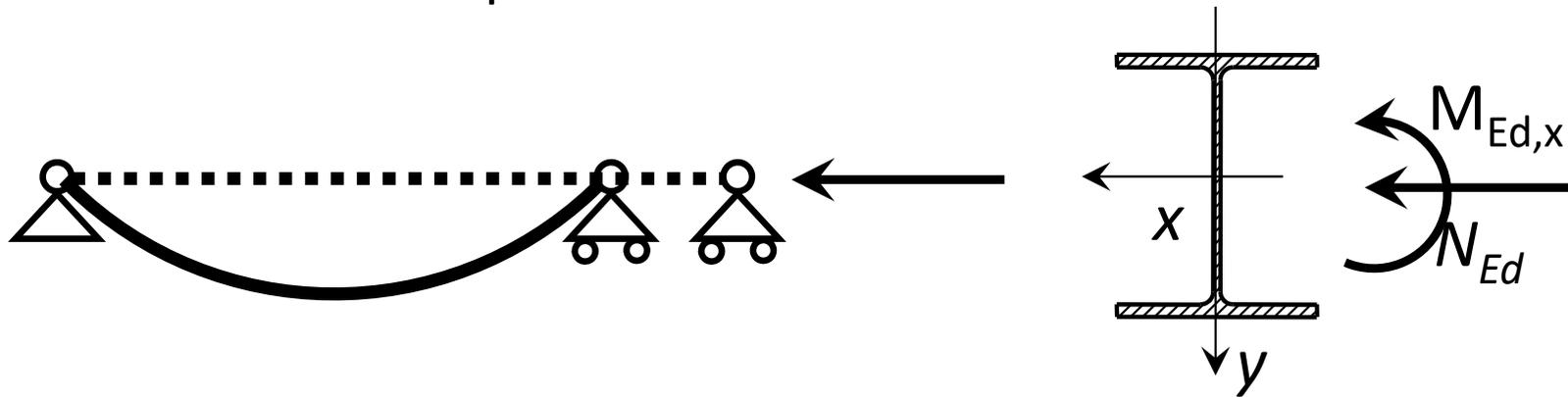
$$\bar{\lambda}_y \Rightarrow \chi_y \Rightarrow N_{b,Rd,y}$$

$$\text{Se } \bar{\lambda}_y > \bar{\lambda}_x \Rightarrow N_{b,Rd,y} < N_{b,Rd,x} \Rightarrow N_{b,Rd} = N_{b,Rd,y}$$

L'instabilità si verifica nel piano ortogonale all'asse y

# Considerazioni

L'asta si instabilizza nel piano di minor resistenza



$$\bar{\lambda}_y > \bar{\lambda}_x \quad \Rightarrow \quad N_{b,Rd,y} < N_{b,Rd,x} \quad \Rightarrow \quad N_{b,Rd} = N_{b,Rd,y}$$

L'instabilità si verifica nel piano ortogonale all'asse y

La presenza di un momento  $M_{Edx}$  favorisce l'instabilità dell'asta quanto un momento  $M_{Edy}$ ? **Probabilmente NO**

# Verifica di resistenza

## Stato limite ultimo

- In alternativa, occorre controllare che:

$$\frac{N_{Ed} \gamma_{M1}}{\chi_x A f_{yk}} + k_{xx} \frac{M_{Ed,x} \gamma_{M1}}{W_x f_{yk}} + k_{xy} \frac{M_{Ed,y} \gamma_{M1}}{W_y f_{yk}} \leq 1$$

$$\frac{N_{Ed} \gamma_{M1}}{\chi_y A f_{yk}} + k_{yx} \frac{M_{Ed,x} \gamma_{M1}}{W_x f_{yk}} + k_{yy} \frac{M_{Ed,y} \gamma_{M1}}{W_y f_{yk}} \leq 1$$

# Verifica di resistenza

## Stato limite ultimo

- In alternativa, occorre controllare che:

$$\frac{N_{Ed} \gamma_{M1}}{\chi_x A f_{yk}} + k_{xx} \frac{M_{Ed,x} \gamma_{M1}}{W_x f_{yk}} + k_{xy} \frac{M_{Ed,y} \gamma_{M1}}{W_y f_{yk}} \leq 1$$

$$\frac{N_{Ed} \gamma_{M1}}{\chi_y A f_{yk}} + k_{yx} \frac{M_{Ed,x} \gamma_{M1}}{W_x f_{yk}} + k_{yy} \frac{M_{Ed,y} \gamma_{M1}}{W_y f_{yk}} \leq 1$$

espressioni analoghe a quella del Metodo A se

$$\frac{M_{eq,Ed}}{\left(1 - \frac{N_{Ed}}{N_{cr}}\right)} = k M_{Ed}$$

# Verifica di resistenza

## Stato limite ultimo

- In alternativa, occorre controllare che:

$$\frac{N_{Ed} \gamma_{M1}}{\chi_x A f_{yk}} + k_{xx} \frac{M_{Ed,x} \gamma_{M1}}{W_x f_{yk}} + k_{xy} \frac{M_{Ed,y} \gamma_{M1}}{W_y f_{yk}} \leq 1$$

$$\frac{N_{Ed} \gamma_{M1}}{\chi_y A f_{yk}} + k_{yx} \frac{M_{Ed,x} \gamma_{M1}}{W_x f_{yk}} + k_{yy} \frac{M_{Ed,y} \gamma_{M1}}{W_y f_{yk}} \leq 1$$

1.  $N_{bRd}$  è calcolato separatamente nelle due direzioni
2. i coefficienti k sono diversi per le due direzioni

# Verifica di resistenza

## Stato limite ultimo

- In alternativa, occorre controllare che:

$$\frac{N_{Ed} \gamma_{M1}}{\chi_x A f_{yk}} + k_{xx} \frac{M_{Ed,x} \gamma_{M1}}{W_x f_{yk}} + k_{xy} \frac{M_{Ed,y} \gamma_{M1}}{W_y f_{yk}} \leq 1$$

$$\frac{N_{Ed} \gamma_{M1}}{\chi_y A f_{yk}} + k_{yx} \frac{M_{Ed,x} \gamma_{M1}}{W_x f_{yk}} + k_{yy} \frac{M_{Ed,y} \gamma_{M1}}{W_y f_{yk}} \leq 1$$

Le espressioni per il calcolo dei k sono complesse

$k_{xx}, k_{yy}, k_{xy}, k_{yx}$  dipendono da:  $\left\{ \begin{array}{l} - \text{snellezza dell'asta} \\ - \text{diagramma del momento} \\ - \text{sforzo normale} \end{array} \right.$

# Fattori d'interazione $k_{xx}$ , $k_{yy}$ , $k_{xy}$ , $k_{yx}$

Sezioni a doppio T di classe 1 e 2

$$k_{xx} = C_{mx} \left( 1 + (\bar{\lambda}_x - 0.2) \frac{N_{Ed}}{N_{b,Rd,x}} \right) \leq C_{mx} \left( 1 + 0.8 \frac{N_{Ed}}{N_{b,Rd,x}} \right)$$

$$k_{xy} = 0.6 k_{yy}$$

$$k_{yx} = 0.6 k_{xx}$$

$$k_{yy} = C_{my} \left( 1 + (2\bar{\lambda}_y - 0.6) \frac{N_{Ed}}{N_{b,Rd,y}} \right) \leq C_{my} \left( 1 + 1.4 \frac{N_{Ed}}{N_{b,Rd,y}} \right)$$

Per presso-flessione retta con  $M_{Ed,x} \neq 0$  si assume  $k_{yx} = 0$

**Table B.1: Interaction factors  $k_{ij}$  for members not susceptible to torsional deformations**

Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
$k_{xx}$	I-sections RHS-sections	$C_{,mx} \left( 1 + 0,6 \bar{\lambda}_x \frac{N_{Ed}}{\chi_x N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{,mx} \left( 1 + 0,6 \frac{N_{Ed}}{\chi_x N_{Rk} / \gamma_{M1}} \right)$	$C_{,mx} \left( 1 + (\bar{\lambda}_x - 0,2) \frac{N_{Ed}}{\chi_x N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{,mx} \left( 1 + 0,8 \frac{N_{Ed}}{\chi_x N_{Rk} / \gamma_{M1}} \right)$
$k_{xy}$	I-sections RHS-sections	$k_{yy}$	$0,6 k_{yy}$
$k_{yx}$	I-sections RHS-sections	$0,8 k_{xx}$	$0,6 k_{xx}$
$k_{yy}$	I-sections	$C_{,my} \left( 1 + 0,6 \bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{,my} \left( 1 + 0,6 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$	$C_{,my} \left( 1 + (2\bar{\lambda}_y - 0,6) \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{,my} \left( 1 + 1,4 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$
	RHS-sections	$C_{,my} \left( 1 + 0,6 \bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{,my} \left( 1 + 0,6 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$	$C_{,my} \left( 1 + (\bar{\lambda}_y - 0,2) \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{,my} \left( 1 + 0,8 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$

For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending  $M_{x,Ed}$  the coefficient  $k_{yx}$  may be  $k_{yx} = 0$ .

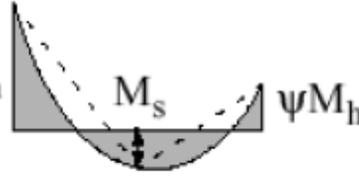
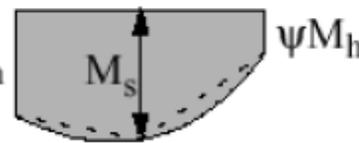
**Table B.1: Interaction factors  $k_{ij}$  for members not susceptible to torsional deformations**

Interaction factors	Type of sections	Design assumptions	
		elastic cross-sectional properties class 3, class 4	plastic cross-sectional properties class 1, class 2
$k_{xx}$	I-sections RHS-sections	$C_{mx} \left( 1 + 0,6 \bar{\lambda}_x \frac{N_{Ed}}{\chi_x N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mx} \left( 1 + 0,6 \frac{N_{Ed}}{\chi_x N_{Rk} / \gamma_{M1}} \right)$	$C_{mx} \left( 1 + (\bar{\lambda}_x - 0,2) \frac{N_{Ed}}{\chi_x N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{mx} \left( 1 + 0,8 \frac{N_{Ed}}{\chi_x N_{Rk} / \gamma_{M1}} \right)$
$k_{xy}$	I-sections RHS-sections	$k_{xy}$	$0,6 k_{yy}$
$k_{yx}$	I-sections RHS-sections		$0,6 k_{xx}$
$k_{yy}$	I-sections	$C_{my} \left( 1 + 0,6 \bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left( 1 + 0,6 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$	$C_{my} \left( 1 + (\bar{\lambda}_y - 0,6) \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left( 1 + 1,4 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$
	RHS-sections		$C_{my} \left( 1 + (\bar{\lambda}_y - 0,2) \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$ $\leq C_{my} \left( 1 + 0,8 \frac{N_{Ed}}{\chi_y N_{Rk} / \gamma_{M1}} \right)$

Nota: nella Circolare è indicato con  $\alpha_m$  quello che qui (sull'EC3) è indicato con  $C_m$

For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending  $M_{x,Ed}$  the coefficient  $k_{yx}$  may be  $k_{yx} = 0$ .

# Fattori di momento uniforme equivalente $C_{mx}$ e $C_{my}$

Moment diagram	range		$C_{mx}$ and $C_{my}$ and $C_{mLT}$	
			uniform loading	concentrated load
	$-1 \leq \psi \leq 1$		$0,6 + 0,4\psi \geq 0,4$	
 $\alpha_s = M_s/M_h$	$0 \leq \alpha_s \leq 1$	$-1 \leq \psi \leq 1$	$0,2 + 0,8\alpha_s \geq 0,4$	$0,2 + 0,8\alpha_s \geq 0,4$
	$-1 \leq \alpha_s < 0$	$0 \leq \psi \leq 1$	$0,1 - 0,8\alpha_s \geq 0,4$	$-0,8\alpha_s \geq 0,4$
		$-1 \leq \psi < 0$	$0,1(1-\psi) - 0,8\alpha_s \geq 0,4$	$0,2(-\psi) - 0,8\alpha_s \geq 0,4$
 $\alpha_h = M_h/M_s$	$0 \leq \alpha_h \leq 1$	$-1 \leq \psi \leq 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
	$-1 \leq \alpha_h < 0$	$0 \leq \psi \leq 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
		$-1 \leq \psi < 0$	$0,95 + 0,05\alpha_h(1+2\psi)$	$0,90 - 0,10\alpha_h(1+2\psi)$

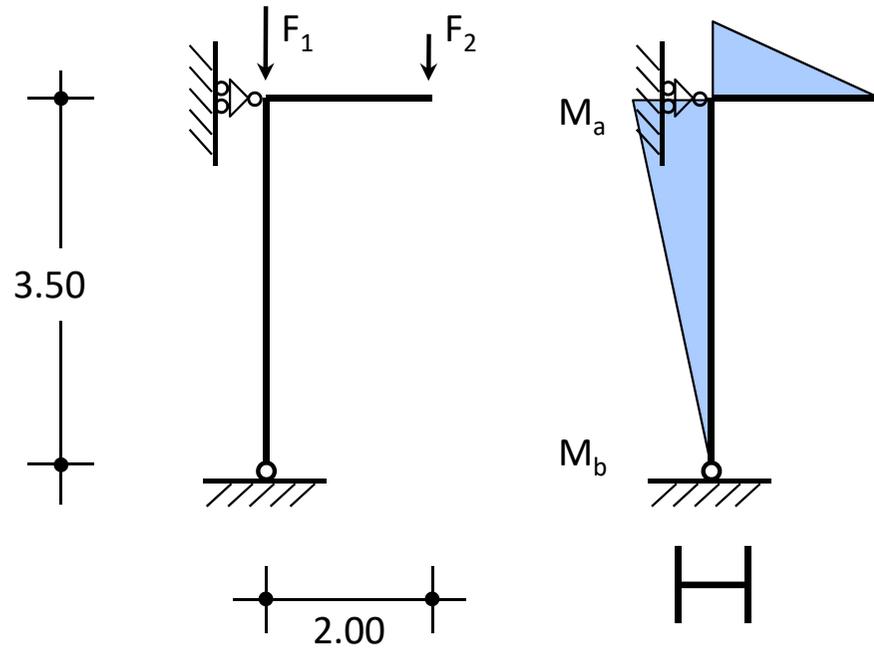
For members with sway buckling mode the equivalent uniform moment factor should be taken  $C_{mx} = 0,9$  or  $C_{my} = 0,9$  respectively.

$C_{mx}$ ,  $C_{my}$  and  $C_{mLT}$  should be obtained according to the bending moment diagram between the relevant braced points as follows:

moment factor	bending axis	points braced in direction
$C_{mx}$	x-x	y-y
$C_{my}$	y-y	x-x
$C_{mLT}$	x-x	x-x

# Verifica di resistenza

## Esempio



$$F_1 = 800 \text{ kN}$$

$$F_2 = 60 \text{ kN}$$

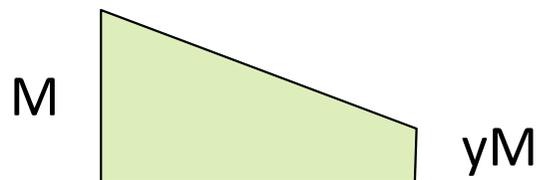
$$M_a = 120 \text{ kNm}$$

$$M_b = 0 \text{ kN}$$

$$N_{Ed} = -860 \text{ kN}$$

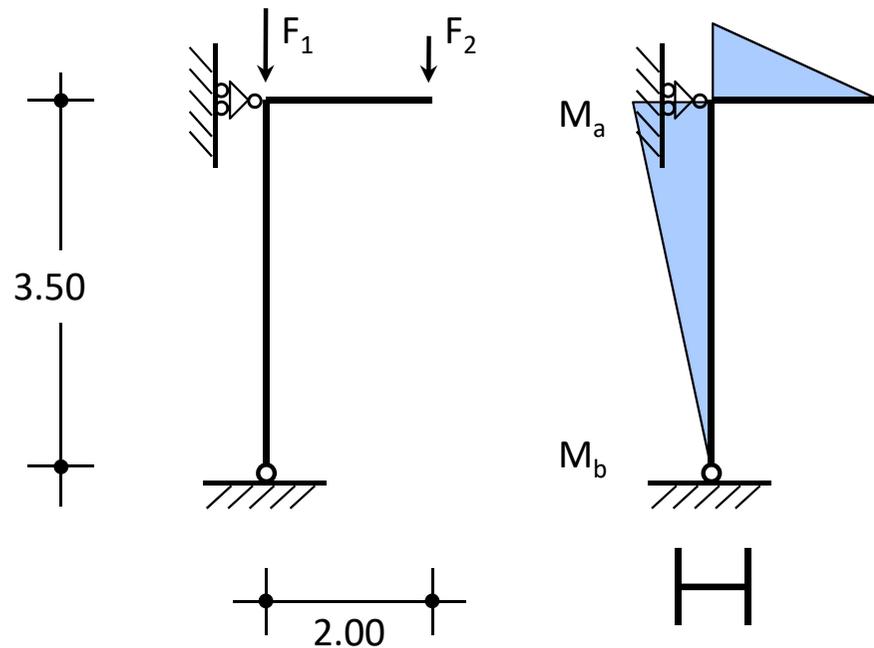
$$\psi = 0$$

$$C_{mx} = 0.6 + 0.4 \psi = 0.6$$



# Verifica di resistenza

## Esempio



$$\bar{\lambda}_x = 0.287$$

$$\chi_x = 0.9689$$

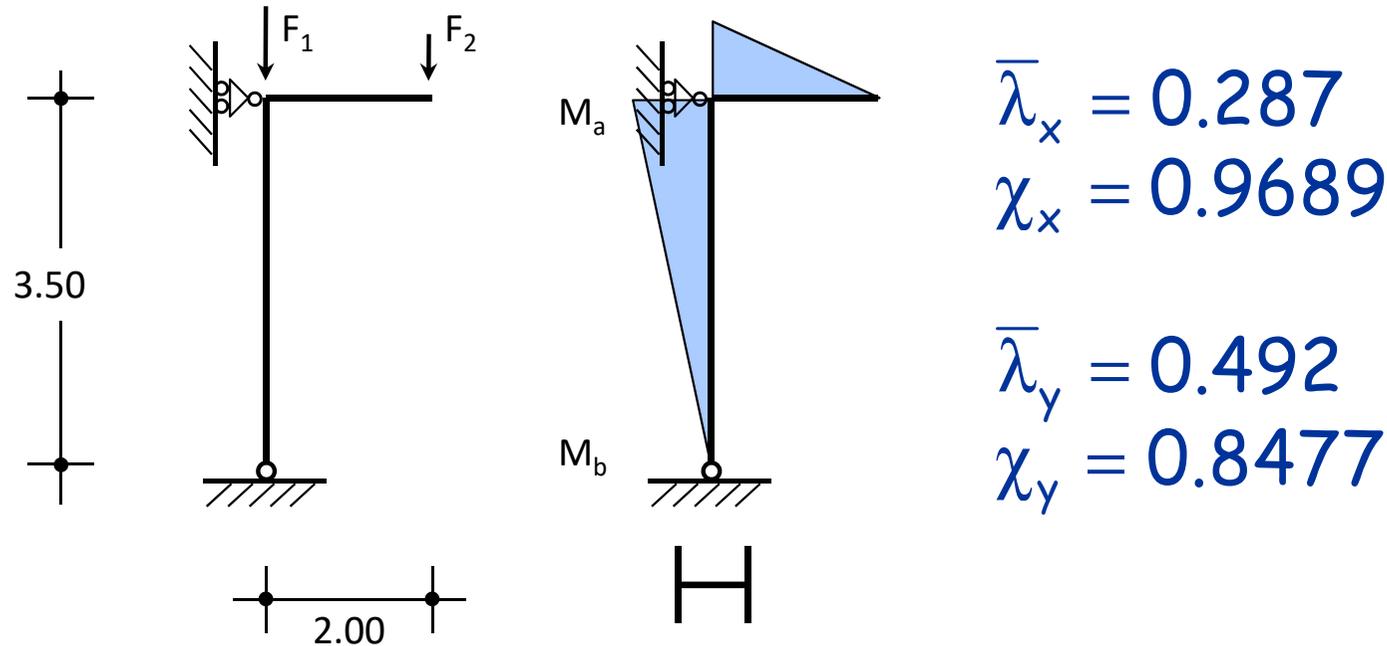
$$\bar{\lambda}_y = 0.492$$

$$\chi_y = 0.8477$$

$$k_{xx} = C_{mx} \left[ 1 + (\bar{\lambda}_x - 0.2) \frac{N_{Ed} \gamma_{M1}}{\chi_x A f_x} \right] = 0.6 \left[ 1 + (0.287 - 0.2) \frac{860}{3231.2} \right] = 0.614$$

# Verifica di resistenza

## Esempio



$$\bar{\lambda}_x = 0.287$$
$$\chi_x = 0.9689$$

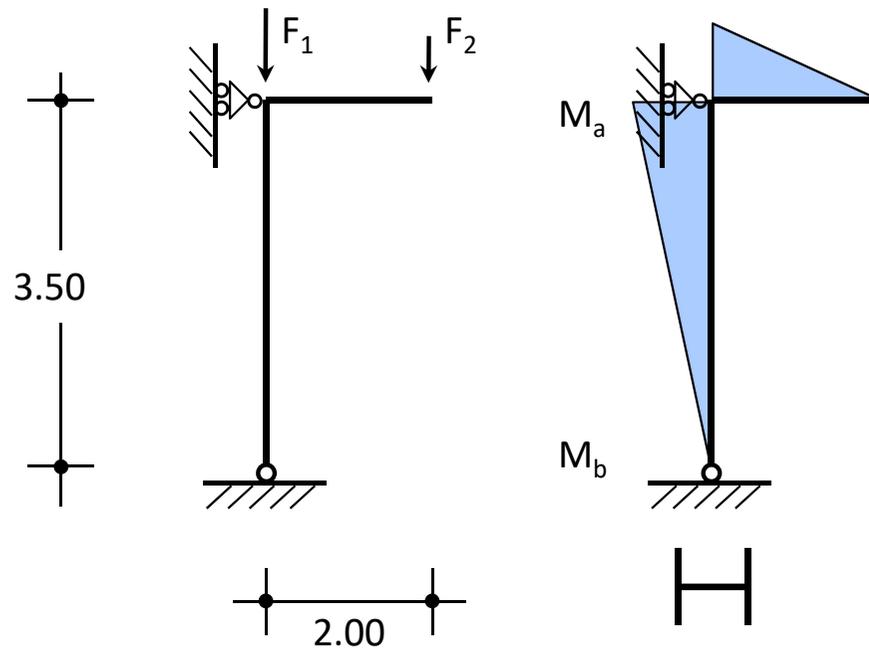
$$\bar{\lambda}_y = 0.492$$
$$\chi_y = 0.8477$$

$$k_{xx} \leq C_{mx} \left[ 1 + 0.8 \frac{N_{Ed} \gamma_{M1}}{\chi_x A f_y} \right] = 0.6 \left[ 1 + 0.8 \frac{860}{3231.2} \right] = 0.728$$

Ok, quindi  $k_{xx} = 0.614$

# Verifica di resistenza

## Esempio

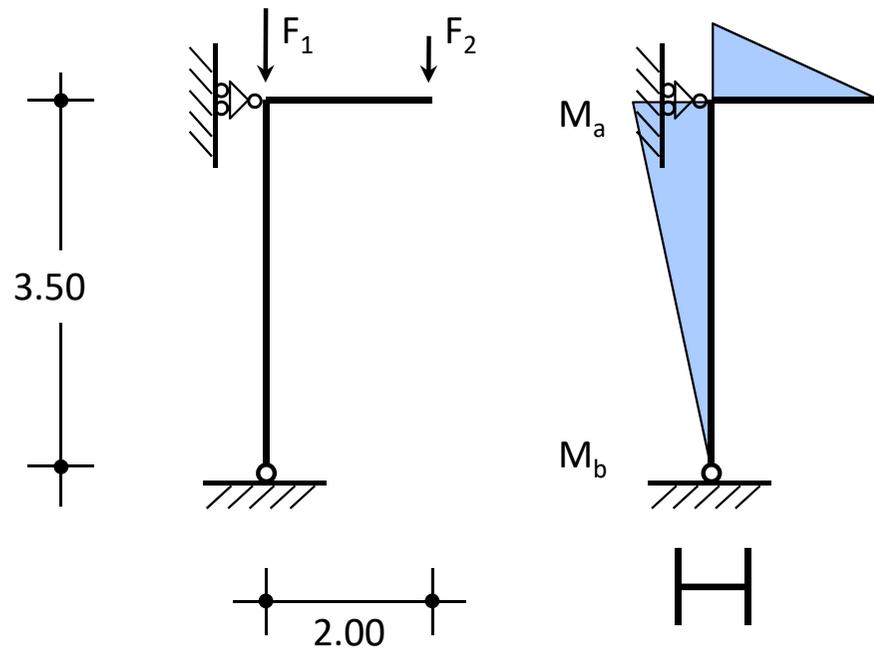


$$\frac{N_{Ed} \gamma_{M1}}{\chi_x A f_y} + \frac{k_{xx} M_{x,Ed} \gamma_{M1}}{W_{pl,x} f_y} = \frac{860}{3231.2} + \frac{0.614 \times 120}{418.1} = 0.266 + 0.176 = 0.442 < 1$$

Con il Metodo A era 0.480

# Verifica di resistenza

## Esempio



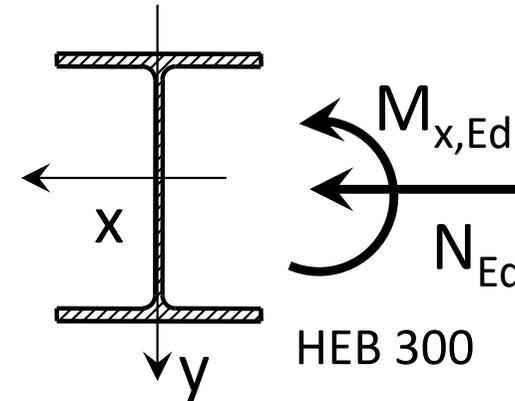
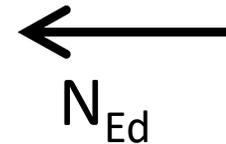
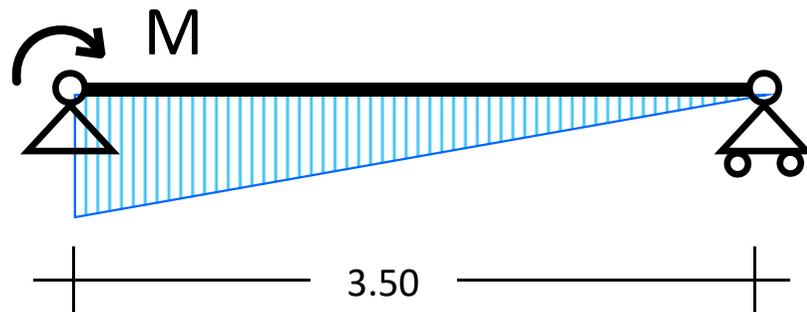
Perché  $k_{yx} = 0$

$$\frac{N_{Ed} \gamma_{M1}}{\chi_y A f_y} + \frac{k_{yx} M_{x,Ed} \gamma_{M1}}{W_{pl,x} f_y} = \frac{860}{2826.8} = 0.304$$

Verifica di stabilità con sforzo normale centrato

# Domini di resistenza

presso-flessione retta (Metodo A)

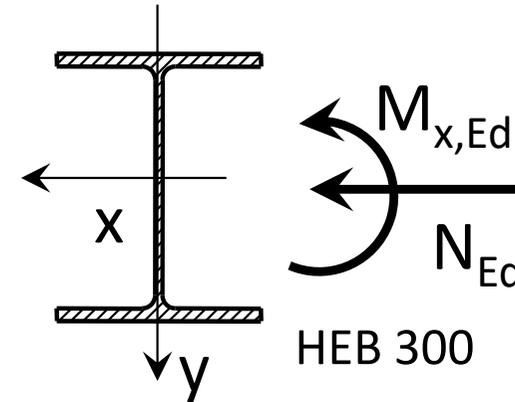
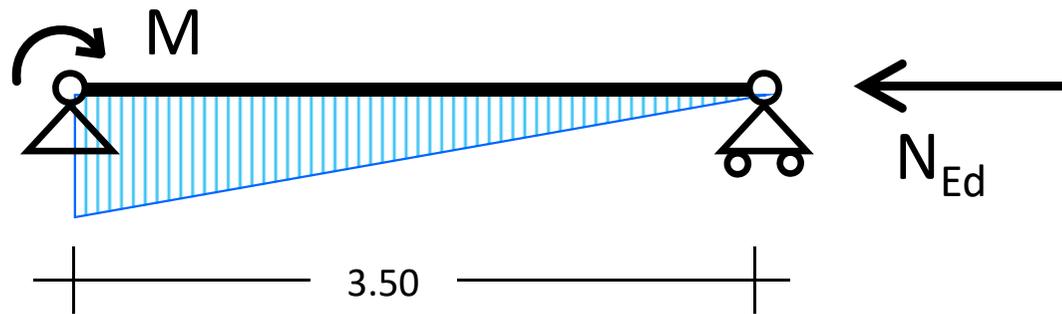


$$\frac{N_{Ed}}{N_{bRd}} + \frac{M_{x,eq,Ed}}{M_{x,Rd} \left( 1 - \frac{N_{Ed}}{N_{cr,x}} \right)} \leq 1$$

$$M_{x,eq,Ed} = 0.6 M_a - 0.4 M_b = M_a \underbrace{\left( 0.6 - 0.4 \frac{M_b}{M_a} \right)}_{C_{mx}}$$

# Domini di resistenza

presso-flessione retta (Metodo A)

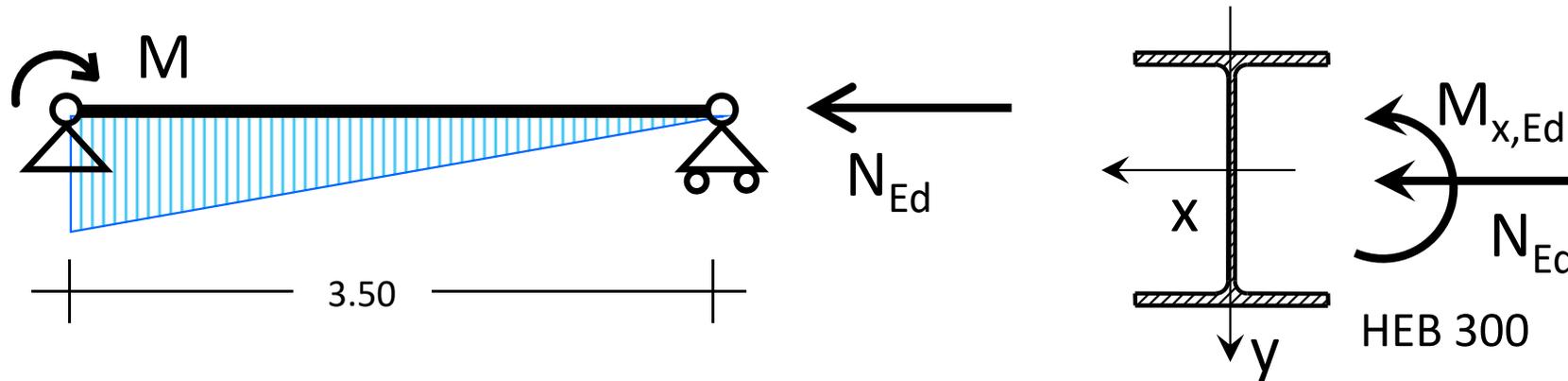


$$\frac{N_{Ed}}{N_{bRd}} + \frac{C_{mx} M_{x,Ed}}{M_{x,Rd} \left( 1 - \frac{N_{Ed}}{N_{cr,x}} \right)} = 1$$

$k_{xx}^*$

# Domini di resistenza

presso-flessione retta (Metodo A)



$$\frac{N_{Ed}}{N_{bRd}} + \frac{k_{xx}^* M_{Ed,x}}{M_{Rd,x}} \leq 1$$

$$k_{xx}^* = \frac{C_{mx}}{1 - \frac{N_{Ed}}{N_{cr,x}}}$$

$$M_{pl,x,Rd} = 418.1 \text{ kNm}$$

$$N_{bRd} = 2826.8 \text{ kN}$$

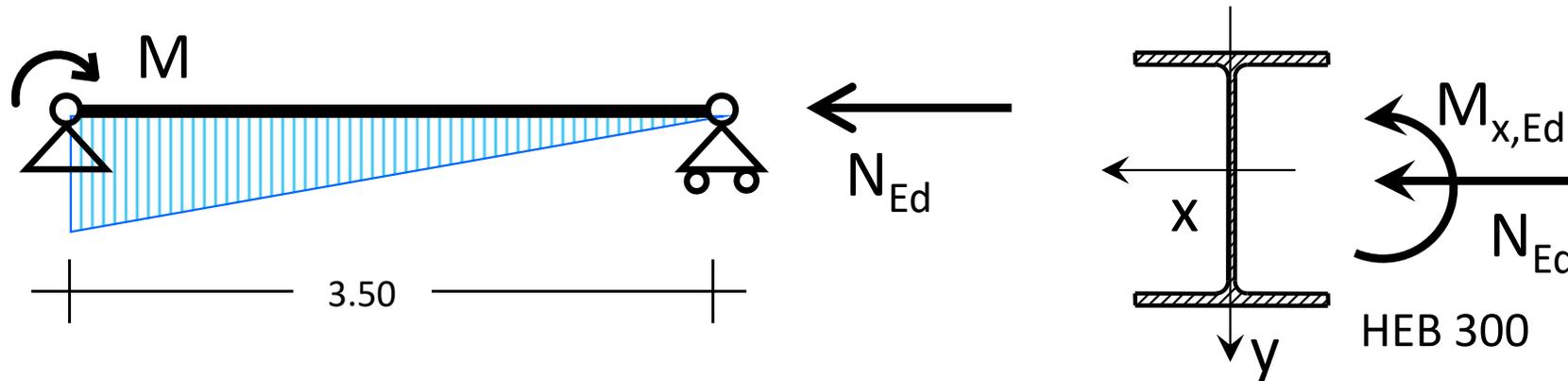
$$N_{cr,x} = 42579.2 \text{ kN}$$

$$\psi = M_B / M_a = 0$$

$$C_{mx} = (0.6 + 0.4 \psi) = 0.6$$

# Domini di resistenza

presso-flessione retta (Metodo A)



$$\frac{N_{Ed}}{N_{bRd}} + \frac{k_{xx}^* M_{Ed,x}}{M_{Rd,x}} \leq 1$$

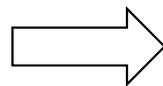
$$M_{pl,x,Rd} = 418.1 \text{ kNm}$$

$$N_{bRd} = 2826.8 \text{ kN}$$

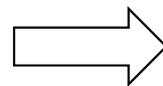
$$N_{cr,x} = 42579.2 \text{ kN}$$

Costruisco il dominio  
per punti

Fisso  $N_{Ed}$



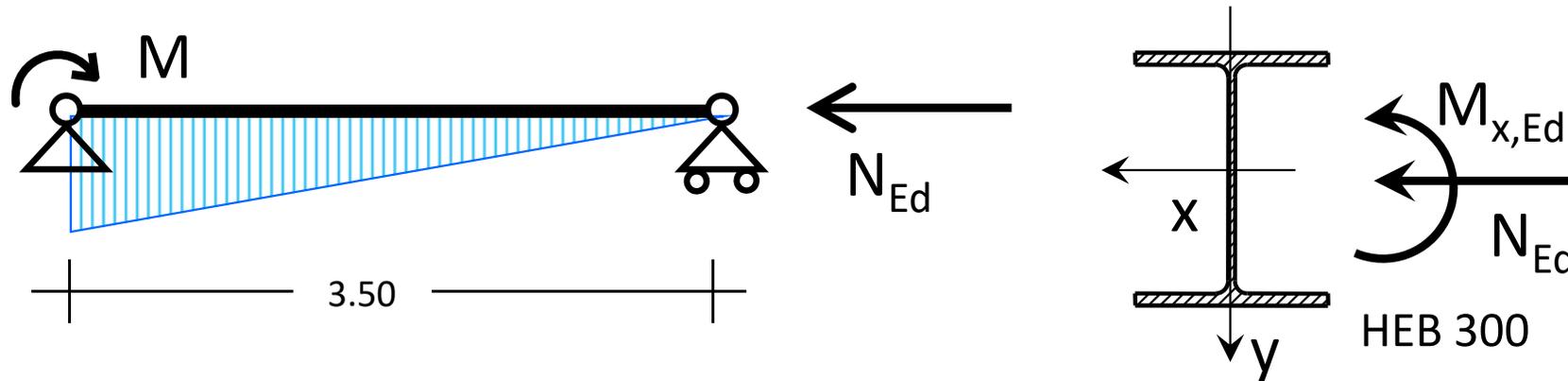
$k_{xx}^*$



$$M_{Ed,x} = \frac{M_{Rd,x}}{k_{xx}^*} \left( 1 - \frac{N_{Ed}}{N_{bRd}} \right)$$

# Domini di resistenza

presso-flessione retta (Metodo B)



$$\frac{N_{Ed}}{N_{bRd,x}} + \frac{k_{xx} M_{Ed,x}}{M_{Rd,x}} \leq 1$$

$$M_{pl,x,Rd} = 418.1 \text{ kNm}$$

$$N_{bRd,x} = 3231.2 \text{ kN}$$

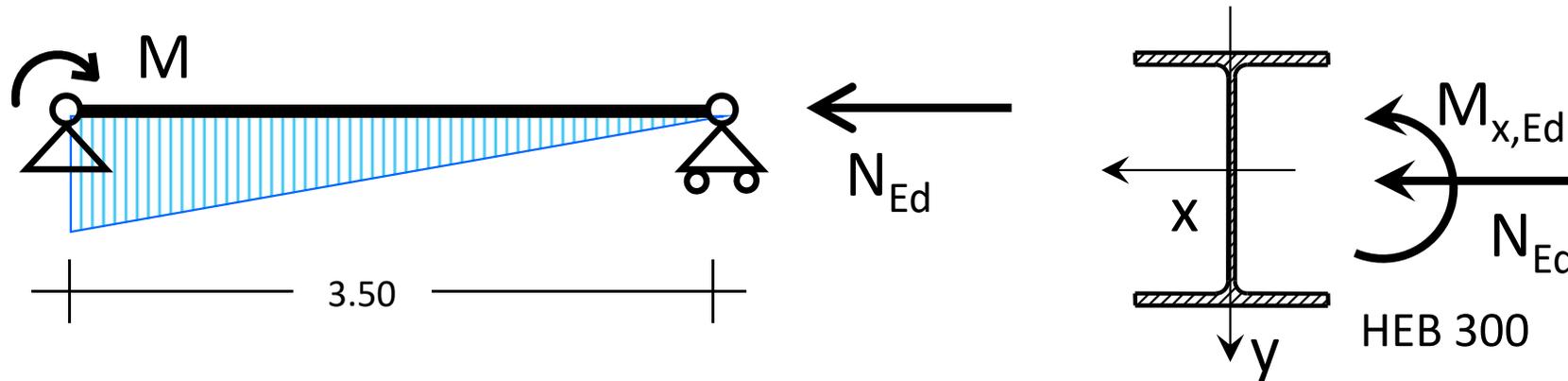
$$k_{xx} = \text{vedi EC3}$$

$$C_{mx} = (0.6 + 0.4 \psi) = 0.6$$

$$\psi = M_B / M_a = 0$$

# Domini di resistenza

presso-flessione retta (Metodo B)



$$\frac{N_{Ed}}{N_{bRd,x}} + \frac{k_{xx} M_{Ed,x}}{M_{Rd,x}} \leq 1$$

$$M_{pl,x,Rd} = 418.1 \text{ kNm}$$

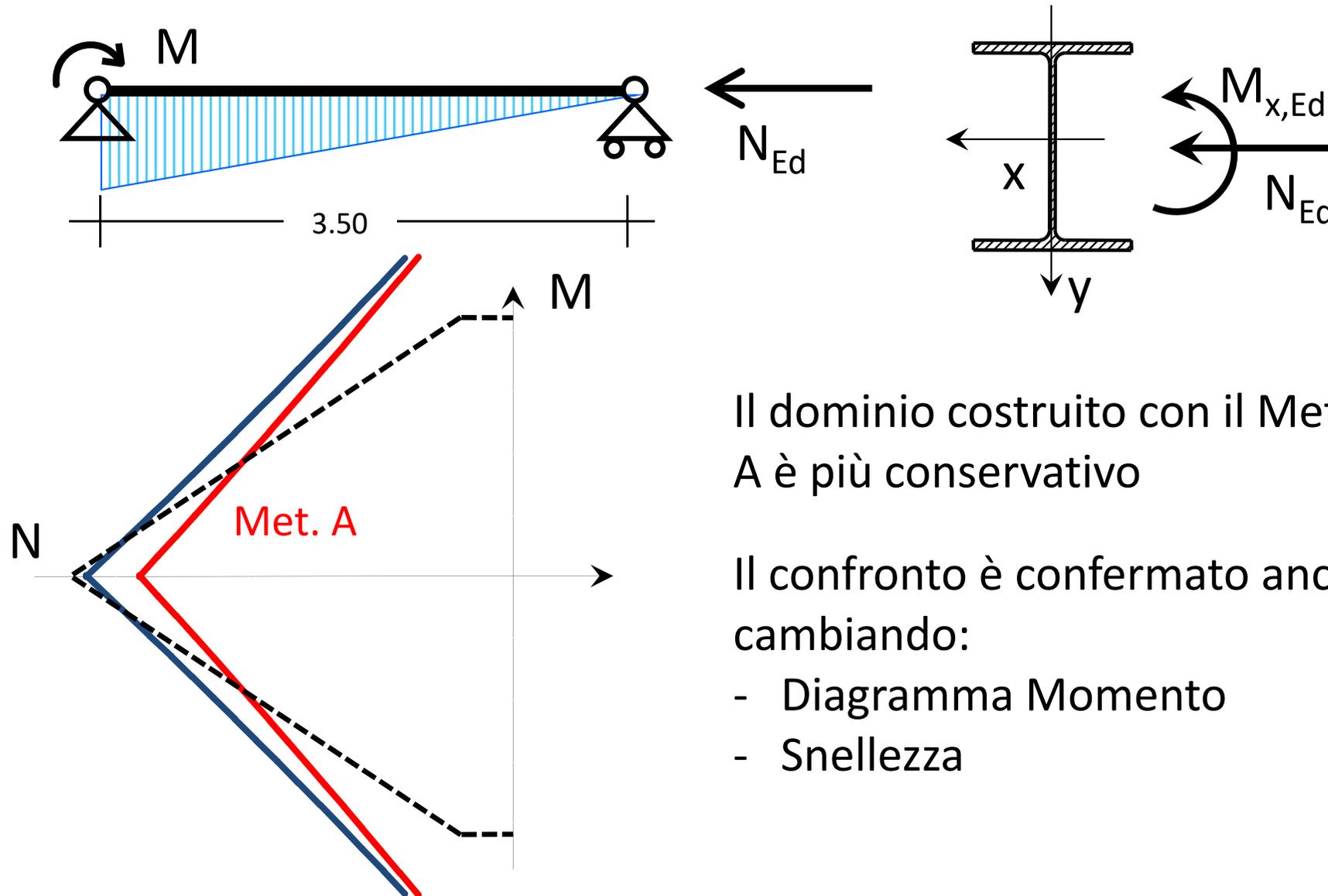
$$N_{bRd,x} = 3231.2 \text{ kN}$$

Costruisco il dominio per punti

Fisso  $N_{Ed}$   $\Rightarrow$   $k_{xx}$   $\Rightarrow$   $M_{Ed,x} = \frac{M_{Rd,x}}{k_{xx}} \left( 1 - \frac{N_{Ed}}{N_{bRd,x}} \right)$

# Domini di resistenza

presso-flessione retta (Metodo A vs Metodo B)



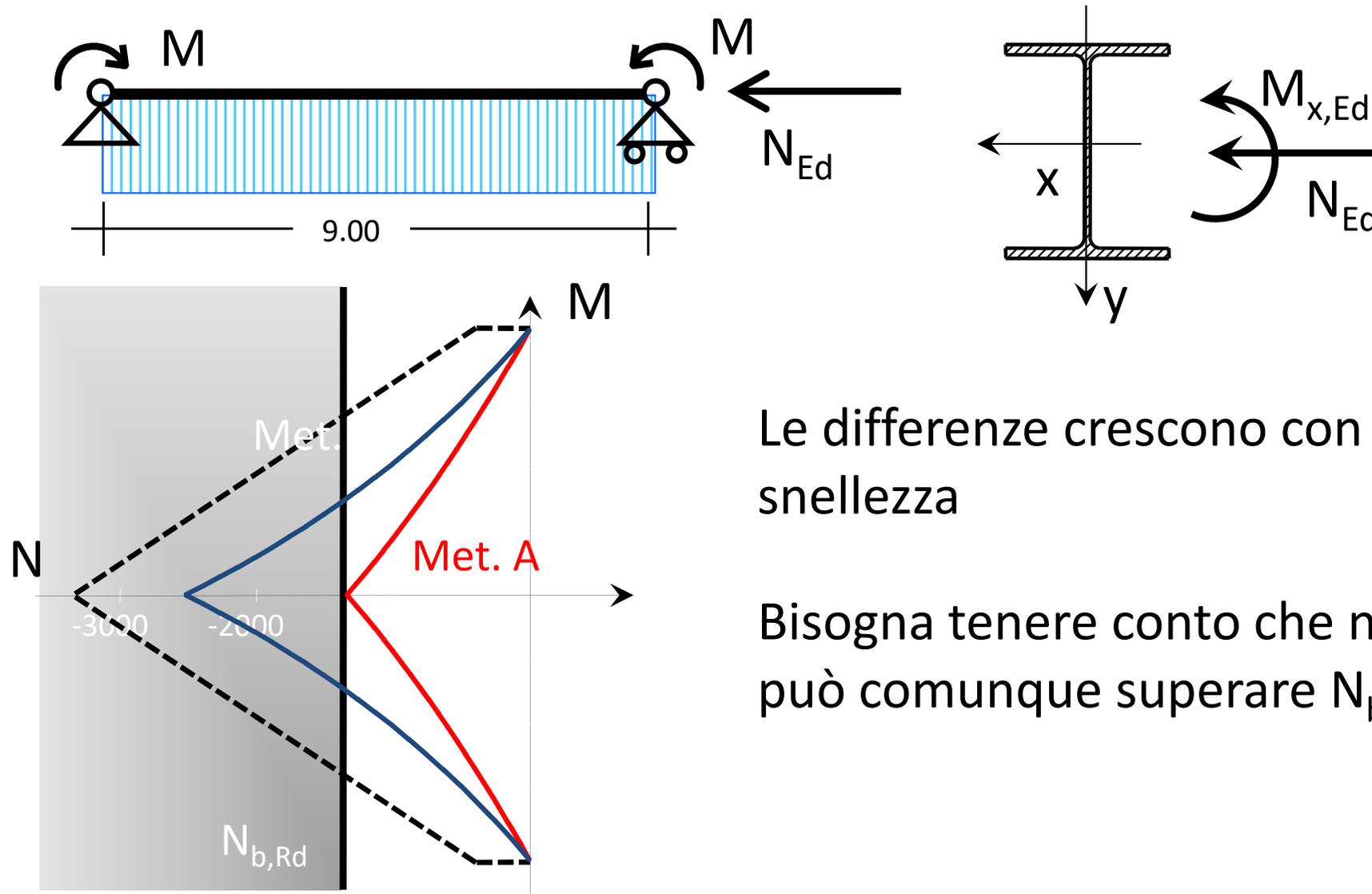
Il dominio costruito con il Metodo A è più conservativo

Il confronto è confermato anche cambiando:

- Diagramma Momento
- Snellezza

# Domini di resistenza

presso-flessione retta (Metodo A vs Metodo B)

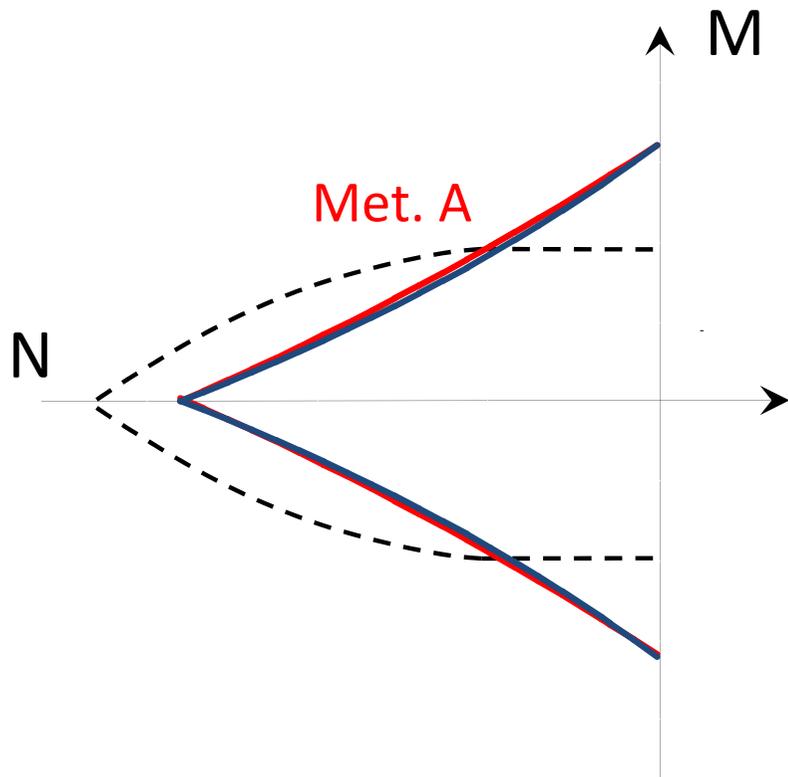
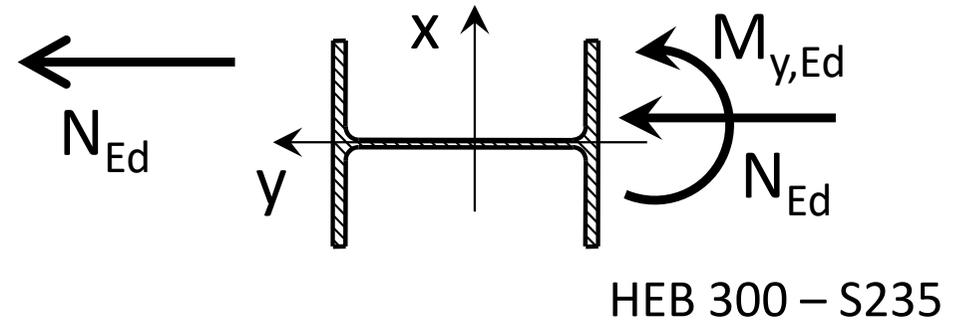
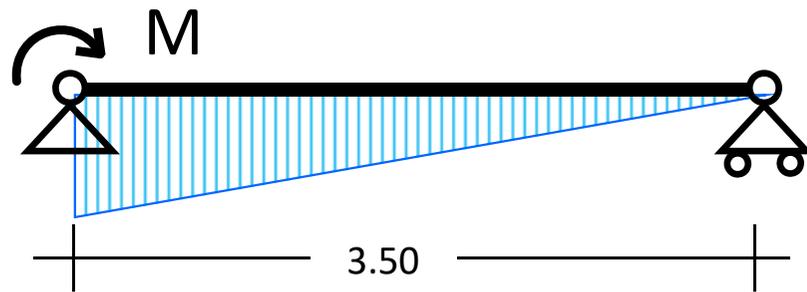


Le differenze crescono con la snellezza

Bisogna tenere conto che non si può comunque superare  $N_{b,Rd}$

# Domini di resistenza

presso-flessione retta (Metodo A vs Metodo B)

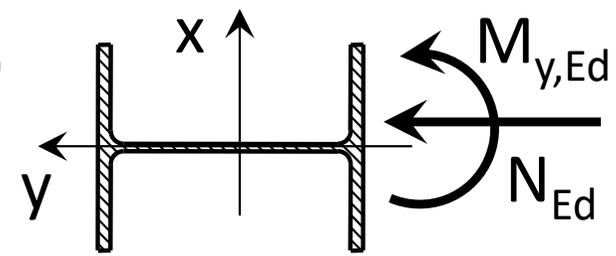
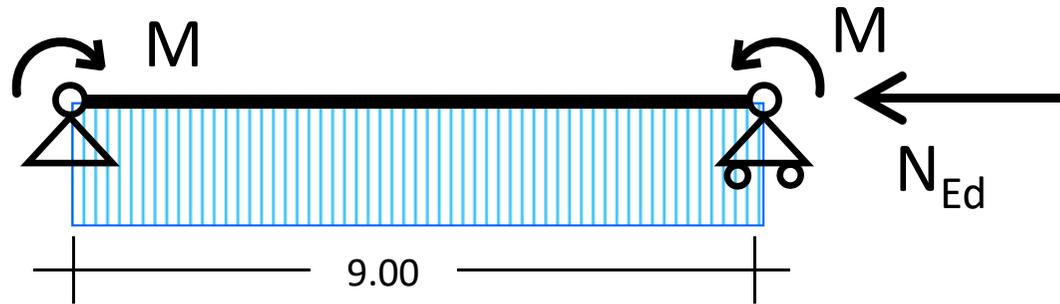


I domini si costruiscono allo stesso modo

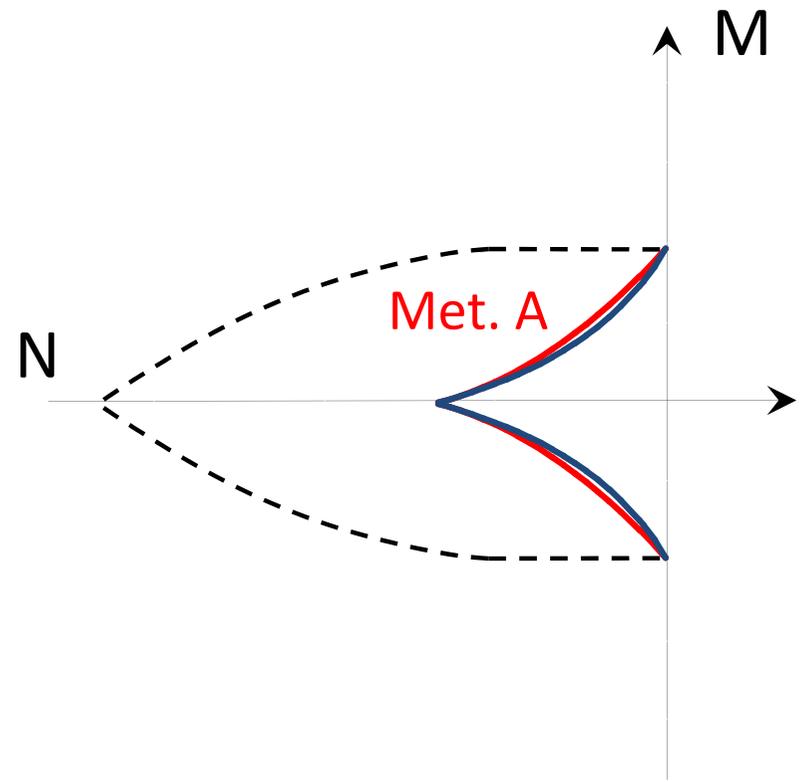
I due domini sono quasi coincidenti

# Domini di resistenza

presso-flessione retta (Metodo A vs Metodo B)



HEB 300 – S235



I due domini sono quasi coincidenti anche in altre condizioni.

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